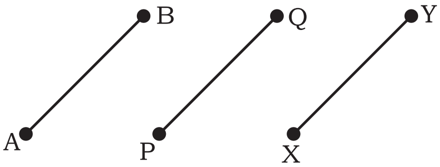
EXERCISE 5.1

1. Which of the following statements are true and which are false? Give reasons for your answers.

(i) Only one line can pass through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.



(iii) A terminated line can be produced indefinitely on both the sides.

(iv) If two circles are equal, then their radii are equal.

(v) In figure, if AB = PQ and PQ = XY, then AB = XY.

2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

(i) parallel lines (ii) perpendicular lines

(iii) line segment (iv) radius of a circle

(v) square

3. Consider two 'postulates' given below:

(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

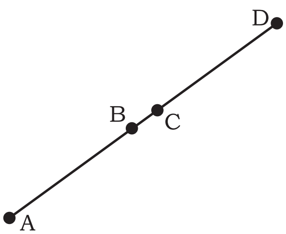
(ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent?

Do they follow from Euclid's postulates? Explain.

4. If a point C lies between two points A and B such that AC = BC, then prove that AC = ½AB. Explain by drawing the figure.

5. In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.



6. In figure, if AC = BD, then prove that AB = CD.

7. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the 5th postulate.)

TEST YOURSELF – EG 1

1. Give the answers of the following problems in a single word:
   (i) What is the least number of distinct points which determine a unique line?
   (ii) In how many points two distinct lines can intersect?
   (iii) In how many points two distinct planes can intersect?
   (iv) Does a ray have any length?
   (v) How many lines can be drawn through a given point?
   (vi) How many planes can be made to pass through a line and a point not on the line?
2. Which of the following statements are true?
   (i) A line segment has no definite length.
   (ii) Two lines always intersect at a point.
   (iii) A ray has only one end-point.
   (iv) The ray AB is the same as ray BA.
   (v) Two intersecting lines are always coplanar.
   (vi) Two lines are always parallel, if they do not intersect.
3. Three distinct points are given in a plane. How many lines can be drawn through them?
4. Four distinct points are given in a plane. How many lines can be drawn?
5. With reference to figure, state which statement is true and which is false:

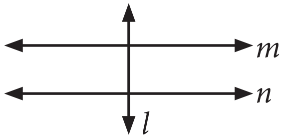
   (i) PQ + QR = PR
   (ii) PR + PS = PS
   (iii) Lines PQ and RS are coincident.
6. If Q lies between P and R and PR = 13, QR = 7, find $PQ^2 + QR$.

EXERCISE 5.2

1. How would you rewrite Euclid's fifth postulate so that it would be easier to understand?
2. Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Test Yourself – EG 2

1. Explain why do you think that Playfair's axiom is (or is not) equivalent to Euclid's fifth postulate.
2. Consider the statement: "There exists at least one triangle in which the sum of the measures of the interior angles is 180º." Do you think that this statement is (or is not) a consequence of Euclid's Fifth Postulate? Explain.



Test Yourself – EG 3

1. Fill in the blanks.
   (i) .... are the assumptions which are obvious universal truths.
   (ii) Things which are equal to the same thing are .... to one another.
   (iii) If equals are subtracted from equals, the remainders are ....
   (iv) Things which are double of the same things are .... to one another.
   (v) Two distinct intersecting lines cannot be .... to the same line.
   (vi) For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to ....
2. Match the following

(I)

Column I (Postulate number)		Column II (Statement)
(1) Postulate 1	(a)	A terminated line can be produced indefinitely
(2) Postulate 2	(b)	All right angles are equal to one another
(3) Postulate 3	(c)	A straight line may be drawn from any one point to any other point
(4) Postulate 4	(d)	A circle can be drawn with any centre and any radius.

(II)

Column I		Column II
(1) Only one line can pass through	(a)	One point
(2) Infinite number of lines can pass through	(b)	Two points
(3) Two distinct lines cannot have more than ..... in common	(c)	Same side
(4) For a line segment .... are required	(d)	Opposite side

3. State Euclid's five postulates.

4. Rewrite Euclid's fifth postulate in simple language.

5. Show that two lines which are parallel to the same line are parallel to each other.

6. If lines AB, AC, AD and AE are parallel to line *l*, then show that the points A, B, C, D, E are collinear.

7. Two points A and B are given. Find how many line segments do they determine.

8. Name the line segments determined by three collinear points A, B and C.

9. Differentiate between "Axioms", "Postulates" and "Theorems".

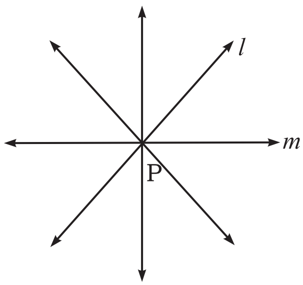
10. Draw supporting diagram for Euclid's five postulates.

ANSWERS

NCERT Exercises and Assignments

Exercise – 5.1

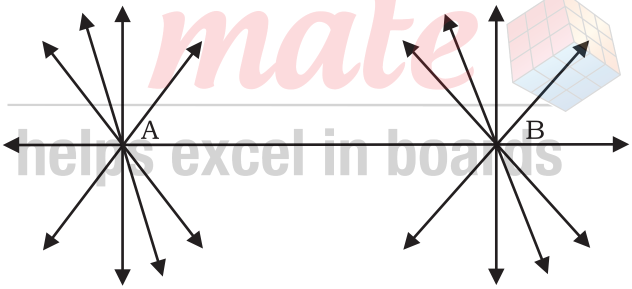
1. (i) False.



Mark a point P on the plane of paper. Using a sharp pencil and a ruler, draw a line *l* passing through it as shown in the figure. Draw another line *m* passing through P. Continuing this process, we can draw as many lines as we wish, each passing through point P. Thus, an infinite number of lines can be drawn passing through a given point.

(ii) False.

Mark two points A and B on the plane of paper. Fold the paper so that a crease passes through A. As explained in part (i) an unlimited number of creases (lines) can pass through A.



Again, fold the paper so that a crease passes through B. Clearly, an unlimited number of creases (lines) can pass through B. Now, fold the paper in such a way that a crease (line) passes through both A and B. We observe that there is just one crease (line) which passes through both A and B.

Thus, through any two points in a plane, exactly one line can be drawn.

(iii) True.

Note that what we call a line segment now a days is what Euclid called a terminated line.

In geometry, by a line, we mean the line in its totality and not a portion of it. A physical example of a perfect line is not possible.

Since a line extends indefinitely in both the directions. So, it cannot be drawn or shown wholly on paper. In practice, only a portion of a line is drawn and arrowheads are marked at its two ends indicating that it extends indefinitely in both directions as shown.

(iv) True.

On superimposing the region bounded by one circle on the other circle, if the circles coincide, then, their centres and boundaries coincide. Therefore, their radii will be equal.

(v) True.

Because things which are equal to the same thing are equal to one another.

2. For the desired definitions, we need the following terms:

(a) point | (b) line

(c) plane | (d) ray

(e) angle | (f) circle

(g) quadrilateral

It is not possible to define first three precisely. However, a good idea of these concepts shall be given.

(a) A small dot made by a sharp pencil on a sheet of paper gives an idea about a point. A point has no dimensions, it has only a position.

(b) A straight crease obtained by folding a paper, a straight string pulled at its two ends, the edge of a ruler are some close examples of a geometrical line. The basic concept about a line is that it should be straight and that it should extend indefinitely in both the directions.

(c) The surface of a smooth wall or the surface of a sheet of paper are close examples of a plane.

(d) A part of a line l, which has only one end point A and contains the point B is called a ray AB.

(e) An angle is the union of two non-collinear rays with a common initial point.

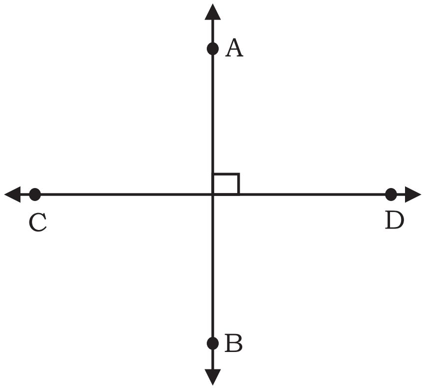
(f) A circle is the set of all those points in a plane whose distance from a fixed point remains constant. The fixed point is called the centre of the circle.

(g) A closed figure made up of four line segments is called a quadrilateral.

(i) **Parallel Lines:** Two lines are said to be parallel when (a) they are non-intersecting; (b) they are coplanar.

In figure, the two lines l_1 and l_2 are parallel.

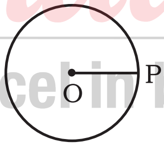
(ii) **Perpendicular Lines:** Two lines AB and CD lying in the same plane are said to be perpendicular, if they form a right angle. We write AB ⊥ CD.



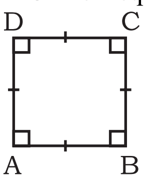
(iii) **Line segment:** A line segment is a part of a line. When two distinct points, say A and B on a line are given, then the part of this line with end-points A and B is called the line segment.

It is represented as $\overline{AB}$. AB and BA denote the same line segment.

(iv) **Radius:** The distance from the centre to a point on the circle is called the radius of the circle. OP is the radius.



(v) **Square:** A quadrilateral in which all the four angles are right angles and all the four sides are equal is called a square. ABCD is a square.



3. There are several undefined terms which the student should list. They are consistent, because they deal with two different situations-

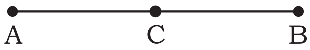
(i) Says that for the given two points A and B, there is a point C lying on the line in between them;

(ii) Says that given A and B, we can take a point C not lying on the line passing through A and B. These 'postulates' do not follow from Euclid's postulates. However, they follow from the axiom stated as given two distinct points, there is a unique line that passes through them.

4. We have a point C lying between two points A and B such that AC = BC.

Adding AC on both sides, we have

$$AC + AC = AC + BC$$

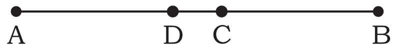


$\therefore$ $2AC = AB$

$\therefore$ $AC = \frac{1}{2}AB$ [$\because$ AC + CB coincides with AB]

5. If possible, let D be another mid-point of AB.

$\therefore$ $AD = DB$ .... (1)



But it is given that C is the mid-point of AB.

$\therefore$ $AC = CB$ .... (2)

Subtracting (1) from (2), we get

$$AC - AD = CB - DB \Rightarrow DC = -DC$$

$$2DC = 0 \Rightarrow DC = 0$$

$\therefore$ C and D coincides.

Thus every line segment has one and only one mid-point.

6. $AC = BD$ .... (1) (Given)

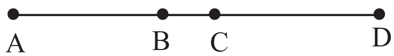
Also $AC = AB + BC$ .... (2) (Point B lies between A and C)

and $BD = BC + CD$ .... (3) (Point C lies between B and D)

Substituting for AC and BD from (2) and (3) in (1), we get

$$AB + BC = BC + CD$$

$\therefore$ $AB + BC - BC = BC + CD - BC$



$\therefore$ $AB = CD$

7. Axiom 5 in the list of Euclid's axioms, is true for any thing in any part of the universe so it is considered as a universal truth.

TEST YOURSELF – EG 1

1. (i) Two (ii) One
(iii) Infinite (iv) No
(v) Infinite (vi) Only one

2. True Statements are :

(iii) A ray has only one end-point.

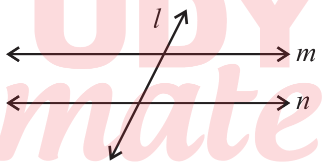
(v) Two intersecting lines are always coplanar.

(vi) Two lines are always parallel, if they do not intersect.

3. One, if they are collinear and three if they are non- collinear.
4. One, if they are collinear, four if three of them are collinear and six, if no three of them are collinear.
5. (i) True (ii) False (iii) False
6. 43

Exercise – 5.2

1. The axiom asserts two facts:
   (i) There is a line through P which is parallel to *l* and
   (ii) there is only one such line.
2. If a straight line *l* falls on two straight lines *m* and *n* such that the sum of the interior angles on one side of *l* is two right angles, then by Euclid's fifth postulate the lines will not meet on this side of *l*. Next, we know that the sum of the interior angles on the other side of line *l* will also be two right angles. Therefore, they will not meet on the other side also. So, the lines m and n never meet and are, therefore, parallel.



TEST YOURSELF – EG 3

1. (i) Axioms (ii) equal
   (iii) equal (iv) equal
   (v) parallel (vi) *l*
2. I. (1 – c), (2 – a), (3 – d), (4 – b) II. (1 – b), (2 – a), (3 – a), (4 – b)
7. One
8. AB, BC and AC