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Sample Paper

(2018-19)

Date : ___________	Physics	Class
Duration : 3 Hrs.		XI
Max. Marks : 70		

Instructions:

- *All questions are compulsory. There are 27 questions in all*
- *This question paper has **four** sections : Section **A**, Section **B**, Section **C**, and Section **D**.*
- *Section **A** contains **five** questions of **one** mark each, Section **B** contains **seven** questions of **two** marks each, Section **C** contains **twelve** questions of **three** marks each and Section **D** contains **three** questions of **five** marks each.*
- *There is no overall choice. However, an internal choice has been provided in **two** questions of **one** marks, **two** questions of **two** marks, **four** questions of **three** marks and **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.*

Section - A

1. Arrange the four fundamental forces of nature in their increasing strength.

2. What happens to coefficient of friction when weight of body is doubled?

3. How the gravitational field at a point inside a solid sphere vary with distance x from its centre, if $x < R$ and $x > R$ (R-radius).

4. The momentum of a body is increased by 50%. What is the percentage change in its K.E.?

OR

What are the important conditions for total internal reflection?

5. What will be the effect on the angle of contact of a liquid, if the temperature increases?

OR

Two boats runing parallel to each other side by side have a tendency to collide with each other why?

Section - B

6. A transverse harmonic wave on a string is described by $y(x, t) = 3 \sin (36\,t + 0.018\,x + \pi/4)$. Where x and y are in cm and t in sec.

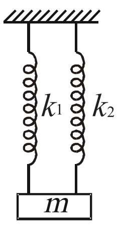
(a) Is this a travelling or a stationary wave? If it is travelling, what is the speed and direction of its propagation?

(b) What is its amplitude and frequency?

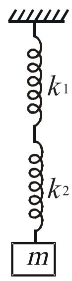
OR

Find the frequency of oscillation in the following arrangement of springs:

(a)



(b)



7. Show that the pressure exerted by a gas is two-third of the average kinetic energy per unit volume of the gas molecules.

8. The orbital velocity v of a satellite depends on its mass m, distance r from the centre of Earth and acceleration due to gravity g. Obtain an expression for orbital velocity using dimensional analysis method.

OR

A physical quantity Q is given by

$$Q = \frac{A^2 . B^{3/2}}{C^4 . D^{1/2}}$$

The percentage error in A, B, C, D are 1%, 2%, 4%, 2%, respectively. Find the percentage error in Q.

9. Explain why

(a) Passengers are thrown forward from their seats when a speeding bus stops suddenly,

(b) It is easier to pull a lawn mower than to push it?

(c) A cricketer moves his hands backwards when holding ?

10. A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C. The temperature of the surrounding is 20°C.

11. n moles of a gass at temperature T expands from volume (V_1) to volume (V_2) isothermally. Obtain the expression for work done.

12. A man can swim with a speed of 4 kmh^{-1} in still water. How long does he take to cross the river 1 km wide, if the river flows steadily at 3 kmh^{-1} and he makes his strokes normal to the river current? Find the distance through which the man goes down the river.

Section - C

13. (a) What are the necessary conditions for an SHM.

(b) A 50 cm long wire of mass 20 g supports a mass of 160 kg. Find the fundamental frequency of the portion of the string between the wall and the pulley. Take g = 10 m/s^2.

14. Obtain an expression for kinetic energy of rotation of a body. Hence define moment of inertia of the body.

OR

A convex lens of focal length 20 cm produced a real image of an object when it is 30 cm away. Find position and magnification of image.

15. Define three coefficients of thermal expansion. Establish a relation between them.

16. A mass m is tied to a string of length l and is rotated in a vertical circle with centre at the other end of the string.

(a) Find the minimum velocity of the mass at the top of the circle so that it is able to complete the circle.

(b) Find the minimum velocity at the bottom of the circle corresponding to the above condition.

OR

Define elastic and inelastic collisions. Write their basic characteristics. A bullet is fired into a block of wood. If gets totally embedded in it and the system moves together as one entity, then state what happens to the initial kinetic energy and linear momentum of the bullet?

17. List the steps and write the work done in them in a Carnot's cycle. Also find the efficiency of the Carnot Engine. **[3]**

OR

(a) At what temperature will the average velocity of oxygen molecules be sufficient so as to escape from the earth? [Given: Escape velocity from the earth is 11.0 km/sec and the mass of one molecule of oxygen is 5.34×10^{-26} kg.] **[2]**

(b) State the assumptions of kinetic theory of gases. **[1]**

18. Find an expression for the pressure exerted by a gas using the kinetic theory of gases.

19. In an experiment, refractive index of glass was measured as 1.45, 1.56, 1.54, 1.44, 1.54 and 1.53. Calculate

(i) mean value of refractive index (ii) mean absolute error

(iii) fractional error (iv) percentage error

20. What do you understand by escape velocity? Derive an expression for the escape velocity.

OR

Explain with the help of lens maker's formula. Why does a convex lens behave as:

(a) Converging when immersed in water (μ = 1.33) and

(b) A diverging lens when immersed in CS_2 solution (μ = 1.6).

21. Read each statement below carefully and state with reasons and examples if it is true or false:

A particle in one-dimensional motion

(a) with zero speed at an instant may have non-zero acceleration at that instant

(b) with zero speed may have non-zero velocity

(c) with constant speed must have zero acceleration

(d) with positive value of acceleration must be speeding up

22. A monkey of mass 40 kg climbes on a rope which can stands a maximum tension of 600 N. In which of the following cases will the rope break : the monkey

(a) climbs up with an acceleration of 6 ms^{-2}. (b) climbs down with an acceleration of 4 ms^{-2}.

(c) climbs up with constant velocity of 5 ms^{-1}.

(d) falls down the rope nearly freely under gravity?

Take g = 10 ms^{-2}. Ignore the mass of the rope.

23. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine. Specific heat of aluminium = 0.91J/g/°C.

24. State carefully if the work done in following cases are positive or negative:

(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) Work done by gravitational force in the above case.

(c) Work done by friction on a body sliding down an inclined place.

(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

(e) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Section - D

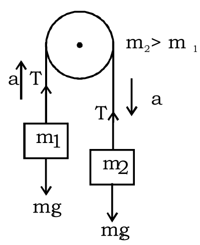
25. What do you mean by Banking of roads. What is the need of banking a circular road. Discuss the motion of car on banked circular road (with friction).

A vehicle running with speed 72 km/hr applies brakes producing a retardation of 3 m/s^2 while taking a turn of radius 100 m. Find the acceleration.

OR

(a) Distinguish between static friction, limiting friction and kinetic friction. How do they vary with the applied force, explain by diagram.

(b) Two objects of masses m_1 and m_2 are tied at the two ends of a light inextensible string passing over a frictionless pulley. Find expressions for the acceleration of the system and the tension in the string.



26. (a) Derive an expression for the rise of liquid in capillary tube of uniform diameter and sufficient length.

(b) A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ.

OR

State and prove Bernoulli's theorem in a flowing ideal liquid. State the assumptions used.

27. (a) State Newton's formula for the velocity of sound in air. Discuss Laplace's correction.

(b) A steel wire 70 cm long has a mass of 7 g. If the wire is under a tension of 100 N, what is the speed of transverse waves in the wire?

OR

(a) Write the differences between traveling waves and standing waves.

(b) A train, standing at the outer signal of railway station blows a whistle of frequency 400 Hz in still air. What is the frequency of the whistle for a platform observer when the train

(i) approaches the platform observer with a speed of 10 m/s

(ii) recedes from the platform with a speed of 10 m/s?

(iii) What is the speed of sound in each case?

[Speed of sound in still air = 340 m/s]

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Hints/Solution to Sample Paper

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1. Gravitational force < weak nuclear forces < electromagnetic forces < strong nuclear forces.

2. Coefficient of friction does not change as it depends on the nature of surface and not on the weight of the body.

3. $E_g \propto x$ if $x < R$ and $E_g \propto \frac{1}{x^2}$ if $x > R$

4. 125%

OR

(a) Light should travel from denser to rarer medium.

(b) Angle of incidence should be greater than critical angle.

5. Surface tension of liquid decreases with increase in temperature. So, the curvature of the meniscus decreases and the angle of contact increases.

OR

According to Bernoulli's principle, the pressure between the boats is low as the velocity between the boats is high. Because of the pressure difference the boats collide.

6. (a) It is a travelling wave

direction → opposite to the x-axis.

(b) $\omega = 2\pi f = 36$

$f = 36/2\pi = 5.73$ Hz

Amplitude = 3 cm

OR

(a) Restoring force = $-k_1x - k_2x$; $k_{eq} = k_1 + k_2$

$$\therefore \; f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

(b) Here, extensions are different. Total extension = $x = x_1 + x_2$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \quad f = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{m(k_1 + k_2)}}$$

7. Pressure, $P = 1/3\, \rho v^2$

Multiplying and dividing by 2 we get,

$$P = \frac{2}{3} \times \frac{1}{2}\rho v^2 = \frac{2}{3} \text{ (Average KE per unit volume).}$$

Hence, the pressure exerted by a gas is equal to two-thirds of average kinetic energy of translation per unit volume of the gas

8. Let orbital velocity of satellite be given by the relation

$v = km^a r^b g^c$ where k is a dimensionless constant and a, b and c are the unknown powers. Writing dimensions on two sides of equation, we have

$$[M^0L^1T^{-1}] = [M]^a\,[L]^b\,[LT^{-2}]^c = [M^aL^{b+c}] = [M^a\,L^{b+c}\,T^{-2c}]$$

Applying principle of homogeneity of dimensional equation, we find that

$a = 0$ ...(*i*)

$b + c = 1$ ...(*ii*)

$-2c = -1$ ...(*iii*)

On solving these equations, we find that

$a = 0$, $b = +\frac{1}{2}$ and $c = +\frac{1}{2}$

$\therefore \quad v = kr^{\frac{1}{2}} g^{\frac{1}{2}}$

or $\quad v = k\sqrt{rg}$.

OR

% error in Q $= 2\left(\frac{dA}{A}\times 100\right)+\frac{3}{2}\left(\frac{dB}{B}\times 100\right)+4\left(\frac{dC}{C}\times 100\right)+\frac{1}{2}\left(\frac{dD}{D}\times 100\right)$

$= 2\times 1+\frac{3}{2}\times 2+4\times 4+\frac{1}{2}\times 2$ = 2 + 3 + 16 + 1 = 22%

9. (a) If the speeding bus stops suddenly, the lower part of the passengers, in contact with bus, would be at rest. However, upper part of the body would still be in motion. Therefore, passengers would move forward. This occurs due to inertia of motion.

(b) When a lawn mower is pulled, the force acts along the handle, which has two components. The upwardly directed vertical component reduced the weight of mower, however, horizontal component of the force helps to push forward the mower. Whereas, when a lawn mower is pushed, force is applied downwards and thereby, vertical component, directed downward, increased the weight of mower, which creates difficulties in pushing the mower. Therefore, pushing of mower is difficult than pulling the same.

(c) When a cricketer holds a catch, the impulse received at hand = $F \times t$ = change in linear momentum of the ball = constant. By moving the hand backward, the cricketer increases the time of impact and reduces the force, so as the reaction, thereby reduces the chances of hurting severely.

10. From Newton's law of cooling,

$\frac{dT}{dt} = -K(T - T_0)$, where T and T_0 are the temperatures of the body and the surrounding, respectively.

If the temperature of the body decreases from T_1 to T_2 in time t,

$$\int_{T_1}^{T_2} \frac{dT}{T - T_0} = -\int_0^t K dt$$

$\Rightarrow \quad \log(T - T_0)\Big|_{T_1}^{T_2} = -Kt$

$\Rightarrow \quad \log_e \frac{T_2 - T_0}{T_1 - T_0} = -Kt$

$\Rightarrow \quad 2.303 \log_{10} \frac{T_2 - T_0}{T_1 - T_0} = -Kt$

$\Rightarrow \quad 2.303 \log_{10} \frac{T_1 - T_0}{T_2 - T_0} = Kt$

$\Rightarrow \quad t = \frac{2.303}{K} \log_{10} \frac{T_1 - T_0}{T_2 - T_0}$

Here, T_1 = 80°C, T_1 = 50°C, T_0 = 20°C; t = 5 min = 5 × 60 = 300 sec

$\therefore \quad 5\times 60 = \frac{2.303}{\text{K}} \log_{10} \frac{80-20}{50-20} = \frac{2.303}{\text{K}} \log_{10}(2)$ ... (1)

Also, if T_1 = 60°C, T_2 = 30°C, T_0 = 20°C, t = ?

$t = \frac{2.303}{K} \log_{10} \frac{60-20}{30-20} = \frac{2.303}{K} \log_{10}(4)$ ... (2)

Dividing (2) by (1), $\frac{t}{5\times 60} = \frac{\log_{10}(4)}{\log_{10}(2)} = \frac{0.6012}{0.3010} = 2$

t = 5 × 60 × 2 = 10 × 60s = 10 mins

11. n = number of moles undergoing isothermal process

P_1, V_1, T = initial thermodynamic variables

P_2, V_2, T = final thermodynamic variables

At any instant during expansion, let the pressure of the gas be P.

During small displacement dy of the piston

Work done dW = PAdy = PdV

Total work done by the gas in expansion from initial volume V_1 to final volume V_2 is W = $\int_{V_1}^{V_2} P(dV)$

From the standard gas equation,

PV = nRT

or P = nRT/V

$\therefore$ $W = n\int_{V_1}^{V_2} \frac{RT}{V}\, dV$

$= nRT \int_{V_1}^{V_2} \frac{1}{V}\, dV$

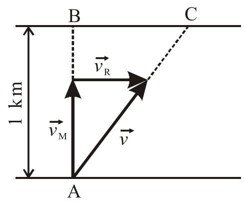
$W = nRT\left[\log_e V\right]_{V_1}^{V_2}$

$W = nRT\ [\log_e V_2 - \log_e V_1]$

$= nRT \log_e\left(\frac{V_2}{V_1}\right)$ ...(1)

$\boxed{W = 2.3026\, nRT \log_{10} \frac{V_2}{V_1}}$ ...(2)

12. In figure, $\vec{v}_M$ and $\vec{v}_R$ represent the velocities of man and river. Clearly $\vec{v}$ is the resultant of these velocities. If the man begins to swim along AB, he will be deflected to the path AC by the flowing river.



Time taken to cover distance AC with velocity $\vec{v}$ will be same as the time taken to cover distance AB with velocity $\vec{v}_M$.

$\therefore$ Time taken by the man to cross the river is

$t = \frac{AB}{v_M} = \frac{1\ km}{4\ km\ h^{-1}} = \frac{1}{4} h = 15 \text{ min}.$

Distance through which the man goes down the river is

$BC = v_R \times t = 3\ kmh^{-1} \times \frac{1}{4} h = 0.75$

13. (*a*) Restoring force must be directly proportional to the displacement and it should be directed towards the mean position.

(*b*) Here, mass/length m = $\frac{20 \times 10^{-3}}{50/100}$ = 0.04 kg/m

T = 160 kg = 160 × 10 N = 1600 N

Length that vibrates, L = 50 – 10 = 40 cm = 0.4 m

$\therefore$ $\upsilon = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.4}\sqrt{\frac{1600}{0.04}}$ = 250 Hz

14. $v_1 = r_1\omega,\ v_2 = r_2\omega$

K.E. of m_1 = $\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1(r_1\omega)^2 = \frac{1}{2}m_1 r_1^2\omega^2$

$\therefore$ K.E. of m_2 = $\frac{1}{2}m_2 r_2^2\omega^2$ and so on.

$\therefore$ Total K.E. of rotation = $\frac{1}{2}m_1 r_1^2\omega^2 + \frac{1}{2}m_2 r_2^2\omega + ...$

$\frac{1}{2}\omega^2(m_1 r_1^2 + m_2 r_2^2 + ...)$ $\Rightarrow$ $\frac{1}{2}\omega^2\left(\sum_{i=1}^{n} m_i r_i^2\right) = \frac{1}{2}\omega^2 I$

where $I = \sum_{i=1}^{n} m_i r_i^2$ = Moment of inertia of the body about the given axis of rotation.

If ω = 1, then K.E. of rotation = $\frac{1}{2}\ I \times \omega^2 = \frac{I}{2}$

$\therefore$ I = 2 × K.E. of rotation

Moment of inertia of a body about a given axis is equal to twice the

K.E. of rotation of the body rotating with unit angular velocity about the given axis.

OR

Given,

f = 20 cm

u = –30 cm

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{20} = \frac{1}{v} - \left(-\frac{1}{30}\right)$

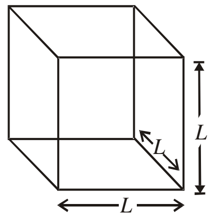
$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30}$$

$v = 60$ cm

$$m = -\frac{v}{u} = \frac{-60}{-30}$$

$m = 2$

15. Three coefficients of thermal expansion are



(i) Coefficient of linear expansion $(\alpha) = \dfrac{\text{increase in length}}{\text{Initial length} \times \text{increase in temperature}}$

(ii) coefficient of superficial expansion $(\beta) = \dfrac{\text{Increase in surface area}}{\text{Initial surface area} \times \text{increase in temperature}}$

(iii) Coefficient of cubical/volume expansion $(\gamma) = \dfrac{\text{Increase in volume}}{\text{original volume} \times \text{increase in temperature}}$

Consider a cube of side L

$\therefore$ $S = L^2$ and $V = L^3$

Let ΔT be the temperature increase of the cube when heated, ΔL be the corresponding increase in length, ΔS be the increase in surface area and ΔV be the increase in volume.

$\therefore$ $\Delta L = L\alpha\Delta T$, $\Delta S = S\beta\Delta T$, $\Delta V = V\gamma\Delta T$

New side of the cube = $L + \Delta L$

$\therefore$ New surface area = $S + \Delta S = (L + \Delta L)^2 \Rightarrow S + \beta S\Delta T = (L + \alpha L\Delta T)^2 = L^2 (1 + \alpha\Delta T)^2$

$\Rightarrow$ $S(1 + \beta\Delta T) = L^2 (1 + 2\alpha\Delta T + \alpha^2\Delta T^2)$ [$\because$ $S = L^2$ and neglecting $\alpha^2\Delta T^2$ as it is small]

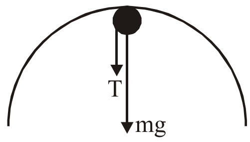
$\Rightarrow$ $1 + \beta\Delta T = 1 + 2\alpha\Delta T \Rightarrow \beta = 2\alpha$

New volume = $V + \Delta V = (L + \Delta L)^3 = (L + L\alpha\Delta T)^3$

$\Rightarrow$ $V + V\gamma\Delta T = L^3 (1 + \alpha\Delta T)^3 \Rightarrow V(1 + \gamma\Delta T) = L^3(1 + \alpha\Delta T)^3$ [$\because$ $V = L^3$ and neglecting $\alpha^2\Delta T^2$ and $\alpha^3\Delta T^3$ as they are small]

$\Rightarrow$ $1 + \gamma\Delta T = 1 + 3\alpha\Delta T \Rightarrow \boxed{\gamma = 3\alpha}$

16. At all positions, there are two forces acting on the mass :



its own weight and the tension in the string.

Here, radius of the circle = l

(a) At the top: Let v_t = velocity at the top

Net force towards centre = $\dfrac{mv_t^2}{l}$ (centripital force)

$$T + mg = \frac{mv_t^2}{l} \Rightarrow T = \frac{mv_t^2}{l} - mg$$

For the movement in the circular path, the string should remain tight i.e., the tension must be positive at all positions.

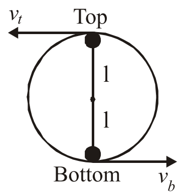
As the tension is minimum at the top, $T_{top} \geq 0$

$\Rightarrow$ $\dfrac{mv_t^2}{l} - mg \geq 0$

$\Rightarrow$ $v_t \geq \sqrt{lg}$

$\Rightarrow$ Minimum or critical velocity at the top = $\sqrt{lg}$

(b) Let v_b be the velocity at the bottom. As the particle goes up, its K.E. decreases and GPE increases.



$\Rightarrow$ loss in K.E. = gain in GPE

$\Rightarrow$ $\dfrac{1}{2}mv_b^2 - \dfrac{1}{2}mv_t^2 = mg(2l)$

$v_b^2 = v_t^2 + 4gl$

$(v_b)_{min} = \sqrt{(v_t^2)_{min} + 4gl} = \sqrt{5gl}$

Note: When a particle moves in a vertical circle, its speed decreases as it goes up and its speed increases as it comes down. Hence it is an example of non-uniform circular motion.

OR

Elastic collision. A collision in which there is absolutely no loss of kinetic energy is called elastic collision

Characteristics

(*i*) The linear momentum is conserved

(*ii*) Total energy of system is conserved.

(*iii*) Kinetic energy is conserved.

(*iv*) Forces involved during elastic collision must be conservative forces.

Inelastic collision. A collision in which there occurs some loss of kinetic energy is called inelastic collision.

Characteristics

(*i*) Linear momentum is conserved.

(*ii*) Total energy is conserved.

(*iii*) K.E. is not conserved.

(*iv*) Some or all forces involved may be non-conservative.

Consider, two particles A and B of masses m_1 and m_2 moving with initial velocities u_1 and u_2 along x- axis . They collide and move as one entity. Let V be the common velocity. When they move as single mass.

m_1= Mass of bullet

u_1 = Initial velocity of bullet

m_2= Mass of wood

u_2 = Initial velocity of wood = 0

According to conservation of momentum

$m_1u_1 + m_2u_2 = (m_1 + m_2)V$

$u_2 = 0$

$$v = \frac{m_1 u_1}{m_1 + m_2} \quad \text{... (i)}$$

Initial K.E., $K_i = \frac{1}{2} m_1 u_1^2$

Final K.E., $K_f = \frac{1}{2}(m_1 + m_2)v^2$

$$\frac{K_f}{K_i} = \frac{\left(\frac{1}{2}\right)(m_1 + m_2)v^2}{\frac{1}{2} m_1 u_1^2} = \left(\frac{m_1 + m_2}{m_1}\right)\frac{v^2}{u_1^2} \quad \text{... (ii)}$$

From (*i*) and (*ii*)

$$= \left(\frac{m_1 + m_2}{m_1}\right)\left(\frac{m_1}{m_1 + m_2}\right)^2 = \frac{m_1}{m_1 + m_2}$$

When, $K_f < K$, there is loss in K.E.

17. Consider the system to follow the four set of positions with co-ordinates (V_1, P_1, T_1), (V_2, P_2, T_1), (V_3, P_3, T_2) and (V_4, P_4, T_2). Then the processes and the work done are as given below:

(A) Isothermal expansion : $Q_1 = W_1 = \int_{V_1}^{V_2} PdV = RT_1 \log_e \frac{V_2}{V_1}$

(B) Adiabatic expansion

$$W_2 = \int_{V_1}^{V_2} PdV = \frac{R}{\gamma - 1}(T_1 - T_2)$$

(C) Isothermal compression

$$Q_2 = W_3 = \int_{V_3}^{V_4} PdV = -RT_2 \log_e \frac{V_4}{V_3} = RT_3 \log_e \frac{V_3}{V_4}$$

(D) Adiabatic compression

$$W_4 = -\int_{V_1}^{V_2} PdV = \frac{R}{\gamma - 1}(T_1 - T_2)$$

Total work done by the gas in the complete cycle = W = (W_1 + W_2) + (W_3 + W_4)

But $W_2 = W_4$ in magnitude.

$\therefore$ $W = W_1 - W_3 = Q_1 - Q_2$

or, W = Area (ABCDA)

Expression = $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q}{Q}$

Using the equation of states for the four steps

We get, $P_1V_1 = P_2V_2$ (isothermal expansion) $P_2V_2^{\gamma} = P_3V_3^{\gamma}$ (adiabatic expansion)

$P_3V_3 = P_4V_4$ (isothermal compression) $P_4V_4^{\gamma} = P_1V_1^{\gamma}$ (adiabatic compression)

Multiplying the four equations we get, $(P_1V_1)(P_2V_2^{\gamma})(P_3V_3)(P_4V_4^{\gamma}) = (P_2V_2)(P_3V_3^{\gamma})(P_4V_4)(P_1V_1^{\gamma})$

$V_1 V_2^{\gamma} V_3 V_4^{\gamma} = V_2 V_3^{\gamma} V_4 V_1^{\gamma}$

$V_2^{\gamma-1} V_4^{\gamma-1} = V_3^{\gamma-1} V_1^{\gamma-1}$

$\Rightarrow \quad (V_2V_4)^{\gamma-1} = (V_1V_3)^{\gamma-1}$

$\Rightarrow \quad V_2V_4 = V_1V_3$

$\Rightarrow \quad \frac{V_2}{V_1} = \frac{V_3}{V_4} \Rightarrow \log_e\left(\frac{V_2}{V_1}\right) = \log_e\left(\frac{V_3}{V_4}\right)$

Also $\frac{Q_2}{Q_1} = \frac{RT_2\log_e(V_3/V_4)}{RT_1\log_e(V_2/V_1)} = \frac{T_2}{T_1}$

Q_2 and Q_1 can be compared with T_2 and T_1 and, therefore,

$\therefore \quad \boxed{\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}}$

OR

(a) $\frac{1}{2}mv^2 = \frac{3}{2}kT$

$T = \frac{mv^2}{3k}$

m = 5.34×10^{-26} kg

k = 1.38×10^{-23} JK^{-1}

$$T = \frac{5.34\times10^{-26}\times\left(11\times10^3\right)^2}{3\times1.38\times10^{-23}}$$

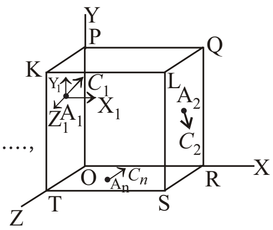
= 1.56×10^5K

(b)
- A gas consists of a large number of molecules which are perfect elastic spheres and are identical for a given gas and different for different gases.
- The molecules of a gas are in continuous, rapid and random motion.
- The size of the gas molecules are very small compared to the distance between them. So, the volume of all the molecules of a gas is negligible compared to the volume of the gas.
- Collisions between molecules and between wall and the molecules are perfectly elastic.
- Molecular density is uniform throughout the gas.
- A molecule moves in straight line between two successive collisions and average distance covered between two successive collisions is called mean free path of the molecules.
- The time of collision is negligible compared to the time between two successive collisions.

18. Consider an ideal gas in a cubical container of volume $V = a^3$

Let $n \to$ no. of molecules of the gas,

$m \to$ mass of the each molecule.



$\therefore$ Total mass of the gas = $M = m \times n$

C_1, C_2 .... C_n are velocities of the molecules A_1, A_2, ..., A_n, respectively.

$(x_1, y_1, z_1), (x_2, y_2, z_2)$, ..., (x_n, y_n, z_n) are the rectangular components of velocities C_1, C_2, ..., C_n, respectively along OX, OY, OZ.

$\therefore \quad x_1^2 + y_1^2 + z_1^2 = C_1^2$ and so on

Initial momentum of A_1 along OX = mx_1

$\therefore$ Change in momentum of A_1 after collision = $-mx_1 - mx_1 = -2mx_1$

$\therefore$ Momentum transferred by A_1 to the wall = $2mx_1$

Molecule A_1 first collides with the wall QLSR and rebounds and then collides with the opposite wall PKTO. Distance covered between these two sucessive collisions = $2a$

$\therefore$ Time between these two collisions $= t = \frac{2a}{x_1}$

$\therefore$ Number of collisions/sec = $\frac{1}{t} = \frac{x_1}{2a}$

$\therefore$ Total momentum transferred to the was/sec $= 2mx_1 \times \frac{x_1}{2a} = \frac{mx_1^2}{a}$

$\therefore$ Force exerted by A_1 on the wall = f_1 = $\frac{mx_1^2}{a}$

Similarly, $f_2 = \frac{mx_2^2}{a}, \ldots, f_n = \frac{mx_n^2}{a} = \frac{m}{a^3}(x_1^2 + x_2^2 + \ldots + x_n^2)$

$\therefore$ Pressure on this wall $= P_x = \dfrac{F_x}{a^2} = \dfrac{m}{a^3}(x_1^2 + x_2^2 + ... + x_n^2)$

Similarly, pressure along OY and OZ are $P_y = \dfrac{m}{a^3}(y_1^2 + y_2^2 + ... + y_n^2)$

$P_z = \dfrac{m}{a^3}(z_1^2 + z_2^2 + ... + z_n^2)$

$\therefore$ Molecular density is uniform, then $P_x = P_y = P_z = P$

$\therefore$ $P_x + P_y + P_z = 3P$

$\Rightarrow$ $P = \dfrac{P_x + P_y + P_z}{3}$

$\therefore$ $P = \dfrac{m}{3a^3}[(x_1^2 + x_2^2 + ... x_n^2) + (y_1^2 + y_2^2 + ... y_n^2) + (z_1^2 + z_2^2 + ... z_n^2)]$

$= \dfrac{m}{3a^3}[C_1^2 + C_2^2 + ... + C_n^2] = \dfrac{mn}{3a^3}\left[\dfrac{C_1^2 + C_2^2 + ...C_n^2}{n}\right]$

$P = \dfrac{M}{3V}C^2 = \dfrac{1}{3}\rho C^2$

19. Mean value of refractive index, $\mu = \dfrac{1.45 + 1.56 + 1.54 + 1.44 + 1.54 + 1.53}{6} = 1.51$

Absolute error measured are:

$\Delta\mu_1 = 1.51 - 1.45 = +0.06$

$\Delta\mu_2 = 1.51 - 1.56 = -0.05$

$\Delta\mu_3 = 1.51 - 1.54 = -0.03$

$\Delta\mu_4 = 1.51 - 1.44 = +0.07$

$\Delta\mu_5 = 1.51 - 1.54 = -0.30$

$\Delta\mu_6 = 1.51 - 1.53 = -0.02$

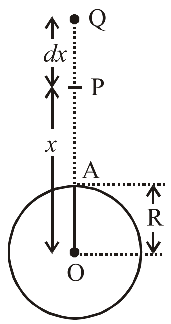
Mean absolute error, $\overline{\Delta\mu} = \sum_{i=1}^{n}\dfrac{|\Delta\mu_i|}{n} = \dfrac{0.06 + 0.05 + 0.03 + 0.07 + 0.03 + 0.02}{6} = 0.04333 = 0.043$

Relative or fractional error $= \pm\dfrac{\overline{\Delta\mu}}{\mu} = \pm\dfrac{0.043}{1.51} = \pm 0.02847 = \pm 0.0285$

Percentage error = ± 0.0285 × 100 = ±2.85%

20. **Escape Velocity:** It is the minimum velocity with which a body should be projected vertically upwards from the surface of the earth so that it crosses the gravitational field of the earth and never comes back on its own.

$F = \dfrac{GMm}{x^2}$ = gravitational force of attraction on a body at P



Work done in taking it from P to Q = dW = Fdx = $\dfrac{GMm}{x^2}dx$

$\Rightarrow$ Work done to take the body to infinity = W

$= \int_R^\infty \dfrac{GMm}{x^2}dx = -GMm\left[\dfrac{1}{x}\right]_R^\infty = -GMm\left[\dfrac{1}{\infty} - \dfrac{1}{R}\right] = \dfrac{GMm}{R}$

This work is done at the expense of the K.E. of the body.

$\Rightarrow$ $\dfrac{1}{2}mv_e^2 = \dfrac{GMm}{R} \Rightarrow v_e = \sqrt{\dfrac{2GM}{R}}$

But $GM = gR^2$ $\qquad \therefore v_e = \sqrt{2gR}$

OR

As $\dfrac{1}{f} = \left(\dfrac{\mu_2}{\mu_1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

For lens material $\mu_2 = 1.5$

(i) If the convex lens is immersed in water (μ = 1.33) its focal length will be positive hence it behaves as converging lens.

(ii) If the convex lens is immersed CS_2 solution (μ = 1.6) its focal length will be negative hence it behaves as diverging lens.

21. (a) **True:** when a body is thrown vertically upwards in the space, then at the highest point, the body has zero speed but has downward acceleration equal to the acceleration due to gravity.

(b) **False:** because velocity is the speed of body in given direction. When speed is zero, the magnitude of velocity of body is zero, hence velocity is zero.

(c) **True:** when a particle is moving along a straight line with constant speed, its velocity remains constant with time. Therefore, acceleration (= change in velocity/time) is zero.

(d) The statement depends upon the choice of the instant of time taken as origin. When the body is moving along a straight line with positive acceleration, the velocity of the body at an instant of time t is $v = u + at$.

The given statement is not correct if a is positive and u is negative, at is the instant of time taken as origin. Then for all the times before the time for which v vanishes, there is slowing down of the particle that is the speed of the particle will decrease with time. It happens when body is projected vertically upwards. But the given statement is true if u is positive, and a is positive, at is the instant of time taken as origin. It is so when the body is falling vertically downwards.

22. (a) When the money climbs up with an acceleration $a = 6\ ms^{-2}$, the tension T in the string must be greater than the weight of the monkey,

$T - mg = ma$

or $T = m\,(g + a) = 40\,(10 + 6) = 640\ N$

(b) When the monkey climbs down with an acceleration,

$a = 4\ ms^{-2}$ $\qquad$ $mg - T = ma$

or $T = m\,(g - a) = 40\,(10 - 4) = 240\ N$

(c) When the monkey climbs up with uniform speed,

$T = mg = 40 \times 10 = 400\ N$

(d) When the monkey falls down the rope nearly freely, $a = g$

$\therefore$ $T = m\,(g - a) = m\,(g - g) = 0$

As the tension in the rope in case (a) is greater than the maximum permissible tension (600 N), so the rope will break in case (a) only.

23. Total energy = $P \times t = 10^4 \times 150 = 15 \times 10^5$ J

As 50% of the heat is lost,

Energy available = $\frac{50}{100} \times 15 \times 10^5 = 7.5 \times 10^5\ J$

$Q = mc\Delta T = 8 \times 10^3 \times 0.91 \times \Delta T$ $\qquad$ $\therefore$ $7.5 \times 10^5 = 8 \times 10^3 \times 0.91 \times \Delta T$

$\Rightarrow$ $\Delta T = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103°C$

24. (a) It is positive, as force and displacement in same direction.

(b) It is negative, as the bucket is moving against the direction of gravitational force.

(c) It is negative, as the friction is always opposite to the direction of motion.

(d) It is positive as the force and displacement are in same direction.

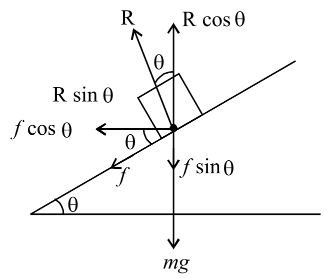
(e) It is negative as the resistive force acts against the direction of motion.

25. When the outer bank of a road is raised with respect to the inner bank so that a component of normal force provides the centrepital force. This is called banking. Equating the forces along horizontal and vertical direction respectively, we get

$$R\sin\theta + f\cos\theta = \frac{mv}{r} \qquad ..(i)$$

$mg + f\sin\theta = R\cos\theta$, where $f = \mu R$

or, $R\cos\theta - f\sin\theta = mg$ $\qquad$ ...(ii)



Dividing equation (i) by equation (ii), we get

$$\frac{R\sin\theta + f\cos\theta}{R\cos\theta - f\sin\theta} = \frac{v^2}{rg}$$

Dividing numerator and denominator of LHS by $R\cos\theta$, we get

$$\frac{\tan\theta + \frac{f}{R}}{1 - \frac{f}{R}\tan\theta} = \frac{v^2}{rg}$$

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg} \qquad \left[\because \mu = \frac{f}{R}\right]$$

$$v^2 = rg\left[\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right]$$

$$\Rightarrow \quad v = \sqrt{rg \cdot \frac{\mu + \tan\theta}{1 - \mu\tan\theta}}$$

If coefficient of static friction is zero

$$v = \sqrt{rg\tan\theta}$$

$$v^2 = rg\tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg}$$