NCERT TEXTUAL EXERCISE (SOLVED)

1. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K, respectively. Express these temperatures in celsius and fahrenheit scales.

Sol. If temperature in kelvin scale is K and that in celsius scale is C, then C = K – 273.15

∴ For neon, C = 24.57 – 273.15 = – 248.58°C

For CO_2, C = 216.55 – 273.15 = – 56.60°C

If temperature in fahrenheit scale is F, then $\frac{F-32}{180} = \frac{K-273.15}{100}$

For neon, $F = \frac{180}{100}(24.57 - 273.15) + 32 = -415.44^\circ F$

For CO_2, $F = \frac{180}{100}(216.55 - 243.15) + 32 = -69.88^\circ F$

2. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B.

Sol. According to the question, Triple point of water = 200 A = 350 B = 273.16 K [Triple point of water in kelvin scale = 273.16 K]

∴ $1A = \frac{273.16}{200}$ K and $1B = \frac{273.16}{350}$ K

If T_A and T_B represent the triple point of water on scales A and B then,

$$\frac{273.16}{200} T_A = \frac{273.16}{350} T_B$$

$$\Rightarrow \frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7}$$

$$\therefore T_A = \frac{4}{7} T_B$$

3. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law: $R = R_0 [1 + \alpha (T - T_0)]$. The resistance is 101.16 Ω at the triple point of water 273.16 K, and 165.5 Ω at the normal melting point of lead 600.5 K. What is the temperature when the resistance is 123.4 Ω ?

Sol. R_0 = 101.6 Ω, T_0 = 273.16 K, R_1 = 165.5 Ω, T_1 = 600.5 K, R_2 = 123.4 Ω, T_2 = ?

Using the relation, $R = R_0 [1 + \alpha (T - T_0)]$

165.5 = 101.6 [1 + α (600.5 – 273.16)] = 101.6 [1 + α × 327.34]

$$\Rightarrow \alpha = \frac{165.5 - 101.6}{101.6 \times 327.34}$$

Also, $123.4 = 101.6\,[1 + \alpha\,(T_2 - 273.16)]$

or, $$123.4 = 101.6\left[1 + \frac{63.9}{101.6 \times 327.34}(T_2 - 273.16)\right]$$

(Substituting the value of α in the equation)

$$\therefore\ T_2 = \frac{(123.4 - 101.6) \times 327.34}{63.9} + 273.16 = 111.67 + 237.16 = 384.83\,\text{K}$$

4. Answer the following:

(a) The triple point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the celsius scale)?

(b) There were two fixed points in the original celsius scale as mentioned above which were assigned the number 0°C and 100°C, respectively. On the absolute scale, one of the fixed points is the triple point of water, which on the kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (kelvin) scale?

(c) The absolute temperature (kelvin scale) T is related to temperature t_c on the Celsius scale by $t_c = T - 273.15$. Why do we have 273.15 in this relation, and not 273.16?

(d) What is the temperature of the triple point of water on an absolute scale whose unit interval size is equal to that of the fahrenheit scale?

Sol. (a) Triple point of water has a unique value (273.16 K) at fixed values of pressure and volume. But melting point of ice and boiling point of water do not have unique value as they depend on pressure.

(b) The other fixed point is the absolute zero.

(c) On the celsius scale, 0°C corresponds to melting point of ice at normal pressure and the same temperature in kelvin scale is 273.15 K. The triple point of water in kelvin scale is 273.16 K.

$\therefore$ By the relation, $t_c = T - 273.15$,

The triple point of water on the celsius scale = 273.16 – 273.15 = 0.01°C

(d) Relation between temperature in fahrenheit scale and absolute scale is

$$\frac{F-32}{180} = \frac{K-273.15}{100} \quad ...(i)$$

For another set of temperature,

$$\frac{F'-32}{180} = \frac{K'-273.15}{100} \quad ...(ii)$$

Subtracting (i) from (ii), we get

$$\frac{F'-F}{180} = \frac{K'-K}{100}$$

$$F'-F\frac{180}{100} \times (K'-K)$$

$\Rightarrow$

If K′ – K = 1K, then $F'-F = \frac{180}{100} \times 1 = \frac{9}{5}$

$\therefore$ The triple point of water (273.16 K) in the new scale is $273.16 \times \frac{9}{5} = 491.69$.

5. Two ideal gas thermometers A and B use oxygen and hydrogen, respectively. The following observations are made:

Temperature	**Pressure thermometer A**	**Pressure thermometer B**
Triple point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of Sulphur	1.797×10^5 Pa	0.287×10^5 Pa

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?

(b) What do you think about the reason for the slightly different answers from A and B? (The thermometers are to tally). What further procedure is needed in the experiment to reduce the discrepancy between the two readings.

Sol. (a) Triple point of water = 273.16 K,

Let T be the melting point of sulphur.

For thermometer A,

$$\frac{P}{P_{tr}} \times 273.16 = \frac{1.797 \times 10^5}{1.250 \times 10^5} \times 273.16 = 392.69 \text{ K} \quad [\because P \propto T]$$

For thermometer B, $T = \dfrac{0.287 \times 10^5}{0.200 \times 10^5} \times 273.16 = 391.98\,K$

(b) The cause of this slight difference in the two temperatures is that oxygen and hydrogen gases are not perfectly ideal.

To reduce the discrepancy, the readings should be taken at a very low pressure where the gases approach to the ideal gas behaviour.

6. A steel tape 1 m long is correctly calibrated for a temperature of 27°C. The length of a steel rod measured by this tape is found to be 63 cm on a hot day when the temperature is 45°C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27°C? Coefficient of linear expansion of steel = 1.2×10^{-5}/°C.

Sol. L_1 = 100 cm and T_1 = 27°C

At 45°C, $L_2 = L_1 + \alpha L_1 \Delta T = 100 + 1.2 \times 10^{-5} \times 100 \times (45 - 27) = 100.0216$ cm

$\therefore$ Length of 1 cm mark at 27°C on this scale becomes at 45°C = $\dfrac{100.0216}{100}$ cm

$\therefore$ Actual length of the steel rod on that day = $\dfrac{100.0216}{100} \times 63 = 63.0136$ cm

Also, length of the same rod at 27°C = 63 × 1 = 63 cm

7. A large steel wheel is to be fitted on to a shaft of the same material. At 27°, the outer diameter of the shaft is 8.7 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using dry ice. At what temperature of the shaft does the wheel slip on the shaft? Assume, coefficient of linear expansion of the steel to be constant over the required temperature; range, $\alpha_{steel} = 1.2 \times 10^{-5}$ K.

Sol. T_1 = 27°C = 273+27 = 300 K, Length at temperature $T_1 = l_1$ = 8.7 cm

Length of temperature $T_2 = l_2$ = 8.69cm, Change in length = $\Delta l = l_2 - l_1 = l_1 \times \alpha \times \Delta T$

$\Rightarrow$ $8.69 - 8.7 = 8.7 \times 1.2 \times 10^{-5} (T_2 - 300)$

$\Rightarrow$ $T_2 - 300 = \dfrac{0.01}{8.7 \times 1.2 \times 10^{-5}} = -95.8$

$\Rightarrow$ $T_2 = 300 - 95.8 = 204.2$ K = – 68.8°C

8. A hole is drilled in a copper sheet. The diamater of the hole is 4.24 cm at 27°C. What is the change in the diameter of the hole when the sheet is heated to 227°C ? Coefficient of linear expansion of copper = 1.7×10^{-5} / °C

Sol. Area of the hole at 27°C = $S_1 = \frac{\pi D_1^2}{4} = \frac{\pi}{4} \times (4.24)^2 \text{ cm}^2$

Area of the hole at 227°C = $S_2 = \frac{\pi D_2^2}{4}$

$\beta = 2\alpha = 2\times1.7\times10^{-5} = 3.4\times10^{-5}$ / °C.

Increase in area = $S_2 - S_1 = S_1\beta\Delta T\, S_1\beta\Delta T$

$\therefore S_2 = S_1 + S_1\beta\Delta T = S_1 (1 + \beta\Delta T)$

$\frac{\pi D_2^2}{4} = \frac{\pi}{4} \times (4.24)^2 = [1 + 3.4 \times 10^{-5} \times (228–27)]$

$D_2^2 = (4.24)^2 \times 1.0068 \Rightarrow D_2 = 4.2544$ cm

$\therefore$ Change in diameter = 4.2544 – 4.24 = 0.0144cm

9. A brass wire 1.8 m long at 27°C is held tightly with little tension between two rigid supports. If the wire is cooled to a temperature of –39°C, what is the tension devoloped in the wire, if the diameter is 2mm? Coefficient of linear expansion of brass = 2×10^{-5}/ °C and Young's modulus of brass = 0.91 $\times 10^{11}$ N/m^2.

Sol. Here, $\ell = 1.8$ m, $T_1 = 27°C$, $T_2 = -39°C$, $r = 1$ mm $= 10^{-3}$m

$\alpha = 2 \times 10^{-5}$/°C $\qquad Y = 0.91 \times 10^{11}$ N/m^2

$\because \quad Y = \frac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \frac{F\ell}{AY}$

Also, $\Delta\ell = \alpha\ell\Delta T \qquad \therefore \frac{F\ell}{AY} = \alpha\ell\Delta T$

$\therefore \quad F = AY\alpha\Delta T = \pi r^2 Y\alpha(T_2 - T_1)$

$F = \frac{22}{7} \times (10^{-3})^2 \times 0.91 \times 10^{11} \times 2 \times 10^{-5}(-39 - 27) = -3.77 \times 10^2$ N

The negative sign indicates that the force is inwards leading to contraction of the wire.

10. A brass rod of length 50 cm and diameter 3 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C, if the original lengths are measured at 40°C? Coefficient of linear expansion of brass and steel are 2.1×10^{-5}/°C and 1.2×10^{-5}/°C, respectively.

Sol. $\Delta\ell_1 = \ell_1\alpha_1\Delta T = 50 \times (2.1 \times 10^{-5})(250 - 40) = 0.2205$ cm

$\Delta\ell_2 = \ell_2\alpha_2\Delta T = 50 \times (1.2 \times 10^{-5})(250 - 40) = 0.126$ cm

$\therefore$ Change in length of the combined rod = $\Delta\ell_1 + \Delta\ell_2$= 0.2205 + 0.126 = 0.3465 cm.

11. The coefficient of volume expansion of glycerine is 49 × 10^{-5}/°C. What is the fractional change in density for a 30°C rise in temperature?

Sol. $\gamma = 49 \times 10^{-5}$/°C, $\Delta T = 30°C$

$V_2 = V_1 + \Delta V = V_1 (1 + \gamma\Delta T)$

$\Rightarrow \quad V_2 = V_1 (1 + 49 \times 10^{-5} \times 30) = 1.0147\, V_1$

$$\because \quad \rho_1 = \frac{M}{V_1} \;\&\; \rho_2 = \frac{M}{V_2} = \frac{M}{1.0147 V_1} = 0.9855\rho_1$$

$$\therefore \text{ Fractional change in density} = \frac{\rho_1 - \rho_2}{\rho_1} = \frac{(\rho_1 - 0.9855\rho_1)}{\rho_1} = 0.0145$$

12. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8 kg. How much is the rise in temperature of the block in 2.5 minuets, assuming 50% of power is used up in heating the machine. Specific heat of aluminium = 0.91 J/g/°C

Sol. Total energy = $P \times t = 10^4 \times 150 = 15 \times 10^5$ J

As 50% of the heat is lost,

$$\therefore \quad \text{Energy available} = \frac{50}{100} \times 15 \times 10^5 = 7.5 \times 10^5 \text{ J}$$

$\because \quad Q = mc\Delta T = 8 \times 10^3 \times 0.91 \times \Delta T \;\therefore\; 7.5 \times 10^5 = 8 \times 10^3 \times 0.91 \times \Delta T$

$$\Rightarrow \quad \Delta T = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103°C$$

13. A block of copper of mass 2.5 g is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt. Specific heat of copper is 0.39 J/*g*/°C. Heat of fusion of water = 335 J/*g*

Sol. Mass of copper = m_1 = 2.5 kg = 2500 *g*, Change in temperature = Δt = 500 – 0 = 500°C

Specific heat of copper = c_1= 0.39 J/g/°C, Lalent heat of fusion of ice = L = 335 J/*g*

Let mass of ice melted = m_2

$\because$ Heat lost by copper = Heat gained by ice $\Rightarrow m_1c_1\Delta t = m_2L$

$$m_2 = \frac{m_1c_1\Delta T}{L} = \frac{2500 \times 0.39 \times 500}{335} = 1500\ g = 1.5 \text{ kg}$$

14. In an experiment on the specific heat of a metal, a 0.20 kg block of a metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal. If heat losses to the surrounding are not negligible, is your answer greater or smaller than the actual value of specific heat of the metal?

Sol. Mass of the metal m_1 = 0.2 kg = 200 *g*

Fall in temperature = ΔT_1 = 150 – 40 = 110°C

Heat lost by the metal = $\Delta Q_1 = m_1 c \Delta T_1$ [*c* = specific heat of metal]

= 200 × *c* × 110 ...(1)

Volume of water = 150 cm^3;

Mass of water = m_2 = 150 *g*

Water equivalent of calorimeter = *w* = 0.025 kg = 25 *g*

Rise in temperature of water and calorimeter = ΔT_2 = 40 – 27 = 13°C

Heat gained by water and colorimeter = $\Delta Q_2 = (m_2 + w)\,\Delta T_2$ = (150 + 25) × 13 = 175 × 13 ...(2)

∵ Heat lost = Heat gained,

200 × *c* × 110 = 175 × 13

$$c = \frac{175 \times 13}{200 \times 110} = 0.1$$

The value of *c* is less than its actual value, as some heat is lost to the surroundings.

15. A child running a temperature of 101°F is given an anti pyretic (i.e., a medicine that lowers fever), which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 min what is the average rate of extra evaporation caused by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child in 30 kg. The specific heat of human body is approximately the same as that of water and latent heat of evaporation of water at that temperature is about 580 cal/g.

Sol. Decrease in temperature = 101– 98 = 3°F = $3 \times \frac{5}{9} = \frac{5}{3}$°C;

Mass of child = *m* = 30 kg

Specific heat of water = specific heat of human body = *c* = 1000 cal/kg/°C

∴ Heat lost by the child = $mc\Delta t = 30 \times 1000 \times \frac{5}{3}$ = 50000 cal.

Let m' be the mass of water evaporated then $m'L = mc\Delta t$

or $m' = \dfrac{5000}{580} = 86.2\,\text{g}$

$\therefore$ Average rate of extra evaporation $= \dfrac{86.2}{20} = 4.31$ g/min

16. A cubical box of thermocole has each side of 30 cm and thickness of 5 cm. 4 kg of ice is put in the box. If out side temperature is 45°C and coefficient of thermal conductivity is 0.01 J/s/m/°C, calculate the mass of ice left after 6 hours. Take latent heat of fusion of ice = 335×10^3 J/kg.

Sol. Length of each side = l = 30 cm = 0.3 m, Thickness = Δx = 5 cm = 0.05 m

Total surface area of the box, $A = 6l^2 = 6 \times 0.3 \times 0.3$, Temperature difference $= \Delta T = 45 - 0 = 45°C$

K = 0.01 J/s/m/°C, Time = Δt = 6 hrs = $6\times60\times60$ s

Latent heat of fusion = $L = 335\times10^3$ J/kg

If m is the mass of ice melted, then $mL = KA\left(\dfrac{\Delta T}{\Delta x}\right)\Delta t = m = \dfrac{KA}{L}\left(\dfrac{\Delta T}{\Delta x}\right)\Delta t$

$= \dfrac{0.01\times0.54\times45\times6\times60\times60}{0.05\times335\times10^3} = 0.313\,\text{kg}$

$\therefore$ Mass of ice left = 4 – 0.313 kg = 3.687 kg

17. A brass boiler has a base area of 0.15 m² and thickness 1 cm. If boils water at the rate of 6 kg/min, when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 609 J/s/m/°C. Heat of vaporisation of water = 2256×10^3 J/g

Sol. $A = 0.15\text{ m}^2$, Δx = 1 cm = 0.01 m

$\because \quad \dfrac{\Delta Q}{\Delta t} = KA\left(\dfrac{\Delta T}{\Delta x}\right)$

$\Rightarrow \quad \dfrac{6\times2256\times10^3}{60} = 6.09\times0.15\left(\dfrac{t-100}{0.01}\right)$

$\Rightarrow \quad 2256\times10^2 = 6.09 \times 0.15 \times \dfrac{t-100}{0.01}$

$\Rightarrow \quad t-100 = \dfrac{2256}{609\times0.15} = 24.7$

$t = 124.7°C$

18. Explain, why:

(a) A body with large reflectivity is a poor emitter.

(b) A brass tray feels much colder than a wooden tray on a chilly day.

(c) An optical pyrometer (for measuring high temperature), caliberated for an ideal black body radiation, give too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in a furnace.

(d) The Earth without its atmosphere would be inhospitably cold.

(e) Heating system based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Sol. (a) A body with large reflectivity will absorb very less amount of heat, so, it will be a poor emitter too.

(b) When we touch a brass tray on a chilly day, heat flows from our body to the tray very fast as brass is a good conductor of heat. So, it appears colder, whereas wood is a bad conductor of heat, and therefore, heat does not flow to wood from our body and we feel it warm.

(c) By Stefan's law, energy radiated when the hot iron is in the open is $E = \sigma T^4$.

Energy radiated by the hot iron when it is in the surroundings of temperature T_0 is

$E' = \sigma(T^4 - T_0^4)$

$\because$ Pyrometer works on the principle that the brightness of a surface depends on its temperature.

$\therefore$ It gives a low value for the temperature, of iron in the open.

(d) The infrared radiation received by the Earth during the day from the Sun is trapped by the atmosphere. The lower layers of the atmosphere reflect the infrared radiation back to the surface of the Earth. So, if the atmosphere in not there, the Earth will be too cold to live in.

(e) Steam at 100°C contains more heat than water at 100°C as latent heat is given to water to convert it to steam. So, heating system based on circulation of steam are more efficient than the system based on circulation of water.

19. A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C. The temperature of the surrounding is 20°C.

Sol. From Newton's law of cooling,

$\frac{dT}{dt} = -K(T - T_0)$, where T and T_0 are the temperatures of the body and the surrounding, respectively.

If the temperature of the body decreases from T_1 to T_2 in time t,

$$\int_{T_1}^{T_2} \frac{dT}{T - T_0} = -\int_0^t K dt$$

$$\Rightarrow \quad \log(T - T_0)\Big|_{T_1}^{T_2} = -Kt$$

$$\Rightarrow \quad \log_e \frac{T_2 - T_0}{T_1 - T_0} = -Kt$$

$$\Rightarrow \quad 2.303 \log_{10} \frac{T_2 - T_0}{T_1 - T_0} = -Kt$$

$$\Rightarrow \quad 2.303 \log_{10} \frac{T_1 - T_0}{T_2 - T_0} = Kt$$

$$\Rightarrow \quad t = \frac{2.303}{K} \log_{10} \frac{T_1 - T_0}{T_2 - T_0}$$

Here, $T_1 = 80°C$, $T_1 = 50°C$, $T_0 = 20°C$; $t = 5$ min $= 5 \times 60 = 300$ sec

$$\therefore \quad 5 \times 60 = \frac{2.303}{K} \log_{10} \frac{80 - 20}{50 - 20} = \frac{2.303}{K} \log_{10}(2) \quad \text{... (1)}$$

Also, if $T_1 = 60°C$, $T_2 = 30°C$, $T_0 = 20°C$, $t = ?$

$$t = \frac{2.303}{K} \log_{10} \frac{60 - 20}{30 - 20} = \frac{2.303}{K} \log_{10}(4) \quad \text{... (2)}$$

Dividing (2) by (1), $\dfrac{t}{5 \times 60} = \dfrac{\log_{10}(4)}{\log_{10}(2)} = \dfrac{0.6012}{0.3010} = 2$

$t = 5 \times 60 \times 2 = 10 \times 60s = 10$ mins