EXERCISE 5.1

1. Express the given complex number in the form $a + i\,b$: $(5i)\left(-\frac{3}{5}i\right)$

Sol. $(5i)\left(-\frac{3}{5}i\right) = -5 \times \frac{3}{5} \times i \times i$

$= -3i^2$

$= -3(-1)$ $\qquad [i^2 = -1]$

$= 3 + 0i$

2. Express the given complex number in the form $a + i\,b$: $i^9 + i^{19}$

Sol. $i^9 + i^{19} = i^{4\times2+1} + i^{4\times4+3}$

$= \left(i^4\right)^2 .i + \left(i^4\right)^4 .i^3$

$= 1 \times i + 1 \times (-i)$ $\qquad [i^4 = 1, i^3 = -i]$

$= i - i$

$= 0 + 0i$

3. Express the given complex number in the form of $a + i\,b$: i^{-39}

Sol. $i^{-39} = i^{-4\times9+3} = \left(i^4\right)^{-9} .i^{-3}$ $\qquad [i^4 = 1]$

$= \frac{1}{i^3} = \frac{1}{-i}$ $\qquad [i^3 = -i]$

$= \frac{-1}{i} \times \frac{i}{i} = i = 0 + i$ $\qquad [i^2 = -1]$

4. Express the given complex number in the form $a + i\,b$: $3(7 + i7) + i(7 + i7)$

Sol. $3(7 + i7) + i(7 + i7) = 21 + 21i + 7i + 7i^2 = 14 + 28i$

5. Express the given complex number in the form $a + i\,b$: $(1 - i) - (-1 + i6)$

Sol. $(1 - i) - (-1 + i6) = 1 - i + 1 - 6i = 2 - 7i$

6. Express the given complex number in the form $a + ib$: $\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$

Sol. $\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$

$= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i$

$= \left(\frac{1}{5} - 4\right) + i\left(\frac{2}{5} - \frac{5}{2}\right)$

$$= \frac{-19}{5} - \frac{21}{10}i$$

7. Express the given complex number in form $a + i\,b$.

$$\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right) - \left(-\frac{4}{3} + i\right)$$

Sol. $$\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right) - \left(-\frac{4}{3} + i\right)$$

$$= \frac{1}{3} + \frac{7}{3}i + 4 + \frac{1}{3}i + \frac{4}{3} - i$$

$$= \left(\frac{1}{3} + 4 + \frac{4}{3}\right) + \left(\frac{7}{3} + \frac{1}{3} - 1\right)i$$

$$= \frac{17}{3} + i\frac{5}{3}$$

8. Express the given complex number in the form $a + i\,b : (1 - i)^4$.

Sol. $$(1-i)^4 = \left[(1-i)^2\right]^2$$

$$= \left[1^2 + i^2 - 2i\right]^2$$

$$= [1 - 1 - 2i]^2$$

$$= 4i^2 = -4 + 0i$$

9. Express the given complex number in the form $a + ib$: $\left(\frac{1}{3} + 3i\right)^3$.

Sol. $$\left(\frac{1}{3} + 3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3} + 3i\right)$$

$$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3} + 3i\right)$$

$$= \frac{1}{27} - 27i + i - 9$$

$$= \left(\frac{1}{27} - 9\right) + i(-27 + 1)$$

$$= \frac{-242}{27} - 26i$$

10. Express the given complex number in the form $a + i\,b$: $\left(-2-\frac{1}{3}i\right)^3$.

Sol. $\left(-2-\frac{1}{3}i\right)^3 = (-1)^3\left(2+\frac{1}{3}i\right)^3$

$$= -\left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8 + \frac{i^3}{27} + 2i\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8 - \frac{i}{27} + 4i - \frac{2}{3}\right]$$

$$= -\left[\frac{22}{3} + \frac{107i}{27}\right]$$

$$= -\frac{22}{3} - \frac{107}{27}i$$

11. Find the multiplicative inverse of the complex number $4 - 3i$.

Sol. Let $z = 4 - 3i$

Then, $\bar{z} = 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

12. Find the multiplicative inverse of the complex number $\sqrt{5} + 3i$.

Sol. Let $z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = \left(\sqrt{5}\right)^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

13. Find the multiplicative inverse of the complex number $-i$.

Sol. Let $z = -i$

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

14. Express the following expression in the form of $a+ib$: $\dfrac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$

Sol. $$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)} = \frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$$

$$= \frac{9-5i^2}{2\sqrt{2}i}$$

$$= \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i}$$

$$= \frac{-7\sqrt{2}i}{2} = 0 - \frac{7\sqrt{2}}{2}i$$

EXERCISE 5.2

1. Find the modulus and the argument of the complex number $z = -1 - i\sqrt{3}$

Sol. $z = -1 - i\sqrt{3}$

Let $z = r(\cos\theta + i\sin\theta)$

Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + \left(-\sqrt{3}\right)^2$$

$\Rightarrow$ $r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$

$\Rightarrow$ $r^2 = 4$

$\Rightarrow$ $r = \sqrt{4} = 2$

$\therefore$ $2\cos\theta = -1$ and $2\sin\theta = -\sqrt{3}$

$\Rightarrow$ $\cos\theta = \dfrac{-1}{2}$ and $\sin\theta = \dfrac{-\sqrt{3}}{2}$

Since both the values of $\sin\theta$ and $\cos\theta$ are negative and $\sin\theta$ and $\cos\theta$ are negative in III quadrant, Argument $= -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$

2. Find the modulus and the argument of the complex number $z = -\sqrt{3} + i$.

Sol. $z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$

$\Rightarrow \quad r^2 = 3 + 1 = 4$

$\Rightarrow \quad r = \sqrt{4} = 2$

$\therefore \quad 2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \quad \cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \quad \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ [As θ lies in the II quadrant]

3. Convert the given complex number in polar form : $1 - i$.

Sol. Let $r\cos\theta = 1$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$

$\Rightarrow \quad r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$

$\Rightarrow \quad r^2 = 2$

$\Rightarrow \quad r = \sqrt{2}$

$\therefore \quad \sqrt{2}\cos\theta = 1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \quad \theta = -\frac{\pi}{4}$ [As θ lies in the IV quadrant]

$\therefore \quad 1 - i = r\cos\theta + ir\sin\theta$

$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$

This is the required polar form.

4. Convert the given complex number in polar form: $-1 + i$.

Sol. Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$

$\Rightarrow$ $r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow$ $r = \sqrt{2}$

$\therefore$ $\sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow$ $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore$ $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As θ lies in the II Quadrant]

It can be written,

$\therefore$ $= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

5. Convert the given number in polar form: $-1 - i$.

Sol. Let, $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$

$\Rightarrow$ $r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow$ $r^2 = 2$

$\Rightarrow$ $r = \sqrt{2}$

$\therefore$ $\sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow$ $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore$ $\theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$ [As θ lies in the III quadrant]

$\therefore$ $-1 - i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\left(-\frac{3\pi}{4}\right) + i\sqrt{2}\sin\frac{-3\pi}{4}$

$$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

6. Convert the given complex number in polar form: – 3.

Sol. Let, $r \cos \theta = -3$ and $r \sin \theta = 0$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-3)^2$$

$\Rightarrow$ $r^2 (\cos^2 \theta + \sin^2 \theta) = 9$

$\Rightarrow$ $r^2 = 9$

$\Rightarrow$ $r = \sqrt{9} = 3$

$\therefore$ $3 \cos \theta = -3$ and $3 \sin \theta = 0$

$\Rightarrow$ $\cos \theta = -1$ and $\sin \theta = 0$

$\therefore$ $\theta = \pi$

$\therefore$ $-3 = r \cos \theta + i\, r \sin \theta = 3 (\cos \pi + i \sin \pi)$

7. Convert the given complex number in polar form: $\sqrt{3} + i$.

Sol. Let, $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left(\sqrt{3}\right)^2 + 1^2$$

$\Rightarrow$ $r^2 (\cos^2 \theta + \sin^2 \theta) = 3 + 1$

$\Rightarrow$ $r^2 = 4$

$\Rightarrow$ $r = 2$

$\therefore$ $2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow$ $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore$ $\theta = \frac{\pi}{6}$ [As θ lies in the I quadrant]

$\therefore$ $\sqrt{3} + i = r\cos\theta + ir\sin\theta$

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

8. Convert the given complex number in polar form: i.

Sol. Let, $r \cos \theta = 0$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 0^2 + 1^2$$

$\Rightarrow \quad r^2(\cos^2\theta + \sin^2\theta) = 1$

$\Rightarrow \quad r^2 = 1$

$\Rightarrow \quad r = 1$

$\therefore \quad \cos\theta = 0$ and $\sin\theta = 1$

$\Rightarrow \quad \theta = \dfrac{\pi}{2}$

$\therefore \quad i = r\cos\theta + ir\sin\theta = \cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}$

This is the required polar form.

EXERCISE 5.3

1. Solve the equation $x^2 + 3 = 0$.

Sol. The given quadratic equation is $x^2 + 3 = 0$.

$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12$

Therefore, the required solutions are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2\times 1} = \frac{\pm\sqrt{12}i}{2}$$

$$= \frac{\pm 2\sqrt{3}i}{2} = \pm\sqrt{3}i$$

2. Solve the equation $2x^2 + x + 1 = 0$.

Sol. The given quadratic equation is $2x^2 + x + 1 = 0$.

$D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\times 2} = \frac{-1 \pm \sqrt{7}i}{4}$$

3. Solve the equation $x^2 + 3x + 9 = 0$.

Sol. The given quadratic equation is $x^2 + 3x + 9 = 0$.

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}i}{2}$$

4. Solve the equation $-x^2 + x - 2 = 0$.

Sol. The given quadratic equation is $-x^2 + x - 2 = 0$.

$D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\times(-1)}=\frac{-1\pm\sqrt{7}i}{-2}$$

5. Solve the equation $x^2 + 3x + 5 = 0$.

Sol. The given quadratic equation is $x^2 + 3x + 5 = 0$.

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm\sqrt{-11}}{2\times1}=\frac{-3\pm\sqrt{11}i}{2}$$

6. Solve the equation $x^2 - x + 2 = 0$.

Sol. The given quadratic equation is $x^2 - x + 2 = 0$.

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-1)\pm\sqrt{-7}}{2\times1}=\frac{1\pm\sqrt{7}i}{2}$$

7. Solve the equation $\sqrt{2}x^2 + x + \sqrt{2} = 0$.

Sol. The given quadratic equation is $\sqrt{2}x^2 + x + \sqrt{2} = 0$.

Therefore, the required solution are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\times\sqrt{2}}=\frac{-1\pm\sqrt{7}i}{2\sqrt{2}}$$

8. Solve the equation $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$.

Sol. The given quadratic equation $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$.

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\left(-\sqrt{2}\right)\pm\sqrt{-34}}{2\times\sqrt{3}}=\frac{\sqrt{2}\pm\sqrt{34}i}{2\sqrt{3}}$$

9. Solve the equation $x^2 + x + \frac{1}{\sqrt{2}} = 0$.

Sol. Therefore, the required solution are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}=\frac{-\sqrt{2}\pm\sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$=\frac{-1\pm\left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

10. Solve the equation $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$.

Sol. The given quadratic equation is $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$.

$\Rightarrow \sqrt{2}x^2 + x + \sqrt{2} = 0$

$\therefore$ Discriminant $(D) = b^2 - 4ac = 1^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$

Therefore, the required solution are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$