EXERCISE – 2.1

1. How many tangents can a circle have?

2. Fill in the blanks:

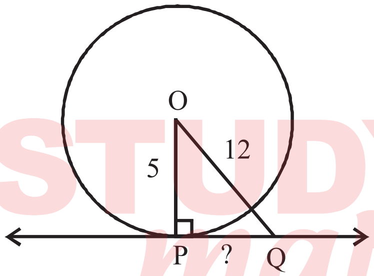
(i) A tangent to a circle intersects it in _________ point(s).

(ii) A line intersecting a circle in two points is called a _________.

(iii) A circle can have _________ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called_____.

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that OQ = 12 cm. Length PQ is ________.



(A) 12 cm (B) 13 cm

(C) 8.5 cm (D) $\sqrt{119}$ cm

4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other is a secant to the circle.

EXERCISE – 2.2

(In questions 1–3, choose the correct option and give justification)

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is _______.

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm

2. If TP and TQ are two tangents to a circle with centre 'O' such that $\angle POQ = 110°$, then $\angle PTQ$ is equal to _______.

(A) 60° (B) 70°

(C) 80° (D) 90°

NCERT TEXTUAL EXERCISE (SOLVED)

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

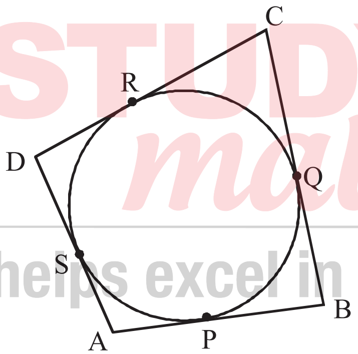
4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

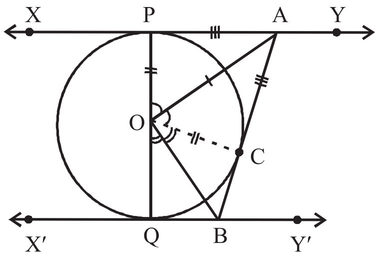
6. The length of a tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

8. A quadrilateral ABCD is drawn to circumscribe a circle as shown in the adjacent figure. Prove that: AB + CD = AD + BC.



9. In the following figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that ∠AOB = 90°.



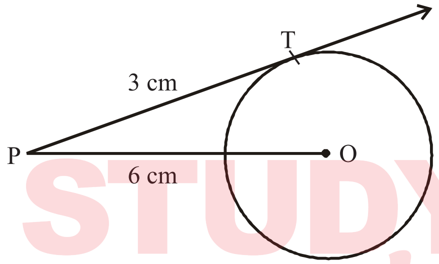
10. Prove that the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segments joining the points of contact at the centre.

11. Prove that the parallelogram circumscribing a circle is a rhombus.

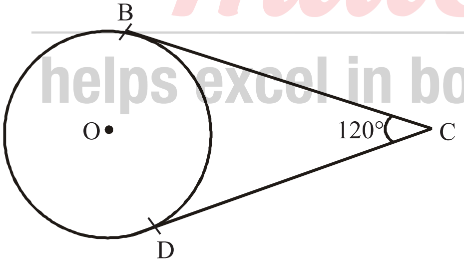
12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.
13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

TEST YOURSELF – CR 1

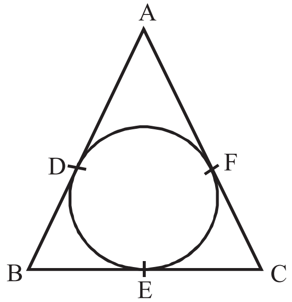
1. O is the centre of the circle and PT is a tangent at T. If PO = 6 cm, PT = 3 cm, calculate the radius of the circle.



2. BC and DC are two tangents of the circle with centre O such that ∠BCD = 120°. Prove that OC = 2BC.

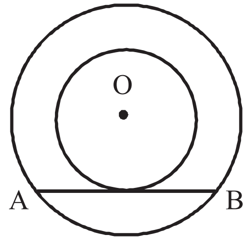


3. In the adjoining figure, if AB = AC, then prove that BE = EC.



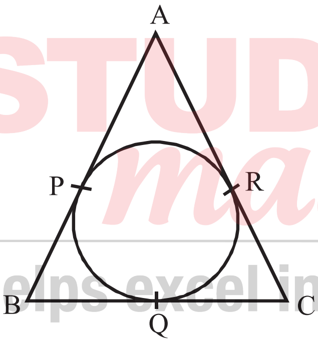
4. Prove that the tangents at the extremities of any chord make equal angles with the chord.

5. Two concentric circles are of radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the smaller circle.

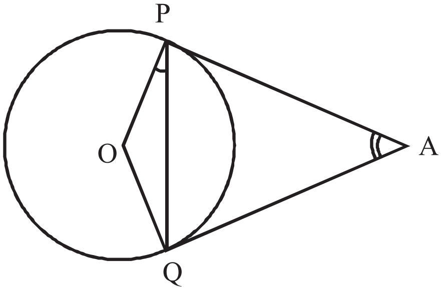


6. The incircle of ΔABC touches the sides AB, BC and CA at the points P, Q and R respectively. Show that:

$AP + BQ + CR = PB + QC + RA = \frac{1}{2}$ (perimeter of ΔABC).

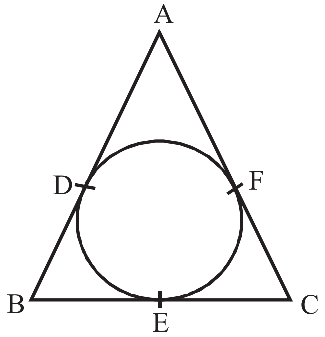


7. Tangents AP and AQ are drawn to a circle with centre O from an external point A. Prove that ∠PAQ = 2∠OPQ.



NCERT TEXTUAL EXERCISE (SOLVED)

8. In the figure given below, AB = 12 cm, BC = 8 cm and AC = 10 cm. Find AD, BE and CF.



9. If all the sides of a rectangle touch a circle, show that it is a square.

ANSWERS

NCERT Textual Exercises and Assignments

Exercise – 2.1

1. Circle can have infinite tangents.

2. (i) A tangent to a circle intersects it in **one and only one** point.

(ii) A line intersecting a circle in two points is called a **secant.**

(iii) A circle can have **two** parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called **point of contact.**

3. OP = 5 cm .... (radius of circle)

OQ = 12 cm .... (given)

In ΔOPQ,

$\angle OPQ = 90^\circ$ .... (radius at the point of contact is $\perp$ to tangent)

$\therefore$ $OQ^2 = OP^2 + PQ^2$ .... (by Pythagoras theorem)

$\therefore$ $12^2 = 5^2 + PQ^2$

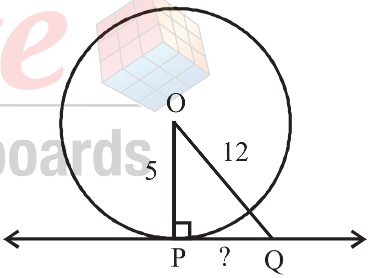
$\therefore$ $144 = 25 + PQ^2$

$\therefore$ $144 - 25 = PQ^2$

$\therefore$ $PQ^2 = 119$

$\therefore$ $PQ = \sqrt{119}$ cm.

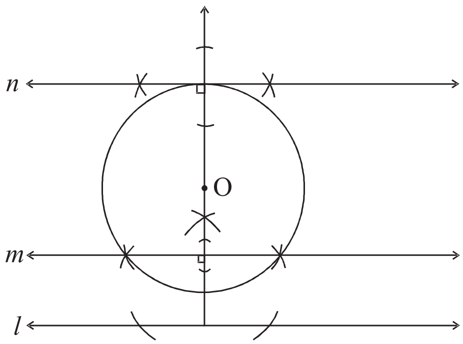
$\therefore$ **Length of PQ =** $\sqrt{119}$ **cm.**



4. Line l is a given line.

Line n is a tangent to the circle and || to line line l.

Line m is a secant to the circle and || to line l.



Note: (i) There is no tangent to a circle passing through a point lying inside the circle.

(ii) There is one and only one tangent to a circle passing through a point lying on the circle.

(iii) There are exactly two tangents to a circle passing through a point lying outside the circle.

The length of the segment of the tangent from an external point 'P' and the point of contact with the circle is called the length of the tangent from the point 'P' to the circle.

Exercise – 2.2

1. (A) 7 cm

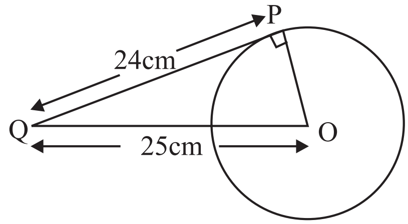
<u>**Justification**</u>

In ΔPOQ,

$\angle QPO = 90°$

... (tangent at any point on the circle is $\perp$ to radius through the point of contact)

$\therefore$ $OQ^2 = OP^2 + PQ^2$.......(by Pythagoras theorem)



$\therefore$ $25^2 = OP^2 + 24^2$

$\therefore$ $625 = OP^2 + 576$

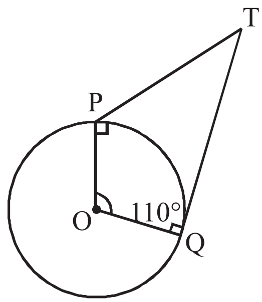
$\therefore$ $OP^2 = 625 - 576$

$\therefore$ $OP^2 = 49$

$\therefore$ $OP = \sqrt{49} = 7$

$\therefore$ **The radius of circle is 7 cm.**

2. (B) 70°



<u>**Justification**</u>

In □OPTQ,

$\angle POQ = 110°$ ... (given)

$\angle OPT = \angle OQT = 90°$

[tangent at any point on the circle is $\perp$ to radius of the circle through the point of contact]

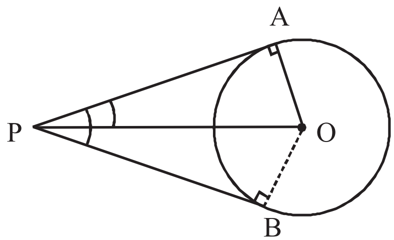
$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360°$ ... (sum of measures of all angles of □ is 360°)

$\therefore$ $110° + 90° + 90° + \angle PTQ = 360°$

$\therefore$ $\angle PTQ = 360° - 290°$

$\therefore$ $\angle$**PTQ = 70°**

3. (A) 50°



<u>**Justification**</u>

In □APBO,

$\angle PAO = \angle PBO = 90°$

(Tangent at any point on a circle is $\perp$ to radius of a circle through that point)

$\angle APB = 80°$ ...(given)

$\therefore$ $\angle AOB + \angle APB + \angle PAO + \angle PBO = 360°$

...(sum of measures of all angles of a □ is 360°)

$\therefore$ $\angle AOB + 80° + 90° + 90° = 360°$

$\therefore$ $\angle AOB = 360° - 260°$

$\therefore \quad \angle AOB = 100°$ ...(I)

In ΔAPO and ΔBPO,

(i) $PA = PB$ ...(length of the tangents drawn from an external point to a circle is equal)

(ii) $OA = OB$ ...(radii of the same circle)

(iii) $OP = OP$ ...(common side)

$\therefore \quad \Delta APO \cong \Delta BPO$ ...(by SSS congruency)

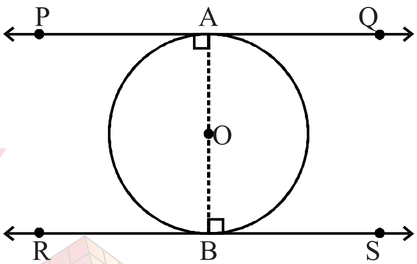
$\therefore \quad \angle POA = \angle POB$ ...(II) ...(cpct)

$\angle AOB = \angle POA + \angle POB$ ...(III) ...(angle addition property)

$\therefore \quad 100 = 2\angle POA$ ... from (I), (II) and (III)

$\therefore \quad \angle POA = \frac{100}{2}$

$\therefore \quad$ **$\angle POA = 50°$**



4. Given:

A circle with centre 'O'

seg AB is the diameter

Lines PQ and RS are tangents to the circle at A and B respectively.

Prove that: line PQ || line RS

Proof: $\angle OAP = \angle OBR = 90°$

.......(tangent at any point on the circle is $\perp$ to radius through the point of contact)

But AB is the diameter of circle. ... (given)

$\therefore$ Points A,O and B are collinear

Hence, $\angle PAB = 90°$ and $\angle RBA = 90°$

$\therefore \quad \angle PAB + \angle RBA = 90° + 90° = 180°$

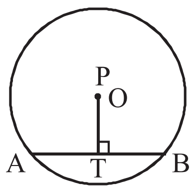
$\therefore$ Interior angles for PQ and RS on transversal AB are supplementary.

$\therefore$ **Line PQ || Line RS** ... (by interior angles test)

5. Given: A circle with centre 'O'. AB is tangent to the circle at T. PT $\perp$ AB.

Prove that: PT passes through centre 'O'

Proof:



OT $\perp$ AB ...(tangent at any point on a circle is $\perp$ to radius of the circle through that point of contact)

But PT $\perp$ AB ... (given)

$\therefore$ OT and PT are $\perp$ to same line AB at same point T on it.

$\therefore$ OT and PT are one and the same line.

....(one and only one $\perp$ can be drawn to a given line at a given point on it.)

$\therefore$ **PT passes through the centre 'O'**

6. OA = 5 cm ... (given)

AB = 4 cm ... (given)

In ΔOBA,

$\angle OBA = 90°$.... (tangent at any point on a circle is $\perp$ to radius of a circle through that point of contact)

$\therefore$ $OA^2 = AB^2 + OB^2$

$\therefore$ $5^2 = 4^2 + OB^2$

$\therefore$ $25 = 16 + OB^2$

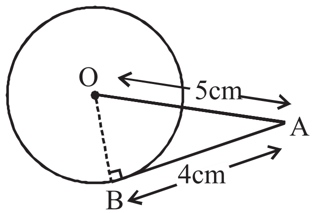
$\therefore$ $25 - 16 = OB^2$

$\therefore$ $9 = OB^2$

$\therefore$ $OB = \sqrt{9}$

$\therefore$ $OB = 3$

$\therefore$ **Radius of the circle = 3 cm**



7. **Given:** Two concentric circles with centre O and radii 5 cm and 3 cm. Chord AB of larger circle touches the smaller circle at P.

To Find: AB

Construction: Join OP and OB.

Solution:

OP = 3 cm ....(radius of smaller circle)

OB = 5 cm ....(radius of larger circle)

In ΔOPB,

$\angle OPB = 90°$ ....(I) ....(tangent at any point on the circle is $\perp$ to radius of the circle through the point of contact)

$\therefore$ $OB^2 = OP^2 + PB^2$ ....(by Pythagoras theorem)

$\therefore$ $5^2 = 3^2 + PB^2$

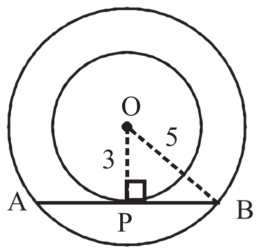
$\therefore$ $25 = 9 + PB^2$

$\therefore$ $PB^2 = 25 - 9$

$\therefore$ $PB^2 = 16$

$\therefore$ $PB = \sqrt{16}$

$\therefore$ PB = 4 cm



$\therefore$ $OP \perp AB$ ....[from (I)]

$\therefore$ $PB = \frac{1}{2}AB$....($\perp$ from centre of circle to the chord bisects the chord)

$\therefore$ $4 = \frac{1}{2}AB$

$\therefore$ **AB = 8 cm**

8. Given: A $\square$ABCD is drawn to circumscribe a circle.

Prove that: AB + CD = AD + BC

Proof:

AP = AS ....(I)

BP = BQ ...(II)

CR = CQ ...(III) $\left\{\begin{array}{l}\text{Length of the tangents drawn from}\\ \text{an external point to the circle is equal}\end{array}\right\}$

DR = DS ...(IV)

AP + BP + CR + DR = AS + BQ + CQ + DS ....adding (I), (II), (III) and (IV)

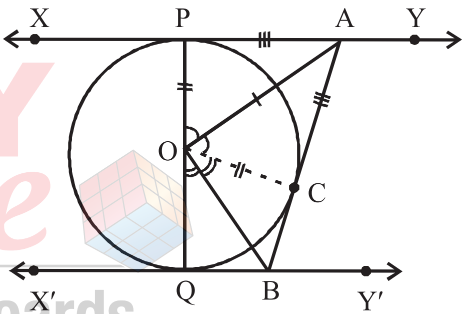
$\therefore$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\therefore$ **AB + CD = AD + BC**

9. Given: XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B.

Prove that: $\angle$AOB = 90°

Construction: Join OC.



Proof:

In ΔOPA and ΔOCA,

OP = OC... (radii of same circle)

OA = OA... (common side)

AP = AC... (length of the tangents drawn from an external point to the circle is equal)

$\therefore$ $\Delta OPA \cong \Delta OCA$ ... (by SSS congruency)

$\therefore$ $\angle POA = \angle COA$ ...(I) ... (cpct)

$\therefore$ $\angle POC = \angle POA + \angle COA$ ...(II) ... (angle addition property)

$\therefore$ $\angle POC \cong 2\angle COA$ ...(III) ... from (I) and (II)

$\therefore$ Similarly, we can prove that

$\therefore$ $\angle QOC \cong 2\ \angle COB$ ...(IV)

$\therefore$ $\angle POC + \angle QOC = 2\ \angle COA + 2\ \angle COB$ ...(V)... adding (III) and (IV)

But, $\angle POC + \angle QOC = 180°$ ...(VI) ... (angles in a linear pair)

$\therefore$ $2\ \angle COA + 2\ \angle COB = 180°$ ... from (V) and (VI)

$\therefore$ $2\ (\angle COA + \angle COB) = 180°$

$\therefore$ $2\ \angle AOB = 180°$ ... ($\because$ $\angle COA + \angle COB = \angle AOB$)

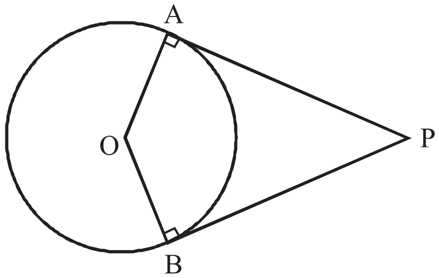
$\therefore$ $\angle AOB = \dfrac{180}{2}$

$\therefore$ $\boldsymbol{\angle AOB = 90°}$

10. Given: A circle with centre 'O'. PA and PB are the tangents drawn from an external point 'P' to the circle.

Prove that: $\angle APB + \angle AOB = 180°$

Proof: $\angle OAP = \angle OBP = 90°$ ...(I) ...(tangent at any point to the circle is $\perp$ to radius of the circle through the point of contact)



In □OAPB,

$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$ ...(II)

...(sum of measures of all angles of □ is 360°)

$\therefore$ $90° + \angle APB + 90° + \angle AOB = 360°$ ...[from (I) and (II)]

$\therefore$ $\angle APB + \angle AOB = 360° - 180°$

$\therefore$ $\boldsymbol{\angle APB + \angle AOB = 180°}$

11. Given: A parallelogram ABCD is circumscribing a circle.

Prove that: □ ABCD is a rhombus.

Proof:

AP = AS ... (I)

BP = BQ ... (II)

CR = CQ ... (III)

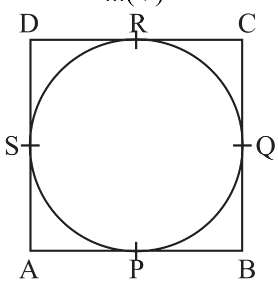
DR = DS ... (IV)

{Length of the tangents drawn from an external point to the circle is equal}

AP + BP + CR + DR = AS + BQ + CQ + DS...adding (I), (II), (III) and (IV)

$\therefore$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\therefore$ AB + CD = AD + BC ...(V)



□ABCD is a parallelogram ...(given)

AB = CD ...(VI) ...(opposite sides of a || are equal)

AD = BC ...(VII)

∴ 2AB = 2BC ...[from (V), (VI) and (VII)]

∴ AB = BC

∴ **□ABCD is a rhombus.** ...(A parallelogram is a rhombus, if its adjacent sides are equal.)

12. **Given:** A ΔABC is drawn to circumscribe a circle of radius 4 cm. seg BD and seg DC are divided by the point of contact D.

BD = 8 cm, CD = 6 cm.

To Find: Sides AB and AC.

Construction: Let Q and R be point of contact of AC and AB respectively. Join point O to points C, Q, A, R and B.

Solution:

CD = CQ = 6 cm
BD = BR = 8 cm
{Length of the tangents drawn from an external point to the circle is equal}

AQ = AR

∴ Let AQ = AR = x cm

BC = BD + CD

∴ BC = 8 + 6 = 14 cm

∴ a = BC

= 14 cm

AC = AQ + CQ

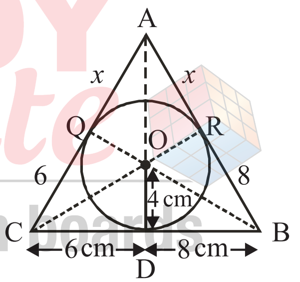
∴ b = AC

= (x + 6) cm

AB = AR + BR

∴ c = AB

= (x + 8) cm



(S) Semi perimeter of ΔABC = $\dfrac{AB + BC + AC}{2}$

$= \dfrac{x + 8 + 14 + x + 6}{2}$

$= \dfrac{(2x + 28)\text{ cm}}{2}$

= (x + 14) cm

Area (ΔABC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)(x+14-14)(x+14-x-6)(x+14-x-8)}$

$= \sqrt{(x+14)(x)(8)(6)}$

$= \sqrt{(48)\,(x)\,(x+14)}$

OD ⊥ BC ......(tangent at any point on the circle is ⊥ to radius of circle through the point of contact)

OD = 4 cm

$A(\Delta OBC) = \frac{1}{2} \times BC \times OD$

$= \frac{1}{2} \times 14 \times 4$

$= 28\ cm^2$

Similarly, we can show that

$ar(\Delta OAC) = \frac{1}{2}\ (x + 6) \times 4$

$ar(\Delta OAB) = \frac{1}{2}\ (x + 8) \times 4$

$ar(\Delta ABC) = ar(\Delta OBC) + ar(\Delta OAC) + ar(\Delta OAB)$

$\sqrt{(48)\times x\times(x+14)} = 28 + \frac{1}{2}(x + 6) \times 4 + \frac{1}{2}(x + 8) \times 4$

$\sqrt{(48)\times x\times(x+14)} = 28 + 2x + 12 + 2x + 16$

$\sqrt{(48)\times x\times(x+14)} = 56 + 4x$

$\sqrt{(48)\times\ x\times\ (x+14)} = 4\ (14 + x)$

$48 \times x \times (x + 14) = 4^2\ (14 + x)^2$ .....(squaring on both sides)

$48 \times x\ \ (x + 14) = 16\ (14 + x)^2$

$3x\ (x + 14) = (14 + x)^2$ .....(dividing by 16)

$3x = 14 + x$ [dividing by $(x + 14)$]

⇒ $2x = 14$

∴ $x = 7$

∴ AC = AQ + CQ

$= x + 6$

$= 7 + 6$

AC = 13 cm.

∴ AB = AR + BR

$= x + 8$

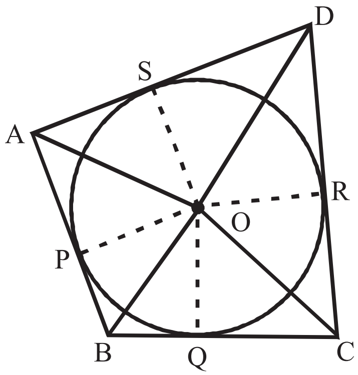
$= 7 + 8$

AB = 15 cm.

13. Given:

(i) A circle with centre O.

(ii) The circle touches the sides AB, BC, CD and AD of □ABCD at points P, Q, R and S respectively.



To Prove:

(i) $\angle AOB + \angle COD = 180°$

(ii) $\angle AOD + \angle BOC = 180°$

Construction: Draw seg OP, seg OQ, seg QR and seg OS.

Proof: In ΔAPO and ΔASO,

$\angle APO = \angle ASO = 90°$ (radius is perpendicular to the tangent)

$OA \cong OA$ (common side)

$OP \cong OS$ (radii of the same circle)

$\therefore$ $\Delta APO \cong \Delta ASO$ (By RHS rule)

$\therefore$ $\angle AOP \cong \angle AOS$ (cpct)

Let, $\angle AOP = \angle AOS = a°$ ....(i)

Similarly,

$\angle BOP = \angle BOQ = b°$ ....(ii)

$\angle COQ = \angle COR = c°$ ....(iii)

$\angle DOR = \angle DOS = d°$ ....(iv)

$\angle AOB + \angle BOC + \angle COD + \angle AOD = 360°$

[$\because$ Sum of the measures of all angles at a point is 360°]

$\therefore$ $a + b + b + c + c + d + a + d = 360$

$\therefore$ $2a + 2b + 2c + 2d = 360$

$\therefore$ $2(a + b + c + d) = 360$

$\therefore$ $(a + b) + (c + d) = 180$

$\therefore$ **$\angle AOB + \angle COD = 180$.**

$(a + d) + (b + c) = 180$

$\therefore$ **$\angle AOD + \angle BOC = 180$.**

TEST YOURSELF – CR 1

1. $3\sqrt{3}$ cm

5. 16 cm

8. 7 cm, 5 cm, 3 cm