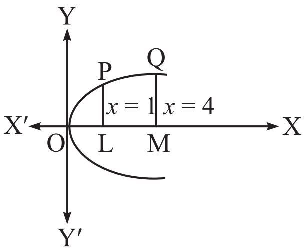
EXERCISE 8.1

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.



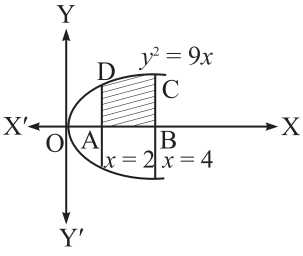
Sol. The curve $y^2 = x$ is a parabola with vertex at the origin. Axis of x is the line of symmetry, which is the axis of the parabola and the area of the region bounded by the curve, $x = 1$, $x = 4$ and the x-axis.

$$= \text{Area LMQP} = \int_1^4 y\,dx = \int_1^4 \sqrt{x}\,dx$$

$$= \frac{2}{3}[x^{3/2}]_1^4 = \frac{2}{3}[4^{3/2} - 1^{3/2}] = \frac{2}{3}[8 - 1] = \frac{14}{3} \text{ sq. units.}$$

2. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and x-axis in the first quadrant.

Sol. The given curve is $y^2 = 9x$, is a parabola with vertex at (0, 0) and axis along the x-axis. It is symmetrical about the x-axis as it contains only even powers of y, $x = 2$ and $x = 4$ are straight lines parallel to y-axis at positive distances of 2 and 4 units from it respectively.



$\therefore$ Required area = area ABCD

$$= \int_2^4 y\,dx = \int_2^4 \sqrt{9x}\,dx = 3\int_2^4 \sqrt{x}\,dx$$

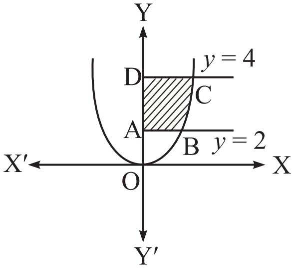
$$[\because y^2 = 9x \Rightarrow y = \pm 3\sqrt{x}, \text{ for the upper portion } y = 3\sqrt{x}\,]$$

$$= \left[\frac{x^{3/2}}{\frac{3}{2}}\right]_2^4 = 3\times\frac{2}{3}[x^{3/2}]_2^4 = 2[4^{3/2} - 2^{3/2}]$$

$= 2[8-2\sqrt{2}] = (16-4\sqrt{2})$ sq. units.

3. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

Sol. The given curve $x^2 = 4y$ is a parabola with a vertex at (0, 0). Since it contains only even powers of x, it is symmetrical about the y-axis; $y = 2$ and $y = 4$ are straight lines parallel to the x-axis at a positive distance of 2 and 4 from it respectively.



Required area = Area ABCD

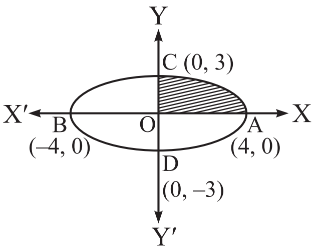
$$= \int_2^4 x\,dx = \int_2^4 2\sqrt{y}\,dy = 2\int_2^4 \sqrt{y}\,dy = 2\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_2^4 = 2\times\frac{2}{3}[4^{3/2} - 2^{3/2}]$$

$$= \frac{4}{3}\left(8-2\sqrt{2}\right) = \left(\frac{32-8\sqrt{2}}{3}\right) \text{sq. units.}$$

4. Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

Sol. The equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$.

The given ellipse is symmetrical about both the axes, as it contains only even powers of y and x.



Now, $\frac{x^2}{16}+\frac{y^2}{9}=1 \Rightarrow \frac{y^2}{9}=1-\frac{x^2}{16} \quad \Rightarrow \quad y^2 = \frac{9}{16}\,(16-x^2)$

$\Rightarrow \quad y = \pm\frac{3}{4}\left(\sqrt{16-x^2}\right)$

Now, area bounded by the ellipse = 4x (area of the ellipse in the first quardant)

$= 4\text{ (area OAC)} = 4\int_0^4 y\,dx = \int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$ [$\because y \geq 0$ in the first quardant]

Put $x = 4\sin\theta$ so that $dx = 4\cos\theta\, d\theta$,

Now when $x = 0$, $\theta = 0$ and when $x = 4$, $\theta = \frac{\pi}{2}$

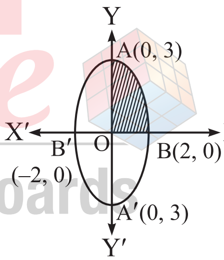
$\therefore$ Required area $= 4\int_0^4 y\,dx$

$= 4\int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$

$= 3\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

5. Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$.

Sol. Given $\frac{x^2}{4}+\frac{y^2}{9}=1 \Rightarrow \frac{y^2}{9}=1-\frac{x^2}{4} \Rightarrow y=\frac{3}{2}\sqrt{4-x^2}$



It is an ellipse centre (0, 0), the length of the semi-major axis = 3 and that of the semi-minor axis = 2.

Its rough sketch is shown in the figure.

$\therefore$ Area bounded by the ellipse

= 4 × area of the region AOB

$= 4\int_0^2 \frac{3}{2}\sqrt{4-x^2}\,dx = 6\int_0^2 \sqrt{4-x^2}\,dx = 6\left[\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$= 3[0 + 8\sin^{-1}1 - 0 - \sin^{-1}0] = 3\left[8\times\frac{\pi}{2}\right] = 12\pi$

$= 6\,[(0 + 2\sin^{-1}1) - (0 + \sin^{-1}(0))] = 6\left[2\times\frac{\pi}{2}\right] = 6\pi$ sq. units.

6. Find the area of the region in the first quadrant enclosed by x-axis, line $x=\sqrt{3}\,y$ and the circle $x^2 + y^2 = 4$.

Sol. Consider the two equations $x^2+y^2=4$ ...(i)

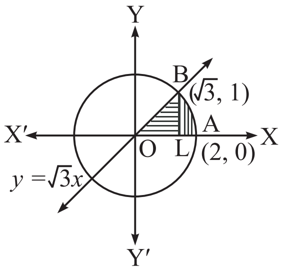
and $x=\sqrt{3}y$. That is $y=\frac{1}{\sqrt{3}}x$ ...(ii)

(1) $x^2+y^2=4$ is a circle with centre O (0, 0) and radius = 4.

(2) $y = \frac{1}{\sqrt{3}}x$ is a straight line passing through (0, 0) and intersecting the circle at $B(\sqrt{3}, 1)$.

The required area = shaded region,

= area OBL + area LBA



$$= \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} x\,dx + \int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{2\sqrt{3}}(3-0) + \left[\left(0 - \frac{\sqrt{3}}{2}\right) + 2\left(\sin^{-1}1 - \sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

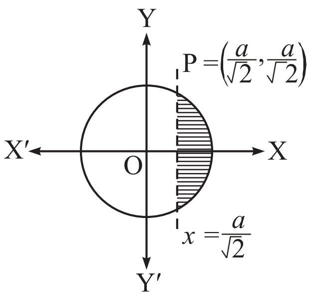
$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq. units.}$$

7. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

Sol. The equations for the given curve are $x^2 + y^2 = a^2$ ...(i)

and $x = \frac{a}{\sqrt{2}}$ ...(ii)

Clearly, (i) represents a circle and (ii) is the equation of a straight line parallel to the y-axis at a distance $\frac{a}{\sqrt{2}}$ units to the right of y-axis.



Solving (i) and (ii), we get $\frac{a^2}{2} + y^2 = a^2 \Rightarrow y^2 = \frac{a^2}{2} \Rightarrow y = \frac{a}{\sqrt{2}}$

$\therefore$ $P\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ is the point of intersection of the curve (1) and (2) in the first quardant.

The smaller region bounded by these two curves is the shaded portion shown in the figure.

$\therefore$ Required Area = Shaded region

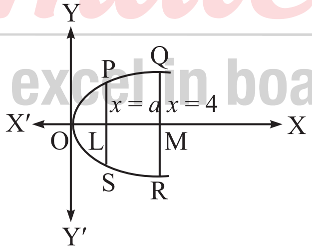
$$= 2\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\,dx = 2\left[\frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$$

$$= \left[\left(0 - \frac{a}{\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}}\right) + a^2\left(\sin^{-1}1 - \sin\frac{1}{\sqrt{2}}\right)\right] = \left[-\frac{a^2}{2} + a^2 + \frac{\pi}{2} - a^2 \times \frac{\pi}{4}\right]$$

$$= \frac{\pi a^2}{4} - \frac{a^2}{2} = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right) \text{ sq. units.}$$

8. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.

Sol. Graph of the curve $x = y^2$ is a parabola as given in the figure. Its vertex is O and the axis is x-axis. QR is the ordinate along $x = 4$.



$\therefore$ Area of the region,

$$A_1 = 2\int_0^4 y\,dx = 2\int_0^4 \sqrt{x}\,dx = 2 \times \frac{2}{3}[x^{3/2}]_0^4 = \frac{4}{3}\left[(4)^{3/2} - 0\right] = \frac{32}{3} \text{ sq. units.}$$

Area of the region bounded by the curve and ordinate $x = a$ is

$$A_2 = 2\int_0^a y\,dx = 2\int_0^a \sqrt{x}\,dx$$

$$= 2 \cdot \frac{2}{3}[x^{3/2}]_0^a = \frac{4}{3} \cdot (a^{3/2} - 0) = \frac{4}{3} \cdot a^{3/2}$$

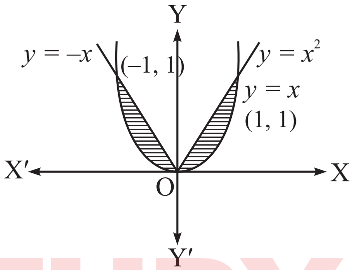
Now, PS $[x = a]$ divides the area of the region ORQ into two equal parts.

$\therefore$ Area of the region ORQ = 2 Area of the region OPS, $\quad\therefore\quad A_1 = 2\,A_2$

$\therefore\ \frac{32}{3} = 2 \cdot \frac{4}{3} a^{\frac{3}{2}}$ $\quad \therefore\ a^{\frac{3}{2}} = 4$ $\quad \therefore a = 4^{\frac{2}{3}}$ units.

9. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

Sol. Clearly $x^2 = y$ represents a parabola with a vertex at (0, 0), positive direction of the y-axis as its axis opens upwards.



$y = |x|$. That is $y = x$ and $y = -x$ represent two lines passing through the origin and making an angle of 45º and 135º with the positive direction of the x-axis.

The required region is the shaded region as shown in the figure. Since both the curves are symmetrical about the y-axis, required area = 2 (shaded area in the first quardant)

$$= 2\int_0^1 (x - x^2)\,dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = 2 \times \frac{1}{6} = \frac{1}{3} \text{ sq. units.}$$

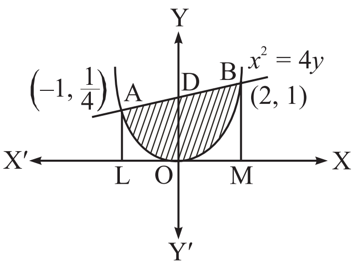
10. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Sol. The given curve is $x^2 = 4y$ ...(i)

which is an upward parabola with a vertex at (0, 0) and is symmetrical about the y-axis.

Equation of the line is $x = 4y - 2$ ...(ii)

Solving (i) and (ii) simultaneously, we get $(4y - 2)^2 = 4y$



$\Rightarrow 16y^2 - 16y + 4 = 4y$

$\Rightarrow 16y^2 - 20y + 4 = 0$

$\Rightarrow 4y^2 - 5y + 1 = 0$

$\Rightarrow (4y - 1)(y - 1) = 0$

$\Rightarrow y = \frac{1}{4}, y = 1$

From (2), when $y = \frac{1}{4}$, $x = 1 - 2 = -1$, when $y = 1, x = 4 - 2 = 2$

$\therefore$ A$\left(-1, \frac{1}{4}\right)$ and B (2, 1) are the points of intersection of (1) and (2). Clearly, from the figures the shaded portion OBDAO is the region of the required area.

Required area = area LOMBDA – area LMBOAL ...(iii)

Now area LMBDA = $\int_{-1}^{2} y\,dx$, (where $x = 4y - 2$)

$\Rightarrow y = \frac{1}{4}(x + 2) = \frac{1}{4}\int_{-1}^{2}(x+2)\,dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2}$

$= \frac{1}{4}\left[(2+4) - \left(\frac{1}{2} - 2\right)\right] = \frac{1}{4}\left(6 + \frac{3}{2}\right) = \frac{15}{8}$ sq. units ...(iv)

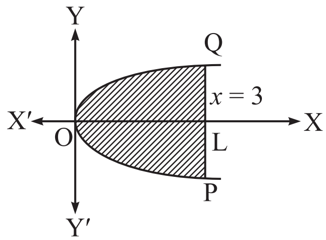
and area LMBOAL = $\int_{-1}^{2} y\,dx$, where $x^2 = 4y$

$= \frac{1}{4}\int_{-1}^{2} x^2 dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2} = \frac{1}{12}[8 - (-1)] = \frac{9}{12} = \frac{3}{4}$ sq. units. ...(v)

From (3) (4) and (5), the required area, $= \frac{15}{8} - \frac{3}{4} = \frac{9}{8}$ sq. units.

11. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

Sol. The curve $y^2 = 4x$ is a parabola as shown in the figure.



Axis of the parabola is the x-axis. The area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$ is A

= Area of region PQO = 2 Area of the region OLQ.

Note: Areas below and above the x–axis are equal.

$$= 2\int_0^3 y\,dx = 2\int_0^3 2\sqrt{x}\,dx = 4\frac{2}{3}\left(x^{\frac{3}{2}}\right)_0^3 = \frac{8}{3}\cdot 3^{\frac{3}{2}} = \frac{8}{3}\cdot\sqrt{27} = 8\sqrt{3} \text{ sq. units}$$

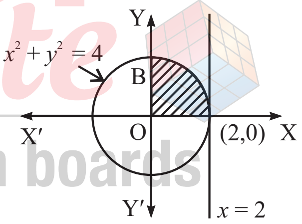
12. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Sol. The area bounded by the circle and the lines $x = 0$ and $x = 2$ in the first quadrant is represented in the figure by shaded region required area

$$= \int_0^2 |y|\,dx = \int_0^2 \sqrt{4 - x^2}\,dx$$



$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$= 0 + 2\sin^{-1}(1) - 0$

$$= 2\times\frac{\pi}{2} = \pi \text{ sq. unit}$$

Thus, the correct option is (a)

13. Area of the regions bounded by the curve $y^2 = 4x$. Y-axis and the line $y = 3$ is

(a) 2 (b) $\frac{9}{4}$

(c) $\frac{9}{3}$ (d) $\frac{9}{2}$

Sol. The area bounded by the curve $y^2 = 4x$. Y-axis and $y = 3$ is represented in the figure by shaded region.

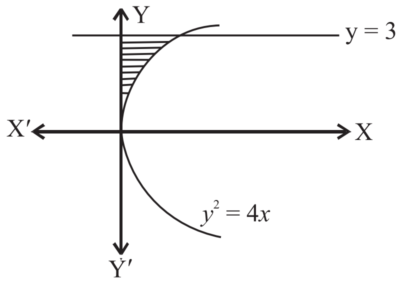
$$\text{Required area} = \int_0^3 |x|\,dy$$

$$= \int_0^3 \frac{y^2}{4}\,dy$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{12}\left(3^3 - 0\right) = \frac{1}{12} \times 27$$

$$= \frac{9}{4} \text{ sq. units}$$

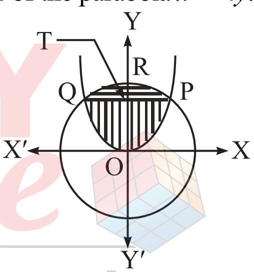
Thus the correct option is (b)



EXERCISE 8.2

1. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Sol. Area is bounded by the circle $4x^2 + 4y^2 = 9$ and interior of the parabola $x^2 = 4y$.



Putting $x^2 = 4y$ in $x^2 + y^2 = \frac{9}{4}$

we get $4y + y^2 = \frac{9}{4}$ or $4y^2 + 16y - 9 = 0$,

$(2y + 9)(2y - 1) = 0; \quad y = \frac{1}{2}, -\frac{9}{2}; \quad y \neq -\frac{9}{2}$

$\therefore y = \frac{1}{2}$, radius of the circle $= \frac{3}{2}$

Area of the region required = Area of the region OPRQ

= [(area of the region OPQ) + (area of the region PQR)]

= 2 × area of the region OPT + 2 × area of the region TPQ

$$= 2 \cdot 2\int_0^{\frac{1}{2}} \sqrt{y}\,dy + 2\int_{1/2}^{3/2} \sqrt{\frac{9 - 4y^2}{4}}\,dy \qquad \left[\because x^2 + y^2 = \frac{9}{4}\right]$$

$$= 4\int_0^{1/2} \sqrt{y}\,dy + 2\int_{1/2}^{3/2} \sqrt{\frac{9}{4} - y^2}\,dy$$

$$= 4 \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^{1/2} + 2\left[\frac{y}{2}\sqrt{\frac{9}{4} - y^2} + \frac{9}{8}\sin^{-1}\left(\frac{y}{3/2}\right)\right]_{1/2}^{3/2}$$

$$= \frac{8}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} + 2\left[\frac{y}{2}\sqrt{\frac{9}{4} - y^2} + \frac{9}{8}\sin^{-1}\frac{2y}{3}\right]_{1/2}^{3/2}$$

$$= \frac{8}{3} \times \frac{1}{2\sqrt{2}} + 2\left[\left(0 + \frac{9}{8}\sin^{-1} 1\right) - \left(\frac{1}{4}\sqrt{2} + \frac{9}{8}\sin^{-1}\frac{2}{3} \times \frac{1}{2}\right)\right]$$

$$= \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{2} + \frac{9}{4}\left(\sin^{-1}1 - \sin^{-1}\frac{1}{3}\right) = \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(1 \cdot \frac{2\sqrt{2}}{3} - 0\right)$$

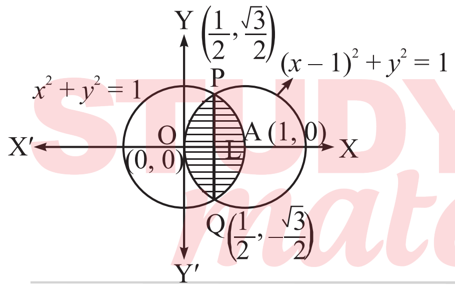
$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3} \text{ sq. units}$$

2. Find the area bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

Sol. The given circles are $x^2 + y^2 = 1$ ...(i)

and $(x - 1)^2 + y^2 = 1$ ...(ii)

Centre of (1) is O (0, 0) and radius =1



Both these circles are symmetrical about the x-axis solving (i) and (ii) we get, $-2x + 1 = 0 \Rightarrow x = \frac{1}{2}$

Then $y^2 = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow y = \frac{\sqrt{3}}{2}$

$\therefore$ The points of intersection are $P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

It is clear from the figure that the shaded portion is the region whose area is required.

$\therefore$ Required area = area OQAPO

= 2 × area of the region OLAP

= 2 × (area of the region OLPO + area of LAPL)

$$= 2\left[\int_0^{1/2} \sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1} \sqrt{1-x^2}\,dx\right]$$

$$= 2\left[\frac{(x-1)\sqrt{1-(x-1)^2}}{2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{1/2}+2\left[\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\sin^{-1}x\right]_{1/2}^{1}$$

$$= -\frac{1}{2}\cdot\frac{\sqrt{3}}{2}+\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(-1) + 0 + \sin^{-1}(1)\left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2}+\sin^{-1}\left(\frac{1}{2}\right)\right)$$

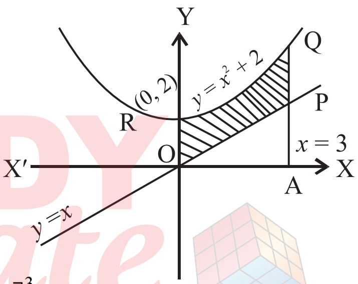
$$= \frac{\sqrt{3}}{4}-\frac{\pi}{6}-\left(-\frac{\pi}{2}\right)+\frac{\pi}{2}-\frac{\sqrt{3}}{4}-\frac{\pi}{6}=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right) \text{ sq. units.}$$

3. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

Sol. Equation of the parabola is $y = x^2 + 2$ or $x^2 = (y - 2)$

its vertex is (0, 2) and axis is the y-axis.

Boundary lines are $y = x$, $x = 0$, $x = 3$. Graphs of the curve and lines have been shown in the figure.



Area of shaded region PQRO

= Area of the region OAQR

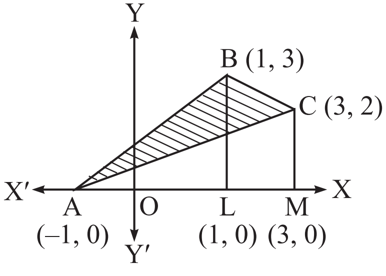
– Area of region OAP

$$= \int_0^3 (x^2+2)\,dx - \int_0^3 x\,dx = \left[\frac{x^3}{3}+2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$$

$$= \left[\left(\frac{27}{3}+6\right)-0\right]-\left(\frac{9}{2}-0\right)=15-\frac{9}{2}=\frac{21}{2} \text{ sq. units.}$$

4. Using integration find the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

Sol. The points A(–1, 0), B(1, 3) and C(3, 2) are plotted and joined. Area of ΔABC



= Area of ΔABL + Area of trapezium BLMC – Area of ΔACM ...(i)

The equation of the line joining the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

The equation of AB, $y - 0 = \frac{3-0}{1+1}(x+1)$ or $y = \frac{3}{2}(x+1)$ ...(ii)

Equation of BC, $y - 3 = \frac{2-3}{3-1}(x-1) = -\frac{1}{2}(x-1) = -\frac{1}{2}x + \frac{1}{2}$

$y = -\frac{1}{2}x + \frac{1}{2} + 3$ or $y = -\frac{x}{2} + \frac{7}{2}$

or $y = \frac{7-x}{2}$ ...(iii)

Equation of CA, $y - 0 = \frac{2-0}{3+1}(x+1) = \frac{1}{2}(x+1)$

or $y = \frac{x+1}{2}$ ...(iv)

$\therefore$ The area of ΔABL $= \int_{-1}^{1} \frac{3}{2}(x+1)\,dx$, [area bounded by AB, $y = \frac{3}{2}(x+1)$, x-axis and $x = 1$].

Area of ΔABL $= \int_{-1}^{1} \frac{3}{2}(x+1)\,dx = \frac{3}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1} = \frac{3}{2}\left[\left(\frac{1}{2} - \frac{1}{2}\right) + 1 + 1\right] = 3$

Area of the trapezium BCML = Area bounded by AC, x-axis, $x = 1$, $x = 3$

$= \int_{1}^{3} \frac{7-x}{2}\,dx = \left[\frac{7}{2}x - \frac{x^2}{4}\right]_{1}^{3}$ $\left[y - \frac{7-x}{2} \text{ from (iii)}\right]$

$= \frac{7}{2}(3-1) - \frac{1}{4}(9-1) = 7 - 2 = 5$

Area of ΔACM = (area bounded by AC, $y = \frac{x+1}{2}$ x-axis and $x = 3$)

$= \int_{-1}^{3} \frac{x+1}{2}\,dx = \frac{1}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{3} = \frac{1}{2}\left[\frac{9-1}{2} + (3+1)\right] = \frac{1}{2}(4+4) = 4$, from (iv)

$\therefore$ The area of ΔABC = 3 + 5 – 4 = 4 sq. units.

5. Using integration find the area of the triangular region whose sides have the equations

$y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Sol. The given lines are

$y = 2x + 1$ ...(i)

$y = 3x + 1$ ...(ii)

$x = 4$ ...(iii)

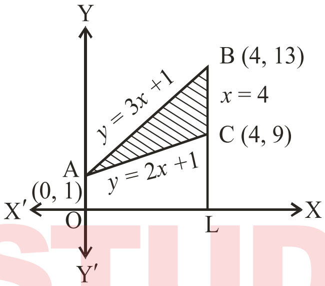
Subtracting (i) from Eq. (ii) we get $0 = x$. That is $x = 0$, putting $x = 0$ in Eq. (i), $y = 1$

$\therefore$ Lines (ii) and (i) intersect at A (0, 1), putting $x = 4$ in Eq. (ii)

$y = 12 + 1 = 13$

The lines (ii) and (iii) intersect at B (4, 13), putting $x = 4$ in Eq. (i)

$y = 8 + 1 = 9$



$\therefore$ Lines (1) and (2) intersect at C (4, 9), area of ΔABC

= Area of the region bounded by AB: $y = 3x + 1$, $x = 0$ $y = 0$ and $x = 4$

$$= \int_0^4 (3x+1)dx = \left[3\frac{x^2}{2} + x\right]_0^4 = \left(\frac{48}{2} - 0\right) + (4 - 0) = 24 + 4 = 28 \text{ sq. units}$$

Area of the trapezium OLCA

= area of the region bounded by AC: $y = 2x + 1$, $x = 0$, $x = 4$, $y = 0$

$$= \int_0^4 (2x+1)\,dx = [x^2 + x]_0^4 = (16 - 0) + (4 - 0) = 20.$$

Putting these values in (4)

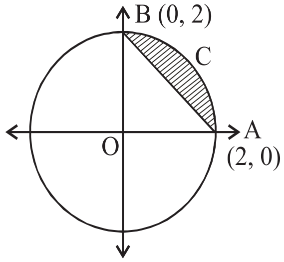
area of ΔABC $= 28 - 20 = 8$ sq. units.

6. Smaller area bounded by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$

(a) $2(\pi - 2)$ (b) $\pi - 2$

(c) $2\pi - 1$ (d) $2(\pi + 2)$

Sol. A circle of radius 2 and centre at O is drawn. The line AB : $x + y = 2$ is passed through (2, 0) and (0, 2). Area of the region ACB



= area of the quadrant OAB – area of ΔOAB ...(i)

Now $x^2 + y^2 = 4$

$\therefore\ y^2 = 4 - x^2$

or $y = \sqrt{4 - x^2}$

$\therefore$ Area of the quadrant OAB

$= \int_0^2 y\,dx = \int_0^2 \sqrt{4 - x^2}\,dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 \quad \left[\because \int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= \left[0 + 2\left(\sin^{-1}\frac{2}{2} - 0\right)\right] = 2 \times \frac{\pi}{2} = \pi$ sq. units

Area of the ΔOAB = area of the region bounded by AB. AB: $x + y = 2$ or $y = 2 - x$ and $x = 0$, $y = 0$

$= \int_0^2 (2 - x)\,dx = \left[2x - \frac{x^2}{2}\right]_0^2 = \left(4 - \frac{4}{2}\right) - (0) = 2$

Putting these values in (i),

area of the region ACB = $\pi - 2$

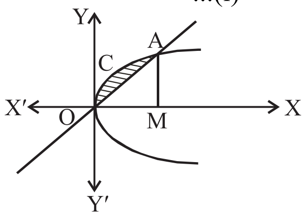
$\therefore$ Option (b) is correct.

7. Area lying between the curves $y^2 = 4x$ and $y = 2x$.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Sol. The curve is $y^2 = 4x$ ...(i)



and the line is $y = 2x$ ...(ii)

From (i) and (ii)

$(4x^2) = 4x$

$\therefore\ x(x-1) = 0$

$x = 0, x = 1$

Curves (1) and (2) intersect at O (0, 0), A (1, 2)

Area of the region OACO = area of the region OMACO – area of OMA ...(iii)

Now, area of the region OMACO = area of the region bounded by the centre OCA

$y = \sqrt{4x}$ x-axis and $x = 1$,

$= \int_0^1 \sqrt{4x}\,dx = 2\cdot\frac{2}{3}[x^{3/2}]_0^1 = \frac{4}{3}\times 1 = \frac{4}{3}$ sq. units.

Area of ΔOMA = area of the region bounded by OA : $y = 2x$, $x = 1$ and x-axis.

$= \int_0^1 2x\,dx = [x^2]_0^1 = 1$

Putting these values in (iii),

$\therefore$ area of region OMACO

$= \frac{4}{3} - 1 = \frac{1}{3}$ sq. units.

$\therefore$ Option (b) is correct.