NCERT TEXTUAL EXERCISE (SOLVED)

EXERCISE – 3.1

1. Fill in the blanks.

(i) The centre of a circle lies in __________ of the circle. (exterior/interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in __________ of the circle. (exterior/interior)

(iii) The longest chord of a circle is a __________ of the circle. (diameter/secant)

(iv) An arc is a __________ when its ends are the ends of a diameter. (semi circle/circle)

(v) Segment of a circle is the region between an arc and __________ of the circle. (chord/radius)

(vi) A circle divides the plane, on which it lies, in __________ parts. (four/three)

2. Write True or False and give reasons for your answer.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius is a diameter of the circle.

(v) A sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

TEST YOURSELF – CR 1

1. Fill in the blanks.

(i) A diameter of circle is a chord that __________ the centre.

(ii) A radius of a circle is a line segment with one end-point __________ and the other end __________

(iii) If we join any two points of a circle by a line segment, we obtain a __________ of the circle.

(iv) Any part of a circle is called an __________ of the circle.

(v) The figure bounded by an arc and the two radii joining the end-points of the arc and the centre, is called a __________ of the circle.

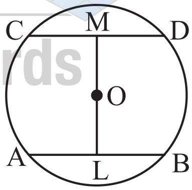
2. Write True or False and give reasons for your answer.
   (i) Each radius of a circle is also a chord of the circle.
   (ii) Each diameter of a circle is also a chord of the circle.
   (iii) The centre of a circle bisects each chord of the circle.
   (iv) A segment is the region between the chord and its corresponding arc.
   (v) A chord of a circle is a segment having its end-points on the circle.

EXERCISE – 3.2

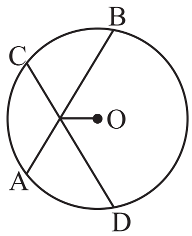
1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.
2. Prove that if chords of congruent circles subtend equal angles at their centres, the chords are equal.

TEST YOURSELF – CR 2

1. If two arcs of a circle are congruent, their corresponding chords are equal.
2. If two chords of a circle are equal, their corresponding arcs are congruent.
3. Let L and M be the middle points of two parallel chords AB and CD of a circle. Prove that the line LM passes through the centre of the circle.



4. Prove that diameter of a circle perpendicular to one of the parallel chords of a circle is perpendicular to the other and bisects it.
5. Prove that the equal chords of a circle subtend equal angles at the centre.
6. Let AB and BC be two chords of a circle whose centre is O. If ∠ABO = ∠CBO, prove that AB = BC.
7. In the figure O is the centre of the circle and PO bisects ∠BPD. Prove that AB = CD.
   [**Hint:** Draw OM ⊥ AB and ON ⊥ CD.]



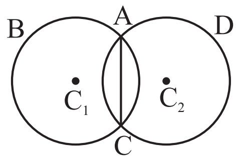
8. In a circle, the chord PQ is bisected by the diameter AB. If AQ || PB, show that PQ is also the diameter of the circle.
9. If the line joining the midpoints of two chords of a circle passes through the centre of the circle, prove that these chords are parallel.
10. The diameter PQ of a circle bisects two chords AB and CD at two points M and N respectively. Prove that AB || CD.

EXERCISE – 3.3

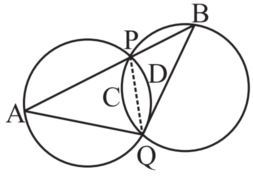
1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?
2. Suppose you are given a circle. Give a construction to find its centre.
3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

TEST YOURSELF – CR 3

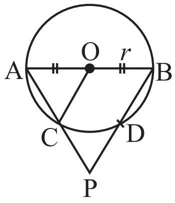
1. A line segment AB = 5 cm. Draw a circle of radius 4 cm passing through points A and B. Can you draw a circle of radius 2 cm passing through A and B? Give reasons in support of your answer.
2. Prove that two different circles cannot intersect each other at more than two points.
3. Two equal circles intersect each other at A and C. Find the ratio of major arc ABC and the major arc ADC.



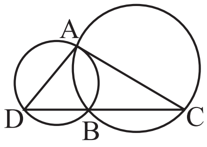
4. Let AB and CD are two chords of a circle on the opposite sides of the centre such that AB = 10 cm, CD = 24 cm and AB || CD; the distance between AB and CD is 17 cm. Find the radius of the circle.
5. Let the ΔABC be right angled at B. On AC, a point D is taken so that AD = DC and AB = BD. Find ∠CAB.
6. Two equal circles intersect at P and Q. A straight line passing through P meets the circles at A and B. Prove that QA = QB. [**Hint:** Join PQ.]



7. AB is a diameter of the circle C(O, r). Chord CD is equal to the radius of the circle. AC and BD when produced meet at a point P. Prove that $\angle APB = 60°$.



8. Two circles intersect at two points A and B, and AD and AC are respective diameters of the circles as given in the figure. Prove that D, B and C are collinear.



9. If the chord of a circle is equal to its radius, find the angles subtended by the chord at a point on the major arc and also on the minor arc.

10. Prove that the circle drawn on any one of equal sides of an isosceles triangle as diameter bisects the base.

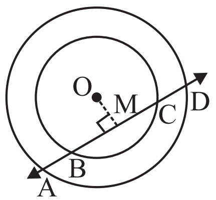
EXERCISE – 3.4

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure).



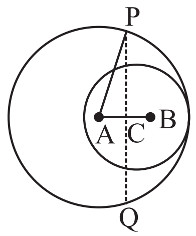
5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. The distance between Salma and Mandip and Reshma and Salma is 6 m, what is the distance between Reshma and Mandip?

6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

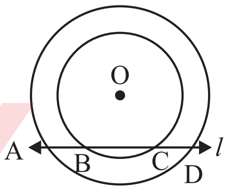
TEST YOURSELF – CR 4

1. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find the distance between their centres.

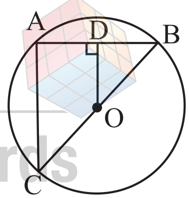
2. Two circles with centres A and B and of radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of segment AB meets the bigger circle in P and Q, find the length Of PQ.



3. Two concentric circles with centre O having A, B, C and D as the points of intersection with the line l as shown. If AD = 12 cm and BC = 8 cm, find the lengths of AB, CD, AC and BD.

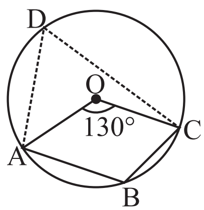


4. In the given figure, OD is perpendicular to the chord AB of a circle whose centre is O. If BC is a diameter, show that CA = 2OD.

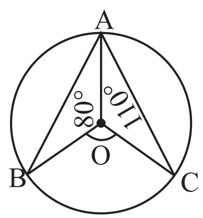


5. Let AB and CD are two chords of a circle such that AB = 6 cm, CD = 12 cm and AB || CD. If the distance between AB and CD is 3 cm, find the radius of the circle.

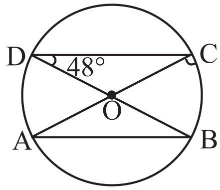
6. In the given figure, find the measure of ∠ABC.



7. (i) Let O is the centre of circle. ∠AOB = 80° and ∠AOC = 110°, find ∠BAC.

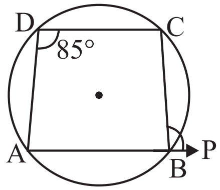


(ii) Let O is the centre of circle and $\angle BDC = 48°$, find $\angle ACB$.

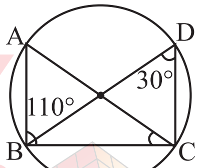


8. Find the values of the following.

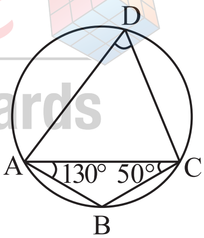
(i) If $\angle ADC = 85°$, find $\angle CBP$.



(ii) If $\angle BDC = 30°$ and $\angle ABC = 110°$, find $\angle BCA$.



(iii) If $\angle BAC = 30°$ and $\angle BCA = 50°$, find $\angle ADC$.

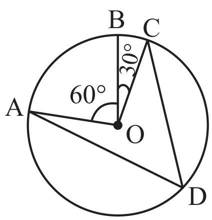


9. In ΔABC, $BE \perp AC$ and $CF \perp AB$ which intersect each other at P. If $\angle A = 40°$, find $\angle BPC$.

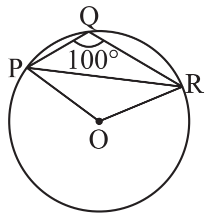
EXERCISE – 3.5

1. In figure, A, B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $AOB = 60°$. If D be a point on the circle other than the arc ABC, find $\angle ADC$.

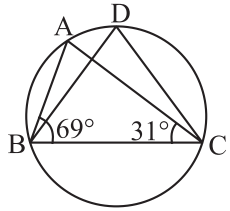
2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



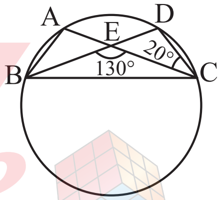
3. In figure, ∠PQR = 100º, where P, Q and R are points on a circle with centre O. Find ∠OPR.



4. In figure, ∠ABC = 69º and ∠ACB = 31º, find ∠BDC.



5. In figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130º and ∠ECD = 20º. Find ∠BAC.

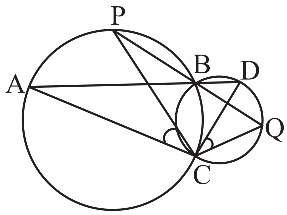


6. Let ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70º and ∠BAC is 30º, find ∠BCD. Further, if AB = BC, find ∠ECD.

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A and D, and P and Q respectively (see figure). Prove that ∠ACP = ∠QCD.



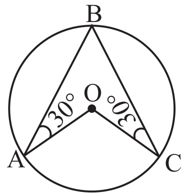
10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

11. Let ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.

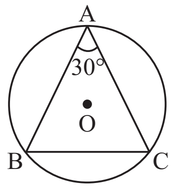
12. Prove that a cyclic parallelogram is a rectangle.

TEST YOURSELF – CR 5

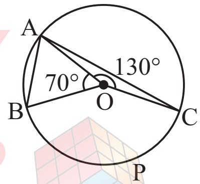
1. Calculate the measure of ∠AOC.



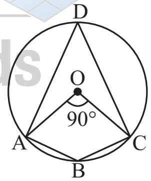
2. Let ABC is a triangle in which ∠BAC = 30º. Prove that BC is the radius of the circle passing through the vertices, whose centre is O.



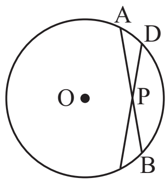
3. Angles subtended by the chords AB and AC of a circle whose centre is O are 70º and 130º respectively. Find ∠BAC and degree measure of arc BPC.



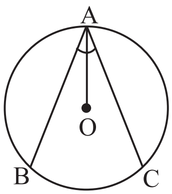
4. Let O be the centre of a circle and the measure of arc ABC is 90º. Determine ∠ADC and ∠ABC.



5. Let AB and CD be two chords of a circle, intersecting each other at P such that AP = CP. Show that AB = CD.

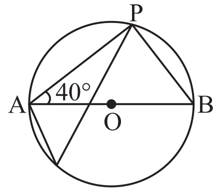


6. Let O be the centre of the circle and BO the bisector of ∠ABC. Show that AB = AC.



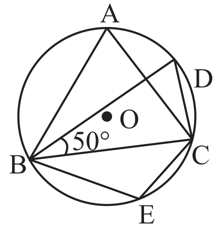
NCERT TEXTUAL EXERCISE (SOLVED)

7. Let AB be the diameter of the circle such that $\angle PAB = 40^\circ$. Find $\angle PCA$.



8. Prove that a cyclic trapezium is isosceles and its diagonals are equal.

9. Let ΔABC be an isosceles triangle with AB = AC and $\angle ABC = 50^\circ$. Find $\angle BDC$ and $\angle BEC$.



10. Let ABCD is a parallelogram. The circle through A, B and C intersects CD produced at E, prove that AE = AD.

EXERCISE – 3.6

OPTIONAL

1. Prove that the line joining centres of two intersecting circles subtends equal angles at the two points of intersection.
2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on the opposite sides of its centre. If the distance between AB and CD be 6 cm, find the radius of the circle.
3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what will be the distance of the other chord from the centre.
4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE within the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.
5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.
6. Let ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

7. If AC and BD be chords of a circle which bisect each other, prove that
(i) AC and BD are diameters (ii) ABCD is a rectangle.

8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^\circ - \frac{1}{2}A$, $90^\circ - \frac{1}{2}B$ and $90^\circ - \frac{1}{2}C$.

9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P and Q lie on the two circles. Prove that BP = BQ.

10. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

ANSWERS

NCERT Exercises and Assignments

Exercise – 3.1

1. (i) interior — (ii) exterior

(iii) diameter — (iv) semicircle

(v) chord — (vi) three parts

2. (i) **True:** As radius is used in two senses—a line segment and a length.

(ii) **False:** As an infinite number of points lie on the circle so we get infinite number of chord on joining any two points.

(iii) **False:** Since these arcs are equal in length so they are called equal arc.

(iv) **True:** As diameter is the longest chord and its length is twice the radius of the circle.

(v) **False:** As the region between an arc and the two radii, joining the centre to endpoints of the arc is known as a sector.

(vi) **True:** Since a circle is the locus of a point which moves in a plane in such a way that its distance from a given fixed point in the plane is always constant. So, a circle lies in a plane.

TEST YOURSELF – CR 1

1. (i) passes through — **(ii)** at the centre, on the circle

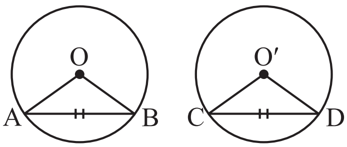
2. (i) False — **(ii)** True

(iii) False — **(iv)** True

(v) True

Exercise – 3.2

1. **Given:** AB and CD are two equal chords of a circles with centres at O and O′ respectively.



To prove: ∠AOB = ∠COD.

Proof: In Δs AOB and CO′D, we have

AO = CO′ [radii of the congruent circles]

BO = DO′ [radii of the congruent circles]

and, AB = CD [given]

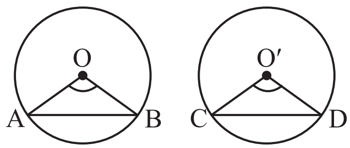
∴ By SSS criterion of congruence, we have

ΔAOB = ΔCO′D

∴ ∠AOB = ∠CO′D [CPCT]

2. **Given:** AB and CD are two chords such that angles subtended by these chords at the centres of the circles O and O′ respectively are equal.

i.e. ∠AOB = ∠CO′D



To prove: AB = CD

Proof: In Δs AOB and CO′D, we have

AO = CO′ [radii of the congruent circles]

BO = DO′ [radii of the congruent circles]

and, ∠AOB = ∠CO′D [given]

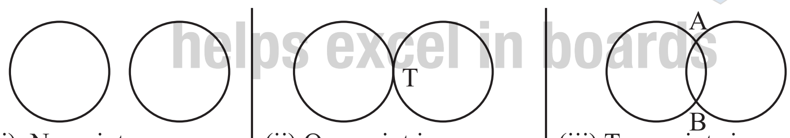
∴ By SAS criterion of congruence, we have

ΔAOB = ΔCO′D

∴ AB = CD [CPCT]

Exercise – 3.3

1.



(i) No point common | (ii) One point in common | (iii) Two points in common

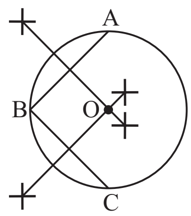
∴ Maximum number of common points is two.

2. **Steps of construction**

(i) Take 3 points A, B and C on the circumference of the circle.

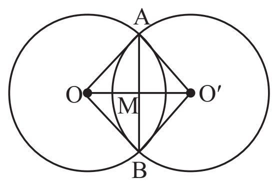
(ii) Join AB and BC.

(iii) Draw the perpendicular bisectors of AB and BC, which intersect each other at O. Then, O is the centre of the circle.



3. **Given:** Two circles, with centres O and O′ intersect at two points A and B so that AB is the common chord of the two circles and OO′ is the line segment joining the centres of the two circles. Let OO′ intersects AB at M.

To prove: OO′ is the perpendicular bisector of AB.



Construction: Draw line segments OA, OB, O′A and O′B.

Proof: In Δs OAO′ and OBO′, we have

OA = OB (radii of the same circle)

O′A = O′B (radii of the same circle)

and, OO′ = OO′ (common)

By SSS criterion of congruence, we have

$\Delta OAO' \cong \Delta OBO'$

$\therefore$ $\angle AOO = \angle BOO'$ (CPCT)

$\therefore$ $\angle AOM = \angle BOM$ ...(i) [$\therefore$ AOO′ = AOM and BOM = BOO′]

In Δs AOM and BOM, we have

OA = OB [radii of the same circle]

$\angle AOM = \angle BOM$ [from (i)]

and, OM = OM [common]

$\therefore$ By SAS criterion of congruence, we have

$\Delta AOM \cong \Delta BOM$

AM = BM and $\angle AMO = \angle BMO$

But $\angle AMO + \angle BMO = 180^\circ$ [linear pair]

$2\angle AMO = 180^\circ$

$\angle AMO = 90^\circ$

Thus, AM = BM and $\angle AMO = \angle BMO = 90^\circ$

Hence, OO′ is the perpendicular bisector of AB.

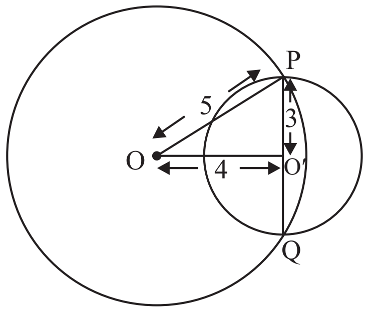
TEST YOURSELF – CR 3

1. A circle of radius 2 cm cannot be drawn as the diameter in this case will be 4 cm which is less than 5 cm.
3. 1 : 1
4. 13 cm
5. 60º
9. 30º and 150º

Exercise – 3.4

1. Let O and O′ be the centres of the circles of radii 5 cm and 3 cm respectively and let PQ be their common chord.

   We have OP = 5 cm and O′P = 3 cm



$\therefore$ By Pythagoras theorem, we have

OO' = 4 cm

As it satisfies Pythagoras theorem,

$\therefore$ $\angle PO'O = 90°$

$\therefore$ $OO' \perp PQ$

$\therefore$ It will bisect the chord.

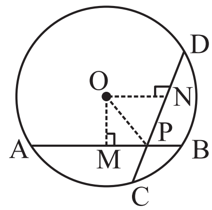
$\therefore$ PQ = 2PO'

= 6 cm

2. **Given:** AB and CD are chords of a circle with centre O. AB and CD intersect at P and AB = CD.

To prove: (i) AP = PD (ii) PB = CP.

Construction: Draw $OM \perp AB$, $ON \perp CD$. Join OP.



$AM = MB = \frac{1}{2} AB$ [perpendicular from centre bisects the chord.]

$NC = ND = \frac{1}{2} CD$ [perpendicular from centre bisects the chord.]

AM = ND and MB = CN ...(I) [$\because$ AB = CD (given)]

In Δs OMP and ONP, we have

OM = ON [equal chords are equidistant for centre]

$\angle OMP = \angle ONP$ [$\because$ each angle = 90º]

OP = OP [common]

$\therefore$ By RHS criterion of congruence, we have

$\Delta OMP \cong \Delta ONP$

$\therefore$ MP = NP ...(II) [CPCT]

Adding (I) and (II), we have

AM + MP = ND + NP

Hence, AP = PD

Subtracting (II) from (I), we have

MB – MP = NC – NP

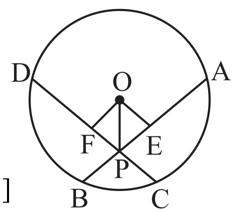
PB = CP

Hence, PB = CP. Proved (ii).

3. **Given:** AB and CD are chords of a circle with centre O. AB and CD intersect at P and AB = CD.

To prove: $\angle OPE = \angle OPF$.

Construction: Draw $OE \perp AB$ and $OF \perp CD$. Join OP.



$\angle OFP = \angle OEP$ [$\because$ each angle = 90º]

OP = OP

OE = OF [Equal chords of a circle are equidistant from the centre.]

∴ By RHS criterion of congruence, we have

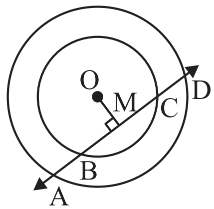
$\Delta OEP \cong \Delta OFP$

∴ $\angle OPE = \angle OPF$ [CPCT]

4. Let OM be the perpendicular from O on line *l*. We know that the perpendicular from the centre of a circle to a chord bisects the chord.

Since BC is a chord of the smaller circle and $OM \perp BC$,

∴ $BM = CM$ ...(i)



Again AD is a chord of the larger circle and $OM \perp AD$,

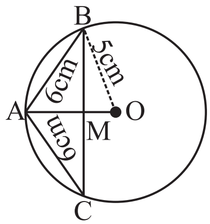
$AM = DM$ ...(ii)

Subtracting (i) from (ii), we get

$AM - BM = DM - CM$

∴ $AB = CD$.

5. Let the three girls Reshma, Salma and Mandip are standing on the circle of radius 5 cm at points B. A and C respectively.



Consider ΔAOB and ΔAOC,

$AO = AO$ [common]

$AB = AC$ [given]

$BO = OC$ [radii of circle]

∴ By SSS criterion of congruence, we have

$\Delta AOB \cong \Delta AOC$

∴ By CPCT, we get

$\angle BAO = \angle CAO$

i.e. $\angle BAM = \angle CAM$

Now consider ΔAMB and ΔAMC.

$AM = AM$ [common]

$AB = AC$ [each of length 6 cm]

$\angle BAM = \angle CAM$

∴ $\Delta AMB \cong \Delta AMC$

By CPCT, we get

$\angle AMB = \angle AMC$ ...(i)

and $BM = CM$

By linear axiom

$\angle AMB + \angle AMC = 180°$

∴ $\angle AMB = \angle AMC = 90°$

Therefore AO is the perpendicular bisector of BC.

Now, M is the midpoint of BC, i.e. $OM \perp BC$.

In right angle triangle ABM, we have

$AB^2 = AM^2 + BM^2$ [Pythagoras theorem]

$\therefore \quad 36 = AM^2 + BM^2$

$\therefore \quad BM^2 = 36 - AM^2$ ...(i)

In the right angled triangle OBM, we have

$OB^2 = OM^2 + BM^2$ (Pythagoras theorem)

$\therefore \quad 25 = (OA - AM)^2 + BM^2$

$\therefore \quad BM^2 = 25 - (OA - AM)^2$

$\therefore \quad BM^2 = 25 - (5 - AM)^2$ ...(ii)

From (i) and (ii), we get

$$36 - AM^2 = 25 - (5 - AM)^2$$

$$11 - AM^2 + (5 - AM)^2 = 0$$

$$11 - AM^2 + 25 - 10\,AM = 0$$

$$10\,AM = 36$$

$$AM = 3.6$$

In the right angled triangle OBM, we have

$$BM^2 = 36 - (3.6)^2$$

$$= 36 - 12.96$$

$$BM = \sqrt{36 - 12.96}$$

$$= \sqrt{23.04}$$

$$= 4.8 \text{ cm}$$

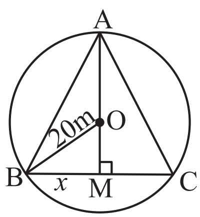
$$BC = 2BM$$

$$= 2 \times 4.8$$

$$= 9.6 \text{ cm}$$

Hence, the distance between Reshma and Mandip = 9.6 cm.

6. Let ABC is an equilateral triangle of side $2x$ metres.



Clearly, $BM = \frac{BC}{2} = \frac{2x}{2} = x$ metres.

In right angled triangle ABM,

$AM^2 = AB^2 - BM^2$ [Pythagoras theorem]

$= (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$AM = \sqrt{3}\,x$

Now, $OM = AM - OA = (\sqrt{3}\,x - 20)$ m

In right angled triangle OBM, we have

$OB^2 = BM^2 + OM^2$ [Pythagoras theorem]

$\therefore \quad 20^2 = x^2 + (\sqrt{3}\,x - 20)^2$

$\therefore \quad 400 = x^2 + 3x^2 - 40\sqrt{3}\,x + 400$

$\therefore \quad 4x^2 - 40\sqrt{3}\,x = 0$

$\therefore \quad 4x\,(x - 10\sqrt{3}) = 0$

Since $x \neq 0$, as so

$\therefore \quad x - 10\sqrt{3} = 0$

$\therefore \quad x = 10\sqrt{3}$

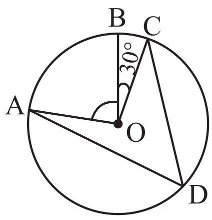
Now, $BC = 2BM = 2x = 20\sqrt{3}$

Hence, the length of each string = $20\sqrt{3}$ m.

Test Yourself – CR 4

1. 13.29 cm
2. $4\sqrt{6}$ cm
3. AB = 2 cm, CD = 2 cm, AC = 10 cm and BD = 10 cm
4. 6.7 cm
6. 105º
7. (i) 85º (ii) 42º
8. (i) 85º (ii) 40º (iii) 80º
9. 140º

Exercise – 3.5



1. Since arc ABC makes $\angle AOC = \angle AOB + \angle BOC = 60^\circ + 30^\circ = 90^\circ$ at the centre of the circle and ADC at a point on the remaining part of the circle.

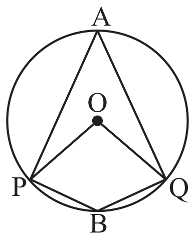
$\therefore \angle ADC = \frac{1}{2}(\angle AOC) = \frac{1}{2} \times 90^\circ = 45^\circ.$

2. Let PQ be chord. Join OP and OQ.

It is given that PQ = OP = OQ [$\because$ here chord = radius]

$\therefore \quad \Delta OPQ$ is equilateral.

$\therefore \quad \angle POQ = 60^\circ$



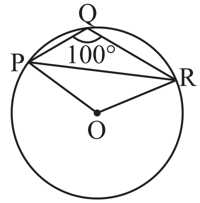
Since arc PBQ makes reflex $\angle POQ = 360^\circ - 60^\circ = 300^\circ$ at the centre of the circle and $\angle PBQ$ at a point in the minor arc of the circle.

$\therefore \quad \angle PBQ = \frac{1}{2}(\text{reflex } \angle POQ) = \frac{1}{2} \times 300^\circ = 150^\circ$

Similarly, $\angle PAQ = \frac{1}{2}(\angle POQ) = \frac{1}{2}(60^\circ) = 30^\circ$

Hence, the angle subtended by the chord on the minor arc = 150º and on the major chord = 30º.

3. Since the angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.



$\therefore$ Reflex $\angle POR = 2\ \angle PQR$

$\therefore$ Reflex $\angle POR = 2 \times 100^\circ = 200^\circ$

$\therefore$ $\angle POR = 360^\circ - 200^\circ = 160^\circ$

In ΔOPR,

$OP = OR$ [radii of the same circle]

$\angle OPR = \angle ORP$ [angles opposite to equal sides are equal]

and $\angle POR = 160^\circ$ [proved above]

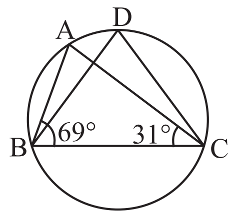
$$\therefore \quad \angle OPR = \angle ORP = \frac{1}{2}(180^\circ - 160^\circ)$$

$$= \frac{1}{2} \times 20^\circ$$

$$= 10^\circ$$

Hence, $\angle OPR = 10^\circ$.

4. In ΔABC,



$\angle BAC + \angle ABC + \angle BCA = 180^\circ$

$\angle BAC + 69^\circ + 31^\circ = 180^\circ$

$\angle BAC = 180^\circ - (69^\circ + 31^\circ)$

$= 180^\circ - 100^\circ$

$= 80^\circ$

Since angles in the same segment are equal,

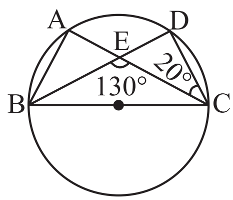
$\therefore$ $\angle BDC = \angle BAC = 80^\circ$.

5. $\angle CED + \angle CEB = 180^\circ$

$\angle CED + 130^\circ = 180^\circ$ [linear pair]

$\angle CED = 180^\circ - 130^\circ = 50^\circ$

In ΔECD,



$\angle EDC + \angle CED + \angle ECD = 180^\circ$

$\therefore$ $\angle EDC + 50^\circ + 20^\circ = 180^\circ$

$\angle EDC = 180^\circ - 50^\circ - 20^\circ$

$= 110^\circ$

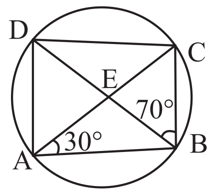
$\angle BDC = \angle EDC = 110^\circ$

Since angles in the same segment are equal,

$\therefore$ $\angle BAC = \angle BDC = 110^\circ$.

6. $\angle BDC = \angle BAC$ [angles in the same segment]

$\angle BDC = 30^\circ$ [$\because \angle BAC = 30^\circ$ (given)]



In ΔBCD, we have

$\angle BDC + \angle DBC + \angle BCD = 180^\circ$ [sum of angles of a triangle]

$30^\circ + 70^\circ + \angle BCD = 180^\circ$ [$\because \angle BCD = 180^\circ - 30^\circ - 70^\circ = 80^\circ$]

If AB = BC, then

$\angle BCA = \angle BAC = 30^\circ$ [angles opp. to equal sides in a Δ arc equal]

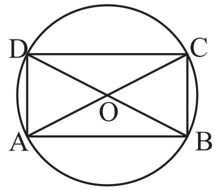
Now, $\angle ECD = \angle BCD - \angle BCE$

$= 80^\circ - 30^\circ = 50^\circ$ [$\angle BCD = 80^\circ$ (found above) and $\angle BCE = \angle BCA = 30^\circ$]

Hence, $\angle BCD = 80^\circ$ and $\angle ECD = 50^\circ$.

7. Diagonals AC and BD of a cyclic quadrilateral are diameters of the circle through the vertices A, B, C and D of the quadrilateral ABCD.

To prove: Quadrilateral ABCD is a rectangle.



Proof: Since all the radii of the same circle are equal,

$\therefore$ OA = OB = OC = OD

$\therefore$ $OA = OC = \frac{1}{2} AC$

and $OB = OD = \frac{1}{2} BD$

$\therefore$ AC = BD

$\therefore$ The diagonals of the quadrilateral ABCD are equal and bisect each other.

$\therefore$ Quadrilateral ABCD is a rectangle.

8. **Given:** Non-parallel sides AD and BC of a trapezium are equal.

To prove: ABCD is a cyclic trapezium.

Construction: Draw DE $\perp$ AB, CF $\perp$ AB

[Hint: In order to prove that ABCD is a cyclic trapezium, it is sufficient to prove that $\angle B + \angle D = 180^\circ$]

Proof: In Δs DEA and CFB, we have

AD = BC [given]

$\angle DEA = \angle CFB$ [each angle is 90º]

DE = CF [distance between two || lines is always equal]

$\therefore$ By RHS criterion of congruence, we have

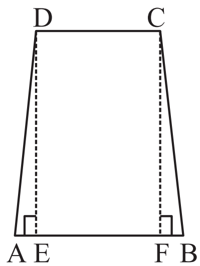
$\Delta DEA = \Delta CFB$

$\therefore$ $\angle A = \angle B$ ...(i) [CPCT]

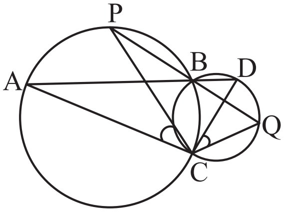
$\angle A + \angle D = 180^\circ$ ...(ii) [interior angles]

$\angle B + \angle D = 180^\circ$ [from (i) & (ii)]

$\therefore$ ABCD is a cyclic trapezium



9. Since angles in the same segment are equal



$\therefore$ $\angle ACP = \angle ABP$ ...(i)

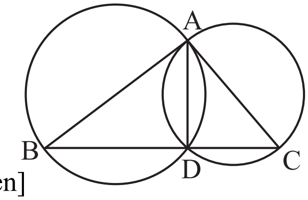
and $\angle QCD = \angle QBD$ ...(ii)

Also $\angle ABP = \angle QBD$ ...(iii) [vertically opposite angles]

$\therefore$ From (i), (ii) and (iii), we have

$\angle ACP = \angle QCD$

10. **Given:** Two circles are drawn with sides AB and AC of ΔABC as diameters. The circles intersect each other at D.



To prove: D lies on BC.

Construction: Join A and D.

Proof: Since AB and AC are the diameters of the two circles, [given]

$\therefore$ $\angle ADB = 90^o$ [angles in a semicircle]

and, $\angle ADC = 90^o$ [angles in a semicircle]

Adding, we get

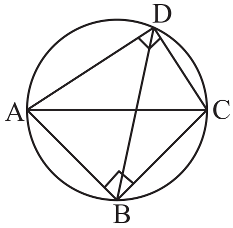
$\angle ADB + \angle ADC = 90^o + 90^o$

$= 180^o$

$\therefore$ BDC is a straight line.

Hence, D lies on BC.

11. Δs ABC and ADC are right angle triangles with common hypotenuse AC. Draw a circle with AC as diameter passing through B and D. Join BD, (as it is a cyclic quadrilateral).



Clearly, $\angle CAD = \angle CBD$ [$\because$ angles in the same segment are equal.]

12. **Given:** ABCD is a parallelogram inscribed in the circle.

To Prove: ABCD is a rectangle.

Proof : Since ABCD is a cyclic quadrilateral,

$\therefore$ $\angle A + \angle C = 180^o$ ...(i)

But $\angle A = \angle C$ ...(ii) [opposite angle of parallelogram]

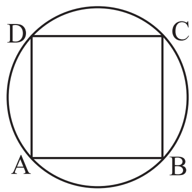
From (i) and (ii), we have

$\angle A = \angle C = 90^o$

Similarly, $\angle B + \angle D = 90^o$

$\therefore$ Each angle of ABCD is of 90^o.

Hence, ABCD is a rectangle.



TEST YOURSELF – CR 5

1. 160^o

3. 80^o

4. 45^o, 135^o

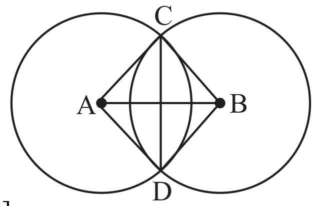
7. 50º

9. 80º, 100º

Exercise – 3.6

OPTIONAL

1. **Given:** Two circles with centres A and B, intersect each other at C and D.



To prove: $\angle ACB = \angle ADB$

Construction: Join AC, AD, BD and BC.

Proof: In Δs ACB and ADB, we have

AC = AD [radii of the same circle]

BC = BD [radii of the same circle]

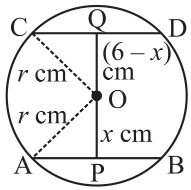
AB = AB [common]

$\therefore$ By SSS criterion of congruence, we have

$\Delta ACB \cong \Delta ADB$

$\therefore$ $\angle ACB = \angle ADB$ [CPCT]

2. Let O be the centre of the given circle and let its radius be r cm. Draw $OP \perp AB$ and $OQ \perp CD$. Since $OP \perp AB$, $OQ \perp CD$ and $AB \parallel CD$. Therefore, points P, O and Q are collinear. So, PQ = 6 cm.



Let $OP = x$. Then, $OQ = (6 - x)$ cm.

Join OA and OC. Then, $OA = OC = r$.

Since the perpendicular from the centre to a chord of the circle bisects the chord.

$\therefore$ AP = PB = 2.5 cm and CQ = QD = 5.5 cm.

In right Δs OAP and OCQ, we have

$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

$\therefore$ $r^2 = x^2 + (2.5)^2$ and $r^2 = (6 - x)^2 + (5.5)^2$ ...(i)

Hence, $x^2 + (2.5)^2 = (6 - x)^2 + (5.5)^2$

$\therefore$ $x^2 + 6.25 = 36 - 12x + x^2 + 30.25$

$\therefore$ $12x = 60$

$\therefore$ $x = 5$

Putting $x = 5$ in (i), we get

$r^2 = 5^2 + (2.5)^2 = 25 + 6.25 = 31.25$

$\therefore$ $r = \sqrt{31.25} = 5.6$ cm (approx.)

Hence, the radius of the circle is 5.6 cm (approx.).

3. Let AB and CD be two parallel chords of a circle with centre O such that AB = 6 cm and CD = 8 cm. Let the radius of the circle be r cm.

Draw OP $\perp$ AB and OQ $\perp$ CD.

Since AB || CD and OP $\perp$ AB, OQ $\perp$ CD. Therefore, points O, Q and P are collinear.

Clearly OP = 4 cm, and P, Q are midpoints of AB and CD respectively.

$\therefore$ AD = PB = $\frac{1}{2}$ AB = 3 cm. [Perpendicular from centre of a circle to a chord bisects the chord]

and, CQ = QD = $\frac{1}{2}$ CD = 4 cm. [Perpendicular from centre of a circle to a chord bisects the chord]

In right angled triangle OAP, we have

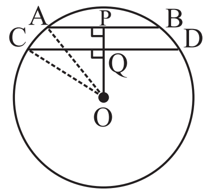
$OA^2 = OP^2 + AP^2$ [Pythagoras theorem]

or $r^2 = 4^2 + 3^2$

$= 16 + 9$

$= 25$

$r = 5$



In right angled triangle OCQ, we have

$OC^2 = OQ^2 + CQ^2$ [Pythagoras theorem]

or $r^2 = OQ^2 + 4^2$

or $25 = OQ^2 + 16$

or $OQ^2 = 9$

$\therefore$ OQ = 3

Hence, the distance of chord CD from the centre is 3 cm.

4. Since an exterior angle of a triangle is equal to the sum of the interior opposite angles.

In ΔBDC, we have

$\angle ADC = \angle DBC + \angle DCB$ ...(i)

Since an angle at the centre is twice the angle at a point on the remaining part of circle.

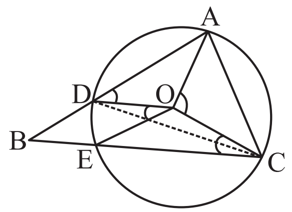
$\therefore$ $\angle ADC = \frac{1}{2} \angle AOC$ ...(ii)

and $\angle DCB = \frac{1}{2} \angle DOE$

From (i) and (ii), we have

$\frac{1}{2} \angle AOC = \angle ABC + \frac{1}{2} \angle DOE$

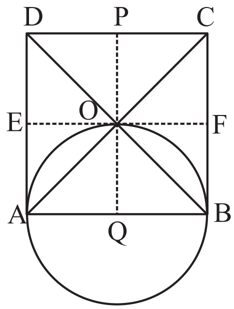
$\therefore$ $\angle ABC = \frac{1}{2} (\angle AOC - \angle DOE)$



Hence, ABC is equal to half the difference of angles subtended by the chords AC and DE at the centre.

5. **Given:** ABCD is a rhombus. AC and BD are its two diagonals which bisect each other at right angle.

To prove: A circle drawn on AB as diameter will pass through O.



Construction: From O draw PQ || AD and EF || AB.

Proof : Since AB = DC

$\therefore$ $\frac{1}{2}$ AB = $\frac{1}{2}$ DC

AQ = DP [$\because$ Q and P are midpoints of AB and CD.]

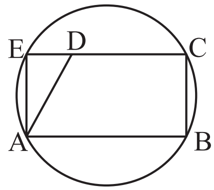
Similarly AE = OQ

$\therefore$ AQ = OQ = QB

$\therefore$ A circle drawn with Q as centre and radius AQ passes through A, O and B.

The circle thus obtained is the required circle.

6. In order to prove that AE = AD, i.e. ΔAED is an isosceles triangle it is sufficient to prove that ∠AED = ∠ADE.



Since ABCE is a cyclic quadrilateral,

$\therefore$ ∠AED + ∠ABC = 180º ...(i)

Now, CDE is a straight line,

$\therefore$ ∠ADE + ∠ADC = 180º ...(ii) (linear pair)

[∠ADC and ∠ABC are opposite angles of a parallelogram i.e. ∠ADC = ∠ABC.]

From (i) and (ii), we get

∠AED + ∠ABC = ∠ADE + ∠ABC

$\therefore$ ∠AED = ∠ADE

$\therefore$ In ΔAED, we have

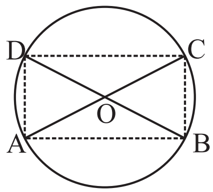
∠AED = ∠ADE

$\therefore$ AD = AE (by isosceles triangle)

7. Let chords AC and BD bisect each other at O.

Join AB, BC, CD, AD.

In ΔOAB and ΔOCD, we get



OA = OC [given]

OB = OD [given]

∠AOB = ∠COD [vertically opposite angles]

$\therefore$ ΔAOB = ΔCOD [SAS criterion]

AB = CD (CPCT)

$\overset{\frown}{AB} = \overset{\frown}{CD}$ ...(i)

[If chords are congruent then corresponding arcs are congruent.]

Similarly, $\overset{\frown}{AD} = \overset{\frown}{CB}$ ...(ii)

Adding (i) and (ii) we get

$$\overset{\frown}{AB} + \overset{\frown}{AD} = \overset{\frown}{CD} + \overset{\frown}{CB}$$

$$\overset{\frown}{BAD} = \overset{\frown}{BCD}$$

$\therefore$ BD divides the circle into two semicircles.

Hence, BD is the diameter.

$\therefore$ $\angle A = \angle C = 90^\circ$ [angle in semicircle]

Similarly, AC is the diameter.

$\therefore$ $\angle B = \angle D = 90^\circ$ [angle in semicircle]

i.e. $\angle A = \angle B = \angle C = \angle D = 90^\circ$

$\therefore$ ABCD is a rectangle.

8. We have, $\angle D = \angle EDF$

$= \angle EDA + \angle ADF$

$= \angle EBA + \angle FCA$ [$\because$ $\angle EDA$ and $\angle EBA$ are the angles in the same segment of the circle]

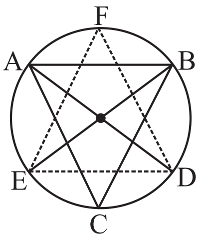
$\angle EDA = \angle EBA.$

Similarly, $\angle ADF$ and $\angle FCA$ are the angles in the same segment and hence,

$\angle ADF = \angle FCA.$

$= \frac{1}{2}\angle B + \frac{1}{2}\angle C$ [$\because$ BE is the internal bisector of $\angle B$ and CF is the internal bisector $\angle C$]

$= \frac{\angle B + \angle C}{2}$



Similarly, $\angle E = \frac{\angle C + \angle A}{2}$

$\angle F = \frac{\angle A + \angle B}{2}$

$\therefore$ $\angle D = \frac{180^\circ - \angle A}{2}$

$\angle E = \frac{180^\circ - \angle B}{2}$

and $\angle F = \frac{180^\circ - \angle C}{2}$

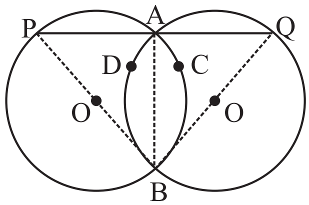
$\therefore$ $\angle D = 90^\circ - \frac{\angle A}{2}$

$\angle E = 90^\circ - \frac{\angle B}{2}$

and $\angle F = 90^\circ - \frac{\angle C}{2}$

$\therefore$ The angles of the DEF are $\angle D = 90^\circ - \frac{1}{2}\angle A$, $\angle E = 90^\circ - \frac{1}{2}\angle B$ and $\angle F = 90^\circ - \frac{1}{2}C$.

9. Let O and O′ be the centre of two congruent circles.



Since AB is a common chord of these circles.

arc ACB = arc ADB

$\therefore$ $\angle BPA = \angle BQA$

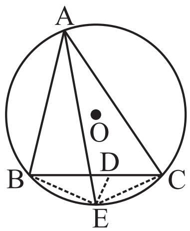
$\therefore$ BP = BQ [Equal arc subtent equal cords.]

10. **Given:** ABC is a triangle inscribed in a circle with centre at O. E is a point on the circle such that AE is the internal bisector of $\angle BAC$ and D is the midpoint of BC.

To prove: DE is the right bisector of BC, i.e.

$\angle BDE = \angle CDE = 90^\circ$

Construction: Join BE and EC.



In ΔBDE and ΔCDE, we have

BE = CE [$\because \angle BAE = \angle CAE$, $\therefore$ arc BE = arc CE, $\therefore$ chord BE = chord CE]

BD = CD [given]

DE = DE [common]

By SSS criterion of congruence, we get

$\Delta BDE \cong \Delta CDE$

$\therefore$ $\angle BDE = \angle CDE$ [CPCT]

Also, $\angle BDE + \angle CDE = 180^\circ$ [linear pair]

$\therefore$ $\angle BDE = \angle CDE = 90^\circ$

Hence, DE is the right bisector of BC.