1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3Å.

Sol. Given, Diameter, d = 3Å, $r = d/2$ = 1.5Å = 1.5×10^{-8} cm

V = molecular volume = $\frac{4}{3}\pi r^3 N$ (Where N–Avogadro's number)

$$= \frac{4}{3} \times \left(\frac{22}{7}\right)(1.5 \times 10^{-8})^3 \times (6.023 \times 10^{23}) = 8.52\ cc$$

V' = actual volume occupied by 1 mole of O_2 at STP = 22400 cc

$$\therefore \quad \frac{V}{V'} = \frac{8.52}{22400} = 3.8 \times 10^{-4}$$

2. Molar volume is the volume occupied by 1 mole of any ideal gas at standard temperature and pressure (STP) which are 1 atmospheric pressure and 0°C. Show that it is 22.4 litres.

Sol. For one mole of an ideal gas, $PV = RT$

$\therefore\ V = \frac{RT}{P}$ [Where, R = 8.31 J mol^{-1} K^{-1}, T = 273 K, P = 1 atmosphere = 1.013×10^5 Nm^{-2}]

$$= \frac{8.31 \times 273}{1.013 \times 10^5} = 0.0224\ \text{m}^3 = 22.4\ \text{litres}$$

3. An oxygen cylinder of volume 30 litre has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm. and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder.

(R = 8.1 J mole^{-1} K^{-1}, molecular mass of O_2 = 32 cc)

Sol. Initially, the volume of the oxygen O_2 cylinder, V_1 = 30 litre = 3×10^{-3} m^3

P_1 = 15 atm = $15 \times 1.01 \times 10^5$ Pa, T_1 = 27 + 273 = 300 K

If n_1 be the moles of O_2 gas in the cylinder, $P_1V_1 = n_1RT_1$

$$\therefore \quad n_1 = \frac{(15 \times 1.01 \times 10^5) \times (30 \times 10^{-3})}{8.3 \times 300} = 18.253$$

M = molecular weight of O_2 = 32g; Initial cylinder mass = $m_1 = n_1M$ = 18.253 × 32 = 584.1 g

Let, n_2 – moles of O_2 left in the cylinder

$$\therefore\ n_2 = \frac{P_2V_2}{RT_2}$$

[where, $V_2 = 30 \times 10^{-3}$m^3, $P_2 = 11 \times 1.01 \times 10^5$ Pa, T_2 = 17 + 273 = 290°K]

$$= \frac{(11 \times 1.01 \times 10^5)(30 \times 10^{-3})}{8.3 \times 290} = 13.847$$

$\therefore$ m_2 – final mass of O_2 in cylinder = 13.847 × 32 = 443.1 g

$\therefore$ Mass of gas taken out

$m_1 - m_2$ = 584.1 – 443.1 = 141 g

4. An air bubble of volume 1 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35°C? Given 1 atm = 1.01×10^5 Pa.

Sol. Given, $V_1 = 1.0\ cm^3 = 1 \times 10^{-6}\ m^3$, T_1 = 12°C = 285 K,

$P_1 = 1\ atm + h_1\rho g = 1.01 \times 10^5 + 40 \times 10^3 \times 9.8 = 493000$ Pa

Let, V_2 = volume of bubble at the surface of the lake

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore\ V_2 = \frac{P_1V_1T_2}{T_1P_2}$$ [Where T_2 = 35°C = 308 K, P_2 = 1atm = 1.01×10^5 Pa]

$$= \frac{(493000) \times (1.0 \times 10^{-6}) \times 308}{285 \times 1.01 \times 10^5} = 5.275 \times 10^{-6}\ m^3$$

5. Estimate the total number of air molecules (inclusive of O_2, N_2, water vapour, and other constituent) in a room of capacity 25 m^3 at a temperature of 27°C and 1 atm pressure.

[Boltzmann constant = $1.38 \times 10^{-23}\ JK^{-1}$]

Sol. Given, $V = 25\ m^3$, T = 27 + 273 = 300 K, $k = 1.38 \times 10^{-23}\ JK^{-1}$.

Now, $PV = nRT = n(Nk)T = N'kT$

where, $nN = N'$ = total no. of air molecules in the given gas

$$N' = \frac{PV}{kT} = \frac{(1.01 \times 10^5) \times 25}{(1.38 \times 10^{-23}) \times 300} = 6.10 \times 10^{26}$$

6. Estimate the average thermal energy of a helium atom at (i) room temperature (27°C) (ii) The temperature of the surface of the Sun (6000 K) (iii) The temperature of 10 million kelvin (the typical core temperature in the case of a star).

Sol. Given, T = 27°C = 300°K

Average thermal energy, = 3/2 kT = $\frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21}$ J

(ii) At, $T = 6000$ K; Avg. thermal energy

$$= \frac{3}{2}kT = \frac{3}{2}\times(1.38\times10^{-23})\times6000 = 1.24\times10^{-19}\text{ J}$$

(iii) At, $T = 10$ million kelvin, $T = 10^7$ K

$$\text{Avg. thermal energy} = \frac{3}{2}kT = \frac{3}{2}\times(1.38\times10^{-23})\times10^7 = 2.1\times10^{-16}\text{ J.}$$

7. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic). The second contains oxygen (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessel contain equal number of respective molecules? Is the root mean square speed of molecules the same in the cases? If no, in which case is v_{rms} the largest?

Sol. At same temperature and pressure, all the vessels have the same volume.

By Avogadro's law, three vessels contain equal number of molecules

Avogadro's no. $N = 6.023\times10^{23}$

As $$v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \quad v_{rms} \propto \frac{1}{\sqrt{m}}$$

Therefore, at a given temperature, rms speed of molecules will not be the same in all the three cases. As neon has the smallest mass, therefore, rms speed will be the largest in case of neon.

8. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at – 20°C? (Atomic mass of Ar = 39.9 u and He = 4 u).

Sol. Let C, C' be the rms velocity of Ar and He at temperatures T and T' K, respectively.

Given, $M = 39.9$, $M' = 4$, $T' = -20 + 273 = 253$ K

$$\text{Now } C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}} \text{ and } C' = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3R\times253}{4}}$$

$$\text{Since } C = C'\text{, therefore, } \sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R\times253}{4}}$$

$$\therefore \quad T = 2523.7\text{ K}$$

9. Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2 atm. and temperature 17°C. Take the radius of a nitrogen molecule to be roughly 1Å. Compare the collision time with the time, the molecule moves freely between two successively collision. (Molecule mass of nitrogen = 28u)

Sol. Given $P = 2$ atm $= 2 \times 1.03 \times 10^5$ Nm^{-2}; $T = 17°C = 290$ K, $d = 2 \times 1 = 2$Å $= 2 \times 10^{-10}$m

$k = 1.38 \times 10^{-23}$ J molecule^{-1}K^{-1}; $M = 28 \times 10^{-23}$ kg

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 \rho} = \frac{1.38 \times 10^{-23} \times 290}{1.414 \times 3.14 \times (2 \times 10^{-10})^2 \times (2.026 \times 10^5)}$$

$$\Rightarrow \quad \lambda = 1.11 \times 10^{-7}\text{ m}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 290}{28 \times 10^{-3}}} = 508.24\text{ m/s}$$

Collision frequency = No. of collisions per second $= \frac{508.24}{1.11 \times 10^{-7}} = 4.58 \times 10^9$

10. From a certain apparatus, the diffusion rate of H_2 has an average value of 28.7 cm^{3}s^{-1}. The diffusion of another gas under the same condition is measured to have an average rate of 7.2 cm^{3}s^{-1}. Identify the gas.

Sol. According to Graham's law of diffusion, $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$,

where, r_1 = diffusion rate of H_2 = 28.7 cm^{3}s^{-1}

r_2 = diffusion rate of unknown gas = 7.2 cm^{3}s^{-1}

M_1 = molecular wt. of H_2 = 2u

$$\therefore \quad \frac{28.7}{7.2} = \sqrt{\frac{M_2}{2}}$$

$\therefore \quad M_2 = 32$, which is molecular mass of O_2 gas.