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Studymate Solutions to CBSE Board Examination 2018-2019

Series : JMS/1 | **Code No. 30/1/1**

Roll No. | | | | | | | |

Candidates must write the Code on the title page of the answer-book.

- *Please check that this question paper contains 18 printed pages.*
- *Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains 30 questions.*
- *Please write down the Serial Number of the question before attempting it.*
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

MATHEMATICS

[Time allowed : 3 hours] *[Maximum marks : 80]*

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 30 questions divided into four Sections – A, B, C and D.

(iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted.

Section – A

1. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4).

Ans. By mid point formula

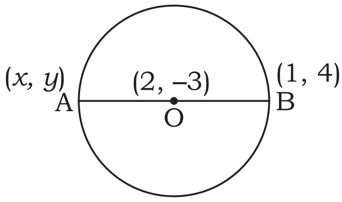
$x = \frac{x_1 + x_2}{2}$; $y = \frac{y_1 + y_2}{2}$

$2 = \frac{x+1}{2}$; $-3 = \frac{y+4}{2}$

$4 = x + 1$; $-6 = y + 4$

$\boxed{x = 3}$; $\boxed{y = -10}$

$\therefore$ A = (3, –10)



2. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

OR

Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.

Ans. $x^2 + 4x + k = 0$

$\therefore D \geq 0$

$b^2 - 4ac \geq 0$

$(4)^2 - 4 \times 1 \times k \geq 0$

$16 - 4k \geq 0$

$k \leq 4$ or $k \geq -4$

OR

$P(x) = 3x^2 - 10x + k$

$\therefore$ Roots are $\alpha, \frac{1}{\alpha}$

Product of zeroes; $\alpha\beta = \frac{c}{a}$

$\alpha \times \frac{1}{\alpha} = \frac{k}{3}$

$1 = \frac{k}{3}$

$\therefore \boxed{k = 3}$

For $k = 3$; the roots are reciprocal.

3. Find A if tan 2A = cot (A –24°)

OR

Find the value of (sin^2 33° + sin2 57°)

Ans. $\tan 2A = \cot(A - 24°)$

$\cot(90 - 2A) = \cot(A - 24°)$ $\{\tan\theta = \cot(90 - \theta)\}$

$\therefore 90° - 2A = A - 24°$

$90 + 24 = A + 2A$

$114 = 3A$

$\therefore \boxed{A = 38°}$

OR

$= \sin^2 33° + \sin^2 57°$

$= \sin^2 33° + \cos^2(90° - 57°)$ $\{\sin\theta = \cos(90 - \theta)\}$

$= \sin^2 33° + \cos^2 33°$

$= 1$

4. How many two digits numbers are divisible by 3?

Ans. A.P. ⇒ 12, 15, ...99

∴ $a = 12$

$d = 3$

$a_n = 99$

∴ $a_n = a + (n - 1)d$

$99 = 12 + (n - 1)3$

$99 = 12 + 3n - 3$

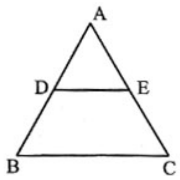
$99 = 9 + 3n$

$90 = 3n$

$\boldsymbol{n = 30}$

∴ 30 two digit number are divisible by 3.

5. In fig 1, DE||BC, AD = 1 cm and BD = 2 cm. What is the ratio of ar (Δ ABC) to the ar (Δ ADE)?

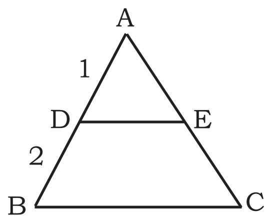


Ans. Given : - DE || BC

AB = 1 cm

BD = 2 cm

To find = $\frac{ar(\Delta ABC)}{ar(\Delta ADE)}$



Sol. In ΔADE and ΔABC

∠ADE = ∠ABC [corresponding angle as DE || BC]

∠DAE = ∠BAC [common]

∴ By AA similarity criteria.

ΔADE ~ ΔABC

∴ $\frac{ar(\Delta ABC)}{ar(\Delta ABC)} = \left(\frac{AB}{AD}\right)^2$ [By area similarly theorem]

$\frac{ar(\Delta ABC)}{ar(ADE)} = \left(\frac{3}{1}\right)^2$

$\frac{ar(ABC)}{ar(ADE)} = \frac{9}{1}$

6. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Ans. $\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

∴ Rational number between $\sqrt{2}$ and $\sqrt{3}$ is 1.5

Section – B

7. Find the HCF of 1260 and 7344 using Euclid's algorithm.

OR

Show that every positive odd integer is of the form $(4q + 1)$ or $(4q + 3)$, where q is some integer.

Ans. 7344 = 1260 × 5 + 1044

1260 = 1044 × 1 + 116

1044 = 116 × 9 + 0

∴ HCF is 116.

OR

Let us start with taking a, where a is a positive odd integer. We apply the division algorithm with a and $b = 4$.

Since $0 \leq r < 4$, the possible remainders are 0, 1, 2 and 3.

That is, a can be $4q$, or $4q + 1$, or $4q + 2$, or $4q + 3$, where q is the quotient.

However, since a is odd, a cannot be $4q$ or $4q + 2$ (since they are both divisible by 2).

Therefore, any odd integer is of the form $4q + 1$ or $4q + 3$.

8. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

OR

If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term.

Ans. $a_{21} = a + 20d$

$= 3 + 20 \times 12$

$= 3 + 240$

$= 243$

Required terms = 243 + 120 = 363

∴ $a_n = 263$

$3 + (n - 1)\ 12 = 363$

$(n - 1)\ 12 = 360$

$n - 1 = 30$

$n = 31$

OR

$S_n = 2n^2 - 4n$

$a_n = S_n - S_{n-1} = 3n^2 - 4n - [3(n - 1)2 - 4\ (n - 1)]$

$= 3n^2 - 4n - [3n^2 + 3 - 6n - 4n + 4]$

$= 3n^2 - 4n - 3n^2 + 10n - 7$

$= 6n - 7$

9. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis? Also find the coordinates of this point on x-axis.

Ans. Let the point $P(x, 0)$ divides

Line segment joining A(1, – 3) and B(4, 5) in ratio $k : 1$.

∴ By section formula; coordinates of P are

$$(x, 0) = \left(\frac{4k+1}{k+1}, \frac{5k-3}{k+1}\right)$$

∴ $0 = \dfrac{5k-3}{k+1}$

⇒ $0 = \dfrac{5k-3}{k+1}$

⇒ $0 = 5k - 3$

$k = \dfrac{3}{5}$

∴ Ratio = 3 : 5

$\therefore$ coordinates are $= \dfrac{4 \times \frac{3}{5} + 1}{\frac{3}{5} + 1} = \dfrac{\frac{12}{5} + 1}{\frac{3}{5} + 1} = \dfrac{\frac{17}{5}}{\frac{8}{5}} = \dfrac{17}{8}$

Coordinates are $\left(\dfrac{17}{8}, 0\right)$.

10. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

Ans. The sample space for tossing of coins thrice is

(HHH), (HHT) (HTH), (HTT),

(THH), (THT), (TTH), (TTT),

(HHH) and (TTT) are probabilities of success.

$\therefore$ No. of outocmes of losing = Total – 2

= 8 – 2 = 6

Total outcomes = 8

P(losing) = $\dfrac{6}{8} = \dfrac{3}{4}$

11. A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

Ans. The sample space for throwing the die once is

(1, 2, 3, 4, 5,6)

(i) (2, 3, 5) are prime numbers

$\therefore$ No. of outcomes favourable = 3

Total outcomes = 6

P(prime no.) = $\dfrac{3}{6} = \dfrac{1}{2}$

(ii) (3, 4, 5) lie between 2 and 6

$\therefore$ No. of favourable outcomes = 3

Total outcomes = 6

P(no. between 2 and 6) $\dfrac{3}{6} = \dfrac{1}{2}$

12. Find c if the system of equations $cx + 3y + (3 - c) = 0$; $12x + cy - c = 0$ has infinitely many solutions?

Ans. $cx + 3y + (3 - c) = 0$ ...(i)

$12x + cy - c = 0$ ...(ii)

$a_1 = c$; $b_1 = c$; $c_1 = (3 - c)$

$a_2 = 12$; $b_2 = c$; $c_2 = -c$

For infinitely many solutions.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{c}{12} = \frac{3}{c} = \frac{3 - c}{-c}$$

$c^2 = 36$ $\quad$ $-3c = 3c - c^2$

$\boxed{c = \pm 6}$ ..(i) $\quad$ $c^2 = 6c$

$c^2 = 6c = 0$

$c(c - 6) = 0$

$\boxed{c = 0, 6}$ ...(ii)

from (i) and (ii) c = 6

Section – C

13. Prove that $\sqrt{2}$ is an irrational number.

Ans. Let us assume, to the contrary, that $\sqrt{2}$ is rational.

So, we can find integers r and s ($\neq 0$) such that $\sqrt{2} = \frac{r}{s}$.

Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$, where a and b are coprime.

So, $b\sqrt{2} = a$.

Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2.

Now, by Theorem 1.3, it follows that 2 divides a.

So, we can write $a = 2c$ for some integer c.

Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$.

This means that 2 divides b^2, and so 2 divides b (again using Theorem 1.3 with $p = 2$).

Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational.

14. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half of their product.

Ans. $x^2 - (k + 6)x + 2(2k - 1)$

Let the zeroes be α and β.

According to the question,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-(k+6))}{(1)} = (k+6)$$

$$\alpha\beta = \frac{c}{a} = \frac{2(2k-1)}{(1)} = (4k-2)$$

Substituting,

$$(k+6) = \frac{1}{2}(4k-2)$$

$$k + 6 = \frac{1}{2} \times 2(2k-1)$$

$\Rightarrow \quad k + 6 = 2k - 1$

$\Rightarrow \quad 2k - k = 6 + 1$

$\Rightarrow \quad k = 7$

15. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

OR

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

Ans. Let the father's present age = x years.

Sum of ages of 2 children = y

ATQ,

$x = 3y$

or $\quad x - 3y = 0 \quad \quad \ldots$(i)

After 5 years,

Father's age = $(x + 5)$

Children's age = $(y + 10)$

$\therefore \quad x + 5 = 2(y + 10)$

$x + 5 = 2y + 20$

$x - 2y = 15 \quad \text{...(ii)}$

From (i) and (ii),

$$\begin{array}{r} x - 3y = 0 \\ x - 2y = 15 \\ (-) \quad (+) \qquad (-) \\ \hline y = 15 \end{array}$$

$\therefore \quad x = 45$

Present age of father = 45 years.

Sum of ages of children = 15 years.

OR

Let the numerator be x

Let the denominator be y

$\therefore$ Fraction is $\frac{x}{y}$

According to the question,

$$\frac{x-2}{y} = \frac{1}{3} \quad \text{...(1)}$$

$$\frac{x}{y-1} = \frac{1}{2} \quad \text{...(2)}$$

$\Rightarrow \quad 3(x - 2) = y$

$\Rightarrow \quad 3x - 6 = y$

$\Rightarrow \quad 3x - y = 6 \quad \text{...(3)}$

$\Rightarrow \quad 2x = y - 1$

$\Rightarrow \quad 2x - y = -1 \quad \text{...(4)}$

Subtracting (4) from (3)

$$\begin{array}{r} 3x - y = 6 \\ \Rightarrow \quad 2x - y = -1 \\ \hline x = 7 \end{array}$$

$\therefore \quad 3(7) - y = 6$

$\Rightarrow \quad 21 - y = 6 \qquad \Rightarrow \quad y = 21 - 6 = 15$

$\Rightarrow \quad y = 15$

$\therefore \quad \frac{7}{15}$

16. Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

OR

The line segment joining the points A(2, 1) and B (5, –8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k.

Ans. Point on y-axis be P(0, y)

Points are A(5, –2) & B(–3, 2)

$PA = PB$ or $PA^2 = PB^2$

$(0 - 5)^2 + (y + 2)^2 = (0 + 3)^2 + (y - 2)^2$

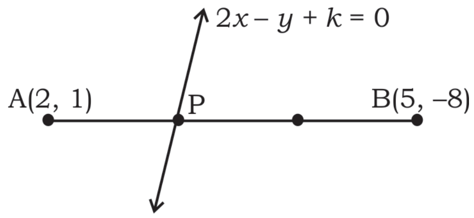
$25 + y^2 + 4 + 4y = 9 + y^2 + 4 - 4y$

$8y = -16$

$y = -2$

$\therefore$ Point P is (0, –2)

OR



P divides AB in ratio 1 : 2

$\therefore$ Coordinates of P are

$\frac{1 \times 5 + 2 \times 2}{1+2}$ and $\frac{1 \times -8 + 2 \times 1}{1+2}$

Point P(3, –2) lies on $2x - y + k = 0$

$\therefore 2(3) - (-2) + k = 0$

$k = -8$

17. Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

OR

Prove that $(1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A) = 2$

Ans. LHS = $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \text{cosec}^2\theta + \cos^2\theta + 2\cos\theta\sec\theta + \sec^2\theta$

$= (\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta) + (\sec^2\theta) + 2\sin\theta\,\text{cosec}\,\theta + 2\cos\theta\sec\theta$

$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2\sin\theta\left(\frac{1}{\sin\theta}\right) + 2\cos\theta\left(\frac{1}{\cos\theta}\right)$

.... [$\because \sin^2\theta + \cos^2\theta = 1$, $\text{cosec}^2\theta = 1 + \cot^2\theta$, $\sec^2\theta = 1 + \tan^2\theta$]

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$

$= 7 + \tan^2\theta + \cot^2\theta$

= RHS

$\therefore$ **LHS = RHS**

Hence proved.

OR

$(1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A) = 2$

$= \left(\frac{1}{1} + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(\frac{1}{1} + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$

$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$

$= \frac{(\sin A + \cos A - 1)(\sin A + \cos A + 1)}{\sin A \cos A}$

$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A}$ $\quad [\because (a-b)(a+b) = a^2 - b^2]$

$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A \cos A}$ $\quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$

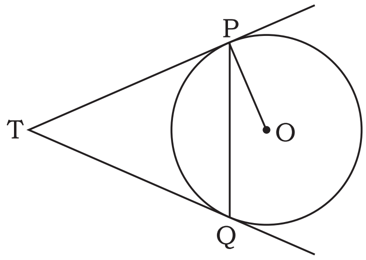
$= \frac{\cancel{1} + 2\sin A\cos A - \cancel{1}}{\sin A \cos A}$ $\quad (\because \sin^2 A + \cos^2 A = 1)$

$= \frac{2\sin A \cos A}{\sin A \cos A}$

= 2

= RHS

18. In Fig. 2, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect point T. Find the length of TP.



Ans. Join OT. Let it intersect PQ at the point R. Then Δ TPQ is isosceles and TO is the angle bisector of ∠PTQ. So, OT ⊥ PQ and therefore, OT bisects PQ which gives PR = RQ = 4 cm.

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2} = 3$ cm

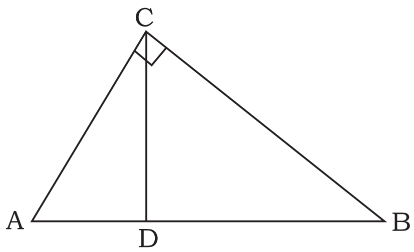
Now, ∠TPR + ∠RPO = 90° = ∠TPR + ∠PTR

So, ∠RPO = ∠PTR

Therefore, right triangle TRP is similar to the right triangle PRO by AA similarity.

This gives $\frac{TP}{PO} = \frac{RP}{RO}$, i.e., $\frac{TP}{5} = \frac{4}{3}$ or $TP = \frac{20}{3}$ cm

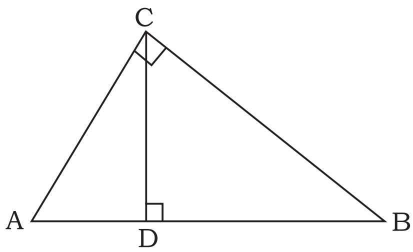
19. In Fig. 3, ∠ACB = 90° and CD ⊥ AB, prove that $CD^2 = BD \times AD$.



OR

If P and Q are the points on side CA and CB respectively of Δ ABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

Ans. Given: In ΔABC, ∠C = 90°



CD ⊥ AB

To Prove $CD^2 = BD \times AD$

Proof:

In ΔACD, ∠D = 90°

So, $AC^2 = CD^2 + AD^2$ [Pythagoras theorem]

$\Rightarrow CD^2 = AC^2 - AD^2$ ...(1)

In ΔBCD, ∠CDB = 90°

So, $BC^2 = CD^2 + BD^2$ [Pythagoras theorem]

$\Rightarrow CD^2 = BC^2 - BD^2$ ...(2)

Adding eqn. (1) & (2)

$2CD^2 = AC^2 + BC^2 - AD^2 - BD^2$

$= AB^2 - AD^2 - BD^2$ $[AC^2 + BC^2 = AB^2]$

$= (AD + BD)^2 - AD^2 - BD^2$

$= AD^2 + BD^2 + 2AD \cdot BD - AD^2 - BD^2$ $[(a + b)^2 = a^2 + b^2 + 2ab]$

$= 2AD \cdot BD$

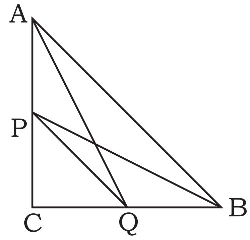
$\Rightarrow \boxed{CD^2 = AD \cdot BD}$

Hence Proved.

OR

Given: In ΔABC, $\angle C = 90°$

P, and Q are paint on sides AC & BC respectively.



To Prove $(AQ^2 + BP^2) = (AB^2 + Pa^2)$

Proof:

In ΔACQ, $\angle C = 90°$

So, $AQ^2 = AC^2 + CQ^2$ [Pythagoras theorem] ...(1)

In ΔPCB, $\angle C = 90°$

So, $BP^2 = PC^2 + BC^2$ [Pythagoras theorem] ...(2)

Adding eqn. (1) & (2)

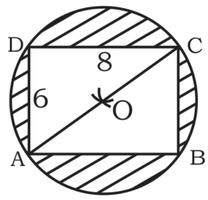
$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + BC^2$

$= (AC^2 + BC^2) + (CQ^2 + PC^2)$

$= AB^2 + PQ^2$ [Pythagoras theorem]

Proved.

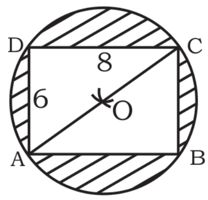
20. Find the area of the shaded region in Fig. 4, if A BCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take n - 3.14)



Ans. Given: ABCD is a rectangle.

CD = 8 cm

AD = 6 cm



In ΔADC

$AC^2 = AD^2 + DC^2$

$AC^2 = 6^2 + 8^2$

$AC^2 = 36 + 64$

$AC^2 = 100$

$AC = \sqrt{100}$

AC = 10 cm

Now

Shaded area = Area of circle – Area of rectangle ABCD

$= \pi(5)^2 - 6 \times 8$

$= \frac{22}{7} \times 25 - 48$

$= \frac{550}{7} - 48$

$= \frac{550 - 336}{7}$

$= \frac{214}{7} = 30.57 \text{ cm}^2$

21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm of standing water is needed ?

Ans. Height of the water flowing in 30 min = $\frac{10}{2}$ km

= 5 km

= 5,000 m

∴ Volume of water in the canal (cuboid) = $l \times b \times h$

= 5,000 × 6 × 1.5 m³

Standing water required = 8 cm

= $\frac{8}{100}$ m

Volume = area × height

$5{,}000 \times 6 \times 1.5 = \text{area} \times \frac{8}{100}$

∴ Area = $5{,}000 \times 6 \times 1.5 \times \frac{100}{8}$

= $\frac{45{,}00{,}000}{8}$

= 5,62,500 m²

= 56.25 hectare ...(1 hectare = 10,000 m²)

∴ **Area irrigated will be 5,62,500m² or 56.25 hectares.**

22. Find the mode of the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	10	16	12	6	7

Ans.

Class	Frequency
0 – 10	8
10 – 20	10
20 – 30	10 f_0
30 – 40	16 f_1
40 – 50	12 f_2
50 – 60	6
60 – 70	7

Modal class is (30 – 40)

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times 4$$

$$= 30 + \left(\frac{16 - 10}{2 \times 16 - 10 - 12}\right) \times 10$$

$$= 30 + \left(\frac{6}{32 - 10 - 12}\right) \times 10$$

$$= 30 + \left(\frac{6}{10} \times 10\right)$$

= 36

Section – D

23. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Or

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Ans. Let the small tap can fill the tank = x h

The larger tap can fill the tank = $(x - 2)$h

Smaller tap in x hour fills = 1 tank

$\therefore$ In 1 hour = $\frac{1}{x}$ th part will be filled

Larger tap in $(x - 2)$ hours fills = 1 tank

$\therefore$ in 1 hour = $\frac{1}{x-2}$ th part will be filled

(Smaller tap + Larger tap) fills 1 tank = $\frac{15}{8}$ hours

In 1 hour = $\frac{8}{15}$ th part will be filled

Now, according to problem

$$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$$\frac{x-2+x}{x(x-2)} = \frac{8}{15}$$

$$\frac{2x-2}{x^2-2x} = \frac{8}{15}$$

$$\frac{\cancel{2}(x-1)}{x^2-2x} = \frac{\overset{4}{\cancel{8}}}{15}$$

$4x^2 - 8x = 15x - 15$

$4x^2 - 8x - 15x + 15 = 0$

$4x^2 - 23x + 15 = 0$

$4x^2 - 20x - 3x + 15 = 0$

$4x(x - 5) - 3(x - 5) = 0$

$(x - 5)(4x - 3) = 0$

$x - 5 = 0$

$x = 5$

$x = \frac{3}{4}$ which is not possible

Hence, smaller tap will fill the tank = 5 h

Larger tap will fill the tank = 5 – 2 = 3 h

24. If the sum of first four terms of an A.P. is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Ans. Let 'a' and 'd' be the first term and common difference respectively.

$S_4 = 40$

$$\frac{\not{4}^{2}}{\not{2}}[2a+3d]=40 \qquad \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$$

$2a + 3d = 20$ ...(1)

and, $S_{14} = 280$

$$\frac{\not{14}^{7}}{\not{2}}[2a+13d]=280$$

$2a + 13d = 40$ ...(2)

Solving (1) & (2)

$$\begin{array}{r} 2a+3d=20 \\ {}_{(-)}2a+{}_{(-)}13d={}_{(-)}40 \\ \hline -10d=-20 \end{array}$$

$\boxed{d=2} \Rightarrow \boxed{a=7}$

$$\therefore\ S_n = \frac{n}{2}[2\times 7+(n-1)\times 2]$$

$$=\frac{n}{\not{2}}\times \not{2}[7+n-1]$$

$$\boxed{S_n = n^2+6n}$$

25. Prove that : $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$

Ans. $\text{LHS} = \dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$ (dividing each term by cos A)

$$= \frac{(\tan A + \sec A) - 1}{(\tan A - \sec A) + 1} = \frac{\{(\tan A + \sec A) - 1\}(\tan A - \sec A)}{\{(\tan A - \sec A) + 1\}(\tan A - \sec A)}$$

$$= \frac{(\tan^2 A - \sec^2 A) - (\tan A - \sec A)}{\{\tan A - \sec A + 1\}(\tan A - \sec A)}$$

$$= \frac{-1 - \tan A + \sec A}{(\tan A - \sec A + 1)(\tan A - \sec A)}$$

$$= \frac{-1}{\tan A - \sec A} = \frac{1}{\sec A - \tan A}$$

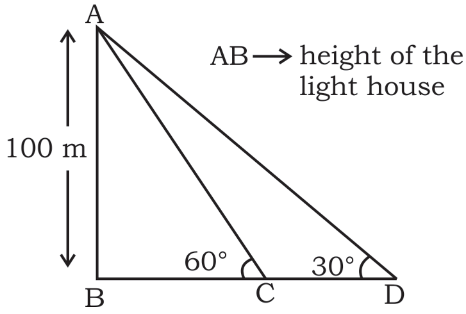
which is the RHS of the identity, we are required to prove.

26. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3}$ = 1.732]

OR

Two poles of equal heights are standing opposite each other on either side of the road. Which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distance of the point from the poles.

Ans.



in ΔABC, $\tan 60° = \frac{AB}{BC}$

$$\sqrt{3} = \frac{100}{BC}$$

$$BC = \frac{100}{\sqrt{3}} \text{ m}$$

in ΔABC, $\tan 30° = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{100}{BD}$$

$$BD = 100\sqrt{3}\text{m}$$

∴ Distance travelled by the boat in 2 minute = CD = BD – BC

$$CD = 100\sqrt{3} - \frac{100}{\sqrt{3}}$$

$$= \frac{300 - 100}{\sqrt{3}}$$

$$CD = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3} \text{ m}$$

∴ Speed of boat = $\frac{\text{Distance travelled}}{\text{Time taken}}$

$$= \frac{200\sqrt{3}}{3} \div 2$$

$$= \frac{200\sqrt{3}}{3} \text{ m / min.}$$

= 57.73 m/min.

OR

Let the height of the pole (AB) = 'h' m

Width of the road (AC) = 80 m

Let the distance of the point E from the pole AB = AE = 'x' m

∴ The distance of the point from the pole CE = EC = $(80 - x)$m

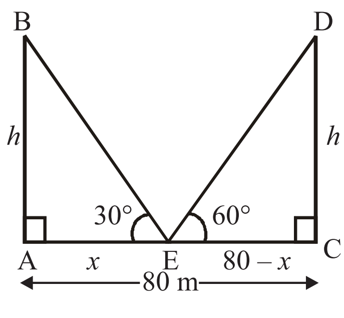
In the first case:

In right ΔBAE,

$$\tan 30° = \frac{AB}{AE}$$

∴ $\frac{1}{\sqrt{3}} = \frac{h}{x}$

∴ $x = \sqrt{3}\, h$ .... (i)



In the second case:

In right ΔDCE, $\tan 60° = \frac{CD}{EC}$

$\therefore \quad \sqrt{3} = \frac{h}{80 - x}$

$\therefore \quad \sqrt{3}\,(80 - x) = h$

$\therefore \quad h = \sqrt{3}\,(80 - \sqrt{3}\,h)$ ..... (From (i))

$\therefore \quad h = 80\sqrt{3} - 3h$

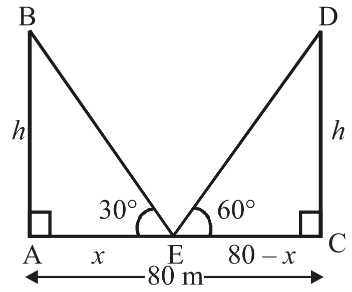
$\therefore \quad h + 3h = 80\sqrt{3}$

$\therefore \quad 4h = 80\sqrt{3}$

$\therefore \quad h = \frac{80\sqrt{3}}{4}$

$\therefore \quad h = 20\sqrt{3}$

$\therefore$ **Height of the poles = $20\sqrt{3}$ m**



Also, $x = \sqrt{3}\,h$

$= \sqrt{3} \times 20\sqrt{3}$

$= 20 \times 3$

$x = 60$ m

and $80 - x = 80 - 60 = 20$ m

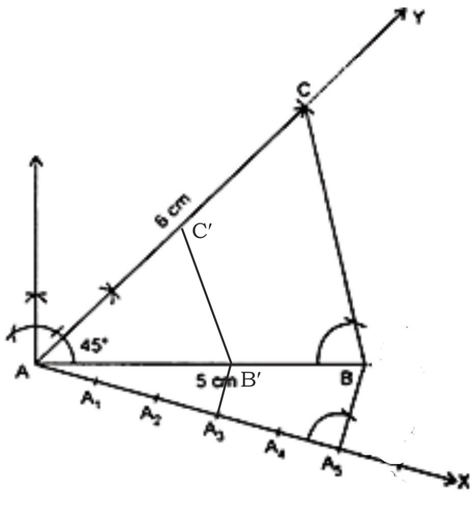
$\therefore$ **Distance of the point from pole AB**

= 60 m and

Distance of the point from the pole CD = 20 m.

27. Construct a ΔABC in which CA = 6 cm, AB = 5 cm and ∠BAC = 45°. Then construct a triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.

Ans. Construct a triangle ABC in which CA = 6 cm, AB = 5 cm and ∠BAC = 45°. Then construct a Δ whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.



Steps of Construction :

1. Draw a line segment AB = 5 cm.
2. Construct ∠A = 45°.
3. Cut on arc AC = 6 cm on Ray AM
4. Join BC, ABC is a triangle with given measurement.
5. Draw an acute angle ∠BAN
7. Join A_5B
8. Construct $\angle AA_3B^1 = \angle AA_5B$.
9. Construct $\angle AB^1C^1 = \angle ABC$
10. ΔAB^1C^1 in the required triangle.

28. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use π = 3.14)

Ans. Volume of frustum = 12308.8 cm^2

R = 20 cm ; r = 120 m.

$$V = \frac{\pi H}{3}(R^2 + r^2 + R.r)$$

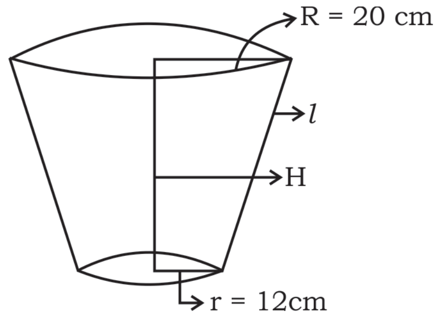
$$\Rightarrow \quad 1230.8 = \frac{3.14}{3} H(20^2 + 12^2 + 20 \times 12)$$

$$\Rightarrow \quad 12308.8 = \frac{3.14}{3} H \ (400 + 144 + 240)$$

$$\Rightarrow \quad 12308.8 = \frac{3.14}{3} H (784)$$

$$\Rightarrow \quad H = \frac{12308.8 \times 3}{3.14 \times 784} = 15 \text{ cm}$$

H = 15 cm



Area of metal sheet = $\pi(R + r)l + \pi r^2$

$$l = \sqrt{H^2 + (R - r)^2} = \sqrt{15^2 + (20 - 12)^2}$$

$$= \sqrt{225 + 64} = \sqrt{289}$$

= 17 cm.

Area = 3.14 (20 + 12) × 17 + 3.14 × 12 × 12

= 3.14(32) × 17 + 3.14 × 144

= 2160.32 cm^2

29. Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides.

Ans. Given a right triangle ABC right angled at B.

To prove $AC^2 = AB^2 + BC^2$

Let us draw BD ⊥ AC).

Now, Δ ADB ~ Δ ABC (By AA similarity)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

or, $AD . AC = AB^2$ ...(1)

Also, ΔBDC ~ ΔABC

So, $\frac{CD}{BC} = \frac{BC}{AC}$

or, $CD . AC = BC^2$ ...(2)

Adding (1) and (2),

$AD . AC + CD . AC = AB^2 + BC^2$

or, $AC (AD + CD) = AB^2 + BC^2$

or, $AC . AC = AB^2 + BC^2$

or, $AC^2 = AB^2 + BC^2$

30. The mean of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-73	Total
Frequency	f_1	5	9	12	f_2	3	2	40

OR

The marks obtained by 100 students of a class in an examination are given below.

Marks	No. of Students
0-5	2
5-10	5
10-15	6
15-20	8
20-25	10
25-30	25
30-35	20
35-40	18
40-45	4
45-50	2

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median.

Ans.

Class	Frequency (f_i)	Cumulative frequency (C.F.)
0-10	f_1	f_1
10-20	5	$f_1 + 5$
20-30	9	$14 + f_1$
30-40	12	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
Total (N)	40	

As, $31 + f_1 + f_2 = 40$

$\Rightarrow \quad f_1 + f_2 = 9$ ...(i)

Median is 32.5, so median class is 30-40.

Here N = 40, $l = 30$, c.f. = $14 + f_1$, $f = 12$, $h = 10$

Using Median = $l + \left(\dfrac{\frac{N}{2} - cf}{f}\right) \times h$

$$32.5 = 30 + \left(\frac{20 - (14 + f_1)}{12}\right) \times 10$$

$\Rightarrow \quad 32.5 - 30 = \left(\dfrac{20 - 14 - f_1}{6}\right) \times 5$

$\Rightarrow \quad 2.5 \times 6 = (6 - f_1) \times 5$

$\Rightarrow \quad 15 = (6 - f_1) \times 5$

$\Rightarrow \quad 3 = 6 - f_1$

$\Rightarrow \quad f_1 = 3$

As, $f_1 + f_2 = 9$ [from eq. (i)]

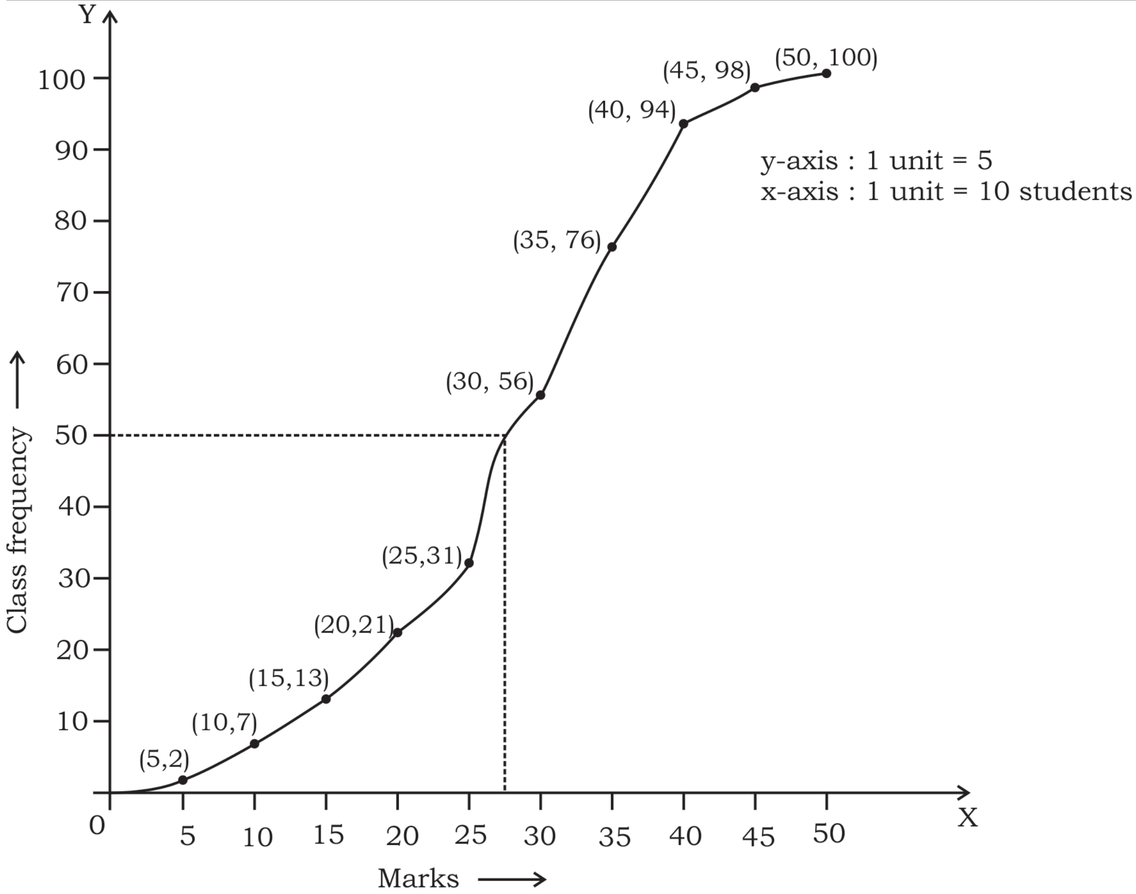
$\Rightarrow \quad 3 + f_2 = 9$

$\Rightarrow \quad f_2 = 6$

Hence, $f_1 = 3$ and $f_2 = 6$.

OR

Marks	No. of students	Marks obtained	of	points on graph
0-5	2	Less than 5	2	(5,2)
5-10	5	Less than 10	7	(10,7)
10-15	6	Less than 15	13	(15,13)
15-20	8	Less than 20	21	(20,21)
20-25	10	Less than 25	31	(25,31)
25-30	25	Less than 30	56	(30,56)
30-35	20	Less than 35	76	(35,76)
35-40	18	Less than 40	94	(40,94)
40-45	4	Less than 45	98	(45,98)
45-50	2	Less than 50	100	(50,100)



n = 100

$\frac{n}{2} = 50$

Median = 25.6 marks

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