NCERT TEXTUAL EXERCISE (SOLVED)

EXERCISE: 14.1

1. Which of the following sentences are statements?

(i) There are 35 days in a month.
(ii) Mathematics is difficult.
(iii) The sum of 5 and 7 is greater than 10.
(iv) The square of a number is an even number.
(v) The sides of a quadrilateral have equal length.
(vi) Answer this question.
(vii) The product of (–1) and 8 is 8.
(viii) The sum of all interior angles of a triangle is 180°.
(ix) Today is a windy day.
(x) All real numbers are complex numbers.

Sol. (i) Statement
(ii) Not a statement
(iii) Statement
(iv) Not a statement
(*v*) Not a statement
(vi) Not a statement
(vii) Statement
(viii) Statement
(ix) Not a statement
(x) Statement

2. Give three examples of sentences which are not statements. Give reasons for the answers.

Sol. The three examples of sentences, which are not statements, are as follows.

(i) He is a Teacher.
It is not evident from the sentence as to whom 'he' is referred to. Therefore, it is not a statement.

(ii) Physics is difficult.
This is not a statement because for some people, physics can be easy and for some others, it can be difficult.

(iii) Where is he going?
This is a question, which also contains 'he', and it is not evident as to who 'he' is. Hence, it is not a statement.

EXERCISE: 14.2

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1. Write the negation of the following statements:

(i) Chennai is the capital of Tamil Nadu.

(ii) $\sqrt{2}$ is not a complex number.

(iii) All triangles are not equilateral triangles.

(iv) The number 2 is greater than 7.

(v) Every natural number is an integer.

Sol. (i) Chennai is not the capital of Tamil Nadu.

(ii) $\sqrt{2}$ is a complex number.

(iii) All triangles are equilateral triangles.

(iv) The number 2 is not greater than 7.

(v) Every natural number is not an integer.

2. Are the following pairs of statements negations of each other?

(i) The number x is not a rational number.
The number x is not an irrational number.

(ii) The number x is a rational number.
The number x is an irrational number.

Sol. (i) The negation of the first statement is 'The number x is a rational number'.

This is same as the second statement. This is because if a number is not an irrational number, then it is a rational number.

Hence, the given statements are negation of each other.

(ii) The negation of the first statement is 'the number x is not a rational number'. This means that the number x is an irrational number, which is same as the second statement.

Hence, the given statements are negations of each other.

3. Find the component statements of the following compound statements and check whether they are true or false.

(i) Number 3 is prime or it is odd.

(ii) All integers are positive or negative.

(iii) 100 is divisible by 3, 11 and 5.

Sol. (i) p : All integers are positive.

q : Number 3 is odd.

p, q are true.

(ii) p : All integers are positive.

q : All integers are negative.

p, q are false.

(iii) The component statements are as follows.

p : 100 is divisible by 3.

q : 100 is divisible by 11.

r : 100 is divisible by 5.

Here, the statements p and q are false and the statement r is true.

EXERCISE: 14.3

1. For each of the following compound statements first identify the connecting words and then break it into component statements.

(*i*) All rational numbers are real and all real numbers are not complex.

(*ii*) Square of an integer is positive or negative.

(*iii*) The sand heats up quickly in the Sun and does not cool down fast at night.

(*iv*) $x = 2$ and $x = 3$ are the roots of the equation $3x^2 - x - 10 = 0$.

Sol. (*i*) Here, the connecting word is 'and'.

p : All rational numbers are real.

q : All real numbers are not complex.

(*ii*) Here, the connecting word is 'or'

p : Square of an integer is positive.

q : Square of an integer is negative.

(*iii*) Here, the connecting word is 'and'

p : The sand heats up quickly in the sun.

q : The sand does not cool down fast at night.

(*iv*) Here, the connecting world is 'and'.

$p : x = 2$ is a root of the equation $3x^2 - x - 10 = 0$.

$q : x = 3$ is a root of the equation $3x^2 - x - 10 = 0$.

2. Identify the quantifier in the following statements and write the negation of the statements.

(*i*) There exists a number which is equal to its square.

(*ii*) For every real number x, x is less than $x + 1$.

(*iii*) There exists a capital for every state in India.

Sol. (i) The quantifier is 'There exists'.

There does not exist a number which is equal to its square.

(ii) The quantifier is 'For every'.

There exist a real number x such that x is not less than $x + 1$.

(iii) The quantifier is 'There exists'.

There exists a state in India which does not have a capital.

3. Check whether the following pair of statements are negation of each other. Give reasons for your answer.

(i) $x + y = y + x$ is true for every real numbers x and y.

(ii) There exist real numbers x and y for which $x + y = y + x$.

Sol. Thus, the given statements are not the negation of each other.

4. State whether the 'Or' used in the following statements is exclusive 'Or' inclusive. Give reasons for your answer.

(i) Sun rises or Moon sets.

(ii) To apply for a driving licence, you should have a ration card or a passport.

(iii) All integers are positive or negative.

Sol. (i) Here, 'or' is exclusive because it is not possible for the Sun to rise and the moon to set together.

(ii) Here, 'or' is inclusive since a person can have both a ration card and a passport to apply for a driving licence.

(iii) Here, 'or' is exclusive because all integers cannot be both positive and negative.

EXERCISE: 14.4

1. Rewrite the following statement with 'if...then' in five different ways conveying the same meaning.

If a natural number is odd, then its square is also odd.

Sol. The given statement can be written in five different ways as follows.

(i) A natural number is odd implies that its square is odd.

(ii) If the square of a natural number is not odd, then natural number is not odd.

(iii) For the square of a natural number to be odd, it is sufficient that the number is odd.

(iv) For a natural number to be odd, it is necessary that its square is odd.

(v) A natural number is odd only if its square is odd.

2. Write the contrapositive and converse of the following statements.

(i) If x is a prime number, then x is odd.

(ii) If the two lines are parallel, then they do not intersect in the same plane.

(iii) Something is cold implies that it has low temperature.

(iv) You cannot comprehend geometry if you do not know how to reason deductively.

(v) x is an even number implies that x is divisible by 4.

Sol. (i) The contrapositive statement:

If a number x is not odd, then x is not a prime number.

The converse statement:

If a number x is odd, then it is a prime number.

(ii) The contrapositive statement:

If two lines intersect in the same plane, then they are not parallel.

The converse statement:

If two lines do not intersect in the same plane, then they are parallel.

(iii) The contrapositive statement:

If something does not have low temperature, then it is not cold.

The converse statement:

If something is at low temperature, then it is cold.

(iv) The contrapositive statement:

If you know how to reason deductively, then you can comprehend geometry.

The converse statement:

If you do not know how to reason deductively, then you cannot comprehend geometry.

(v) The contrapositive statement:

If x is not divisible by 4, then x is not an even number.

The converse statement:

If x is divisible by 4, then x is an even number.

3. Write each of the following statements in the form 'If...then'

(i) You get a job implies that your credentials are good.

(ii) The banana trees will bloom if it stays warm for a month.

(iii) A quadrilateral is a parallelogram if its diagonals bisect each other.

(iv) To get an A^+ in the class, it is necessary that you do all the exercises of the book.

Sol. (i) If you get a job, then your credentials are good.

(ii) If the banana tree stays warm for a month, then it will bloom.

(iii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

(iv) If you want to get an A^+ in the class, then do all the exercises of the book.

4. For the given statements (a) and (b), identify the given statements as contrapositive or converse:

(a) If you live in Delhi, then you have winter clothes.

(i) If you do not have winter clothes, then you do not live in Delhi.

(ii) If you have winter clothes, then you live in Delhi.

(b) If a quadrilateral is a parallelogram, then its diagonals bisect each other.

(i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

(ii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Sol. (a) (i) Contrapositive statement

(ii) Converse statement

(b) (i) Contrapositive statement

(ii) Converse statement

EXERCISE: 14.5

1. Show that the statement:

p : 'If x is real number such that $x^3 + 4x = 0$ then $x = 0$' is true by

(i) direct method

(ii) method of contradiction

(iii) method of contrapositive.

Sol. (i) direct method.

$$x^3 + 4x = 0 \quad \Rightarrow x(x^2 + 4) = 0$$

$$\Rightarrow x^2 + 4 \neq 0, x \in \mathbf{R} \text{ hence, } x = 0$$

(ii) By contradiction:

If $x \in \mathbf{R}$ and $x^3 + 4x = 0$, then $x = 0$.

Let $x \in \mathbf{R}$ but $x \neq 0$

$\Rightarrow x \neq 0$

$\Rightarrow x^3 \neq 0$

$\Rightarrow x^3 + 4x \neq 0$

But $x^3 + 4x = 0$ (given)

$\Rightarrow$ Our assumption is wrong.

$\Rightarrow x = 0$

Hence proved.

(iii) Contrapositive.

$p : x^3 + 4y = 0$

$q : x = 0$

$\sim p : x \neq 0$

$\Rightarrow x^3 \neq 0$

$\Rightarrow x^3 + 4x \neq 0$

$\Rightarrow \sim q \rightarrow \sim p$

Hence proved

2. Show, that the statement 'for any real numbers a and b, $a^2 = b^2$ implies that $a = b$' is not true by giving a counter example.

Sol. Let $a = 2$, $b = -2$ $\Rightarrow$ $a^2 = b^2 = 4$ but $a \neq b$.

Here, the given statement is not true.

3. Show that the following statement is true by the method of contrapositive:

p : If x is an integer and x^2 is also even, then x is also even.

Sol. Let x is not even, i.e. $x = 2n + 1$

$\Rightarrow$ $x^2 = (2n + 1)^2 = 4n^2 + 4n + 1 = 4(n^2 + n) + 1$

$4(n^2 + n) + 1$ is odd, i.e. 'If q is not true then p is not true' is proved

Hence, the given statement is true.

4. By giving counter example, show that the following statement are not true.

(i) p : If all the angles of a triangles are equal then the triangle is obtuse.

(ii) q : The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Sol. (i) Let an angle of triangle be $90^o + \theta$.

Now, Sum of the angles $= 3(90^o + \theta) = 270^o + 3\theta$ which is greater than $180°$

$\therefore$ A triangle having equal angles cannot be obtuse angled triangle.

(ii) The equation $x^2 - 1 = 0$ has one root $x = 1$ which lies between 0 and 2.

$\therefore$ The given statement is not ture.

5. Which of the following statements are true and which are false? In each case give a valid reason for saying so.

(i) p : Each radius of a circle is a chord of the circle.

(ii) q : The centre of a circle bisects each chord of the circle.

(iii) r : Circle is a particular case of an ellipse.

(iv) s : If x and y are integers such that $x > y$, then $-x < -y$.

(v) t : $\sqrt{11}$ is a rational number.

Sol. (i) **False:** The end points of radius do not lie on the circle, therefore, it is not a chord.

(ii) **False:** Only diameters are bisected at the centre. Other chords do not pass through the centre. Therefore, the centre cannot bisect them.

(iii) **True:** Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. It become a circle when $a = b$.

(iv) **True:** By the rule of inequality.

(v) **False:** Since 11 is a prime number, therefore $\sqrt{11}$ is irrational.