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Studymate Solutions to CBSE Board Examination 2018-2019

Series : BVM/1

Code No. 65/1/1

Roll No. | | | | | | | |

Candidates must write the Code on the title page of the answer-book.

- *Please check that this question paper contains 21 printed pages.*
- *Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains 29 questions.*
- *Please write down the Serial Number of the question before attempting it.*
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

MATHEMATICS

[Time allowed : 3 hours] *[Maximum marks : 100]*

General Instructions:

(i) *All questions are compulsory.*

(ii) *The question paper consists of 29 questions.*

(iii) *Marks for each questions are indicated against it.*

(iv) *Questions 1 to 4 in Section-A are Very Short Answer Type Questions carrying one mark each.*

(v) *Questions 5 to 12 in Section-B are Short Answer Type Questions carrying 2 marks each.*

(vi) *Questions 13 to 23 in Section-C are Long Answer I Type Questions carrying 4 marks each.*

(vii) *Questions 24 to 29 in Section-D are Long Answer II Type Questions carrying 6 marks each*

(vii) *Please write down the serial number of the Question before attempting it.*

Section – A

1. If A and B are square matrices of the same order 3, such that |A| = 2 and AB = 2I, write the value of |B|.

Ans. Given |A| = 2

and AB = 2I

$\therefore$ $|AB| = 2^3|I|$

$|A||B| = 2^3$

$\therefore$ |B| = 4

2. If f(x) = x + 1, find $\frac{d}{dx}(fof)(x)$.

Ans. Given f(x) = x + 1

then FOF(x) = f[f(x)]

= f[x + 1]

= x + 1 + 1

x + 2

$\therefore$ $\frac{d}{dx}FOF(x) = \frac{d}{dx}(x + 2)$

= 1

3. Find the order and the degree of the differential equation $x^2\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^4$.

Ans. Order = 2

Degree = 1

4. If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines.

OR

Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$.

Ans. Directions cosines of line are

$l = \cos(90°) = 0$

$m = \cos(135°) = \cos(90° + 45°)$

$= -\sin 45°$

$= -\frac{1}{\sqrt{2}}$

$n = \cos(45°) = \frac{1}{\sqrt{2}}$

OR

Given point (3, 4, 5)

and parallel vector $\vec{b} = 2\hat{i}+2\hat{j}-3\hat{k}$

$\therefore$ Vector equation of line

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k})+\lambda(2\hat{i}+2\hat{j}-3\hat{k})$

Section – B

5. Examine whether the operation * defined on R by a * b = ab + 1 is (i) binary or not. (ii) if a binary operation, is it associative or not?

Ans. (a) $\forall$ a, b $\in$ R

ab $\in$ R

$\Rightarrow$ ab + 1 $\in$ R

$\Rightarrow$ a * b $\in$ R

$\Rightarrow$ Operation is binary.

(b) Let a, b, c $\in$ R

Now, (a * b) * c = (ab + 1) * c = abc + c + 1 ...(i)

a * (b * c) = a * (bc + 1) a(bc + 1) + 1 = abc + a + 1 ...(ii)

By (i) and (ii),

(a* b) * c $\neq$ a * (b * c)

Not Associative.

6. Find a matrix A such that 2A – 3B + 5C = 0, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

Ans. 2A – 3B + 5C = 0

$\therefore$ 2A = 3B – 5C

$$2A = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$2A = \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} - \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix}$$

$$2A = \begin{bmatrix} -6-10 & 6-0 & 0-10 \\ 9-35 & 3-5 & 12-30 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$

7. Find: $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$

Ans. $I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

Let tan x = t

$\sec^2 x \, dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2 + 2^2}} = \log\left|t + \sqrt{t^2 + 4}\right| + C = \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$$

8. Find: $\int \sqrt{1 - \sin 2x}\, dx, \frac{\pi}{4} < x < \frac{\pi}{2}$

OR

Find: $\int \sin^{-1}(2x)\, dx$

Ans. $I = \int \sqrt{1 - \sin 2x}\, dx, \frac{\pi}{4} < x < \frac{\pi}{2}$

$= \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x}\, dx$

$= \int \sqrt{(\sin x - \cos x)^2}\, dx$ ...[$\because$ sin x > cos x for $\frac{\pi}{4} < x < \frac{\pi}{2}$]

$= \int (\sin x - \cos x)\, dx$

= –cos x – sin x + C

OR

$I = \int 1.\sin^{-1}(2x)\, dx$

$= \sin^{-1}(2x)\int 1dx - \int\left\{\frac{d}{dx}\sin^{-1}(2x)\int 1dx\right\}dx$

$= x\sin^{-1}(2x) - \int\frac{1\times 2}{\sqrt{1-4x^2}}\,x\,dx$

$= x\sin^{-1}(2x) - \int\frac{1}{-4}\frac{dt}{\sqrt{t}}$ ...[Let $1 - 4x^2 = t$; $-8xdx = dt$; $2xdx = \frac{dt}{-4}$]

$= x\sin^{-1}(2x) + \frac{1}{4}2\sqrt{t} + C$

$= x\sin^{-1}2x + \frac{1}{2}\sqrt{1-4x^2} + C$

9. Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where 'a' and 'b' are arbitrary constants.

Ans. Family of curve : $y = e^{2x}(a + bx)$

$\frac{y}{e^{2x}} = a + bx$

Differentiating w.r.t. to x.

$\frac{e^{2x}y' - y2e^{2x}}{(e^{2x})^2} = b$

$\frac{y' - 2y}{e^{2x}} = b$

Again differentiating,

$\frac{e^{2x}(y'' - 2y') - (y' - 2y)2e^{2x}}{(e^{2x})^2} = 0$

$\frac{y'' - 2y' - 2(y' - 2y)}{e^{2x}} = 0$

$y'' - 2y' - 2y' + 4y = 0$

$y'' - 4y' + 4y = 0$

Required differential equation.

10. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a}\ \vec{b}\ \vec{c}]$

Ans. Given: $\vec{a}, \vec{b}$ are 2 vector.

Such that

$|\vec{a}| = 1, |\vec{b}| = 1, |\vec{a} + \vec{b}| = 1.$

$\because$ $|\vec{a} + \vec{b}|^2 = |\vec{c}|^2$

$(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = 1$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 1$

$2\vec{a}.\vec{b} = -1$ ...(1)

$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}).(\vec{a} - \vec{b})$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$= 1 + 1 + 1$ {By equation (1)}

$|\vec{a} - \vec{b}|^2 = 3$

$\therefore$ $|\vec{a} - \vec{b}| = \sqrt{3}$

OR

$\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$

$$\therefore \quad \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$

= 2 (–4 –1) – 3(2 + 3) + 1 (1 – 6)

= – 10 – 15 – 5

= – 30

11. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.

Ans. Exp: A die is thrown

∴ S = {1R, 2R, 3R, 4G, 5G, 6G}

A: Number is Even

∴ A = {2R, 4G, 6G}

∴ $P(A) = \frac{3}{6} = \frac{1}{2}$

B: Number marked red.

B = {1R, 2R, 3R}

$P(B) = \frac{3}{6} = \frac{1}{2}$

∴ $P(A) \times P(B)\ \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ...(i)

Now $(A \cap B) = \{2R\}$

$P(A \cap B) = \frac{1}{6}$ ...(ii)

By (i) and (ii)

$P(A \cap B) \neq P(A)\ P(B)$

∴ A and B are not independent.

12. A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of (i) 5 successes? (ii) atmost 5 successes?

OR

The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

Ans. No. of random variable i.e., n = 6.

Success: getting odd no.

∴ $p = P(\text{success}) = \frac{3}{6} = \frac{1}{2}$

∴ $q = 1 - p = \frac{1}{2}$

By Binomial distribution

P(r success) = $^6C_r\ p^r\ q^{n-r}$

(i) P(5 Success) = $^6C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1$

$= 6 \times \frac{1}{2^6}$

$= \frac{6}{64}$

$= \frac{3}{32}$

(ii) P(At most 5 success) = 1 – P(6 success)

$= 1 - {}^6C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$

$= 1 - \frac{1}{64}$

$= \frac{63}{64}.$

OR

Sum of probabilities = 1k + 2k + 3k = 1 or 6k = 1 ⇒ k = $\frac{1}{6}$

The probabiltiy distribution is as given below:

x	***0***	***1***	***2***
P(x)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

Section – C

13. Show that the relation R on **R** defined as R = {(a, b) : a ≤ b}, is reflexive, and transitive but not symmetric.

OR

Prove that the function f : N → N, defined by f(x) = $x^2 + x + 1$ is one-one but not onto. Find inverse of f : N → S, where S is range of f.

Ans. R : **R** → **R**

Define by R = {(a, b) : a ≤ b}

For reflexive:

∵ ∀ x ∈ **R**

x = x

∴ (x, x) ∈ R

∴ Relation is reflexive.

For symmetric:

∵ 1, 3 ∈ **R**

1 < 3 = (1, 3) ∈ R

But 3 < 1 ...[Not true]

⇒ (3, 1) ∉ R

⇒ Relation is not symmetric.

For transitive:

Let (x, y) and (y, z) ∈ R

$\Rightarrow$ $x \le y$ and $y \le z$

$\Rightarrow$ $x \le z$

$\Rightarrow$ $(x, z) \in R$

$\Rightarrow$ Relation is transitive.

OR

$f : N \to N$

Defined by $f(x) = x^2 + x + 1$ ...(i)

For one-one:

Let $x_1, x_2 \in N$

Such that $f(x_1) = f(x_2)$

$\therefore$ $x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\therefore$ $x_1^2 + x_2^2 + x_1 - x_2 = 0$

$\therefore$ $(x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\therefore$ $x_1 + x_2 + 1 \ne 0$ ...[$\because$ x_1 and $x_2 \in N$]

$\therefore$ $x_1 - x_2 = 0$

$\therefore$ $x_1 = x_2$

$\therefore$ Function is one-one.

For onto:

Let $y \in N$ such that

$f(x) = y$

$\therefore$ $x^2 + x + 1 = y$

$\therefore$ $\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 1 = y$

$\therefore$ $\left(x + \frac{1}{2}\right)^2 = y - \frac{3}{4}$

$\therefore$ $x + \frac{1}{2} = \frac{\sqrt{4y-3}}{2}$

$\therefore$ $x = \frac{\sqrt{4y-3}-1}{2} \notin N \ \forall \ y \in N$

$\therefore$ Function is not onto.

Now for $f : N \to S$

Function will be one-one and onto both.

$\therefore$ $f^{-1} : S \to N$

Define by $f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2}$

14. Solve: $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$

Ans. $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$ ...$\left\{\because \tan^{-1} A + \tan^{-1} B = \tan^{-1} \frac{A+B}{1-AB}, AB < 1\right\}$

$\therefore$ $\tan^{-1} \frac{4x+6x}{1-(4x)(6x)} = \frac{\pi}{4}$

$\therefore$ $\frac{10x}{1-24x^2} = \tan\frac{\pi}{4}$

$\therefore$ $10x = 1 - 24x^2$

$\therefore$ $24x^2 + 10x - 1 = 0$

$\therefore$ $24x^2 + 12x - 2x - 1 = 0$

$\therefore$ $12x(2x + 1) - 1(2x + 1) = 0$

$\therefore$ $(12x - 1)(2x + 1) = 0$

$\therefore$ $x = \frac{1}{12}, -\frac{1}{2}$

$\because$ $24x^2 < 1 \Rightarrow x \in \left(\frac{-1}{\sqrt{24}}, \frac{1}{\sqrt{24}}\right)$

$\Rightarrow$ $x = \frac{1}{12}$

15. Using properties of determinants, prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

Ans. LHS = $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$\Delta = \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$

$= (a-1)^2 \begin{vmatrix} a-1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

$= (a - 1)^2 [(a - 1)(1 - 0) - 0 + 0]$

$= (a - 1)^3$

= R.H.S.

16. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$

Ans. $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$

Differentiating w.r.t. to x.

$\frac{d}{dx}\log\left(x^2+y^2\right) = 2\frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right)$

$\frac{1}{x^2+y^2}\frac{d}{dx}\left(x^2+y^2\right) = \frac{2\times 1}{1+\frac{y^2}{x^2}}\frac{d}{dx}\left(\frac{y}{x}\right)$

$\therefore$ $\frac{2x+2yy'}{x^2+y^2} = \frac{2x^2}{x^2+y^2} \times \frac{xy' - y\times 1}{x^2}$

$x + yy' = xy' - y$

$x + y = xy' - yy'$

$y' = \frac{x+y}{x-y}$

$\therefore$ $\frac{dy}{dx} = \frac{x+y}{x-y}$

OR

$x^y - y^x = a^b$

Differentiate w.r.t to x.

$$\frac{d}{dx}x^y - \frac{d}{dx}y^x = 0$$

Let $x^y = P$ and $y^x = Q$

$$\therefore \quad \frac{dP}{dx} - \frac{dQ}{dx} = 0 \qquad \ldots(i)$$

Now, $P = x^y$

taking log

$\log P = y \log x$

Differentiate w.r.t to x

$$\frac{1}{P}\frac{dP}{dx} = \frac{y}{x} + \log x \frac{dy}{dx}$$

$$\therefore \quad \frac{dP}{dx} = x^y\left(\frac{y}{x} + \log x \frac{dy}{dx}\right) \qquad \ldots(ii)$$

$Q = y^x$

Taking log,

$\log Q = x \log y$

Differentiate w.r.t. to x

$$\frac{1}{Q}\frac{dQ}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$$

$$\frac{dQ}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \qquad \ldots(iii)$$

Now by (i),

$$x^y\left(\frac{y}{x} + \log x \frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$

$$x^y \log x \frac{dy}{dx} - xy^{x-1}\frac{dy}{dx} = y^x \log y - yx^{y-1}$$

$$\therefore \quad \frac{dy}{dx} = \frac{y^x \log y - yx^{y-1}}{x^y \log x - xy^{x-1}}$$

17. If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0.$

Ans. $y = (\sin^{-1} x)^2$

Differentiate w.r.t. to x

$$\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1} x)^2$$

$$\frac{dy}{dx} = \frac{2\sin^{-1} x}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1} x$$

Again differentiating with respect to x.

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1 \times -2x}{2\sqrt{1-x^2}}\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

$$\therefore \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

18. Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

Ans. Given: $y = \sqrt{3x-2}$

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}} \text{ = Slope of tngent}$$

$\because$ Tangent is parallel to the line $4x - 2y + 5 = 0$

$$\frac{3}{2\sqrt{3x-2}} = \frac{-4}{-2}$$

$$3 = 4\sqrt{3x-2}$$

$$9 = 16(3x - 2)$$

$$\frac{9}{16} + 2 = 3x$$

$$3x = \frac{41}{16}$$

$$x = \frac{41}{48}$$

at $x = \frac{41}{48}, y = \frac{3}{4}$

Equation of tangent

$$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$

$$\frac{4y-3}{4} = 2\frac{(48x-41)}{48}$$

$$24y - 18 = 48x - 41$$

$$48x - 24y - 23 = 0$$

Equation of normal, at $\left(\frac{41}{48}, \frac{3}{4}\right)$ and slope = $\frac{-1}{2}$

$$y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$$

$$\frac{4y-3}{4} = \frac{-1}{2}\left(\frac{48x-41}{48}\right)$$

$$96y - 72 = -48x + 41$$

$$48x + 96y - 113 = 0$$

19. Find: $\int \frac{3x+5}{x^2+3x-18} fx.$

Ans. Let $I = \int \frac{3x+5}{x^2+3x-18} dx$

$$I = \frac{3}{2}\int \frac{2x + \frac{10}{3}}{x^2+3x-18} dx$$

$$= \frac{3}{2}\int \frac{2x + 3 + \frac{1}{3}}{x^2+3x-18} dx$$

$$I = \frac{3}{2}\int \frac{2x+3}{x^2+3x-18} dx + \frac{3}{2} \times \frac{1}{3}\int \frac{dx}{x^2+3x-18}$$

Let $I = \frac{3}{2}I_1 + \frac{1}{2}I_2$ ...(1)

$$I_1 = \int \frac{2x+3}{x^2+3x-18} dx$$

Let $x^2 + 3x - 18 = t$

$(2x + 3)dx = dt$

$I_1 = \int \frac{dt}{t}$

$= \log |t| + C_1$

$I_2 = \log|x^2 + 3x - 18| + C_1$ ...(2)

Now $I_2 = \int \frac{dt}{x^2 + 3x - 18}$

$= \int \frac{dt}{x^2 + 2\frac{3}{2}x + \frac{9}{4} - \frac{9}{4} - 18}$

$= \int \frac{dt}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}.$

$I_2 = \frac{1}{2\frac{9}{2}} \log\left|\frac{x + \frac{3}{2} - \frac{9}{2}}{x + \frac{3}{2} + \frac{9}{2}}\right| + C_2$

$I_2 = \frac{1}{9}\log\left|\frac{x-3}{x+6}\right| + C_2$ ...(3)

Using (1), (2) and (3), we get

$I = \frac{3}{2}\log|x^2 + 3x - 18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C$ where $C = 9 + C_2$.

20. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx,$ hence evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx.$

Ans. To prove : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

R.H.S = $\int_0^a f(a-x)dx$

Let $a - x = t$

$dx = -dt$

x	0	a
t	a	0

$= -\int_a^0 f(t)dt$

$= \int_0^a f(t)dt$ $\left(\because \int_a^b f(x)dx = -\int_b^a f(x)dx\right)$

$= \int_0^a f(x)dx = \text{L.H.S}$ $\left(\because \int_0^a f(x)dx = \int_0^a f(t)dt\right)$

L.H.S. = R. H.S.

Let, $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ ...(i)

$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx$ ...(ii) $\left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

Adding (i) and (ii), we get,

$2I = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$

$2I = 2\pi\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$ $\left(\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx \text{ if } f(x) = f(2a-x)\right)$

Let $\cos x = t$

$\sin x\, dx = -dt$

x	0	$\pi/2$
t	1	0

$I = -\pi\int_1^0 \frac{dt}{1+t^2}$

$= -\pi\left[\tan^{-1} t\right]_1^0$

$$= -\pi\left[0 - \frac{\pi}{4}\right]$$

$$\boxed{I = \frac{\pi^2}{4}}$$

21. Solve the differential equation: $xdy - ydx = \sqrt{x^2 + y^2}dx$, given that y = 0 when x = 1.

OR

Solve the differential equation: $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition y(0) = 0.

Ans. Given :

$$xdy - ydx = \sqrt{x^2 + y^2}dx$$

$$xdy = (y + \sqrt{x^2 + y^2})dx$$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} \quad ...(1)$$

Let $y = vx$ ...(2)

$$\frac{dy}{dx} = v + \frac{xdv}{dx} \quad ...(3)$$

Using (1), (2), & (3), we get,

$$v + \frac{xdv}{dx} = \frac{vx + \sqrt{x^2 + x^2v^2}}{x}$$

$$v + \frac{xdv}{dx} = v + \sqrt{1 + v^2}$$

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

L.H.S. $= \int \frac{dv}{\sqrt{1 + v^2}}$, Let $v = \tan\theta$

$$= \int \frac{\sec^2\theta dv}{\sec\theta}$$

L.H.S. $= \int \sec\theta d\theta$

$$= \log|\sec\theta + \tan\theta|$$

$$= \log\left|v + \sqrt{1 + v^2}\right|$$

$\therefore \quad \log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log|c|$

So, $\quad v + \sqrt{1 + v^2} = cx$

$$\frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x} = cx$$

$$y + \sqrt{x^2 + y^2} = cx^2 \quad ...(4)$$

Given: y = 0 and x = 1

0 + 1 = c

c = 1

So, solution of differential equation,

$$y + \sqrt{x^2 + y^2} = x^2$$

OR

$$(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$$

$$\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$$

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{2x}{1+x^2} \ \& \ Q=\frac{4x^2}{1+x^2}$$

$$\text{I.F.} = e^{\int Pdx}$$

$$= e^{\int \frac{2x}{1+x^2}dx}$$

$$= e^{\log|1+x^2|}$$

I.F. = $1 + x^2$

So, solution of differential equation,

$$y(1+x^2)=\int \frac{4x^2}{(1+x^2)}(1+x^2)dx$$

$$y(1+x^2)=\int 4x^2dx$$

$$y(1+x^2)=\frac{4}{3}x^3+c.$$

at x = 0, y = 0.

$$0=\frac{4}{3}(0)+c$$

$$\boxed{c=0}$$

$$\therefore y(1+x^2)=\frac{4}{3}(x^3)$$

$$\boxed{y=\frac{4x^3}{3(1+x^2)}}$$

22. If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

Ans. Given:

$$A(\hat{i}+\hat{j}+\hat{k}), B(2\hat{i}+5\hat{j}), C(3\hat{i}+2\hat{j}-3\hat{k}) \text{ and } D(\hat{i}-6\hat{j}-\hat{k})$$

$$\overrightarrow{AB}=(2\hat{i}+5\hat{j})-(\hat{i}+\hat{j}+\hat{k})=\hat{i}+4\hat{j}-\hat{k}$$

$$\overrightarrow{CD}=(\hat{i}-6\hat{j}-\hat{k})-(3\hat{i}+2\hat{j}-3\hat{k})=-2\hat{i}-8\hat{j}+2\hat{k}$$

Angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$

$$\cos\theta=\frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|}=\frac{-2-32-2}{3\sqrt{2}\times 6\sqrt{2}}=\frac{-36}{36}=-1$$

$\theta = \pi$

$\because$ Angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is π.

$\Rightarrow$ $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are opposite and collinear vector.

23. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

Ans. Given lines are

$$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$$

$\Rightarrow \quad \dfrac{x-1}{-3} = \dfrac{y-2}{\frac{\lambda}{7}} = \dfrac{z-3}{2}$ ...(i)

and $\quad \dfrac{7-7x}{3\lambda} = \dfrac{y-5}{1} = \dfrac{6-z}{5}$

$\Rightarrow \quad \dfrac{x-1}{\frac{-3\lambda}{7}} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ ...(ii)

If lines (i) and (ii) are at right angle then,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \frac{\lambda}{7}(1) + 2(-5) = 0$$

$\Rightarrow \quad \dfrac{9\lambda}{7} + \dfrac{\lambda}{7} - 10 = 0$

$\Rightarrow \quad 10\lambda - 70 = 0$

$\Rightarrow \quad \lambda = 7$

Now from (i),

$\dfrac{x-1}{-3} = \dfrac{y-2}{1} = \dfrac{z-3}{2} = \delta$ ...(Say)

Then general point P $(-3\delta + 1, \delta + 2, 2\delta + 3)$.

From (ii),

$\dfrac{x-1}{-3} = \dfrac{y-5}{1} = \dfrac{z-6}{-5} = \mu$ ...(Say)

Then general point in the line is Q $(-3\mu + 1, \mu + 5, -5\mu + 6)$

If lines intersect then,

$-3\delta + 1 = -3\mu + 1$

$\Rightarrow \quad -3\delta + 3\mu = 0$

$\Rightarrow \quad \delta = \mu$ ...(iii)

$\delta + 2 = \mu + 5$

$\Rightarrow \quad \delta - \mu = 3$ ...(iv)

and $\quad 2\delta + 3 = -5\mu + 6$

$\Rightarrow \quad 2\delta + 5\mu = 3$ ...(v)

Solving (iv) and (v)

$2(3 + \mu) + 5\mu = 3$

$6 + 7\mu = 3$

$\mu = \dfrac{-3}{7}, \delta = \dfrac{18}{7}$

Here μ and δ doesnot satisfy (iii).

$\therefore$ Lines are not intersecting

Section – D

24. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations

$x + y + z = 6, x + 3z = 7, 3x + y + z = 12$.

OR

Find the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Ans. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

$|A| = 1(-2) - 1\ (1 - 6) + 1(1)$

$|A| = -2 + 5 + 1$

$|A| = 4$

Cofactor matrix, $C = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$

$$\text{adj}(A) = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Let, $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, which is equal to A.

$\therefore \quad B^{-1} = A^{-1}$

$$B^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$BX = C$

$X = B^{-1}C$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -12+24 \\ 30-14-12 \\ 6+14-12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore \quad x = 3,\ y = 1,\ z = 2.$

OR

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

A = IA

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + 2R_3$ and $R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + 2R_2$ and $R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$$

I = BA

$$B = A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

25. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹70 per square metre for the base and ₹45 per square metre for the sides, what is the cost of least expensive tank?

Ans. Let V be the volume of tank and base dimensions are a and b meter.

Given:

$V = 8\ m^3$

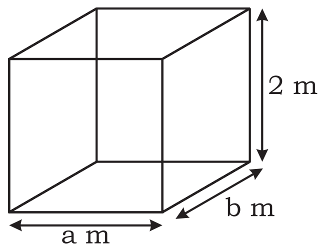
$V = 2 \times a \times b$

$V = 2ab\ m^3$

$2ab = 8$

$ab = 4$

$b = \dfrac{4}{a}$ ...(i)



Area of the base = ab m^2

Cost for building base = ₹ 70ab

Area of the sides = 2(2b) + 2(2a)

Area of the sides = 4(a + b) m^2

Cost for building sides = 45 × 4 (a + b) = ₹ 180(a + b)

Toal cost for tank building = 70ab + 180(a + b)

$$C = 70 \times a \times \frac{4}{a} + 180\left(a + \frac{4}{a}\right)$$ ...[Using (i)]

$$C = 280 + 180\left(a + \frac{4}{a}\right)$$

$$\frac{dC}{da} = 0 + 180\left(1 - \frac{4}{a^2}\right)$$

$$\frac{dC}{da} = 180\left(1 - \frac{4}{a^2}\right)$$

For minima or maxima,

$$\frac{dC}{da} = 0$$

$$180\left(1 - \frac{4}{a^2}\right) = 0$$

a = 2 and b = 2 ...[Using (i)]

$$\frac{d^2C}{da^2} = 180\left(\frac{8}{a^3}\right)$$

$$\left.\frac{d^2C}{da^2}\right|_{a=2} = \frac{180 \times 8}{a^3} > 0$$

$$\frac{d^2C}{da^2} > 0$$

∴ Cost for building minimum at a = 2 m and b = 2 m.

Minimum cost = ₹ $\left[280 + 180\left(2 + \frac{4}{2}\right)\right]$ = ₹ 1,000

26. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

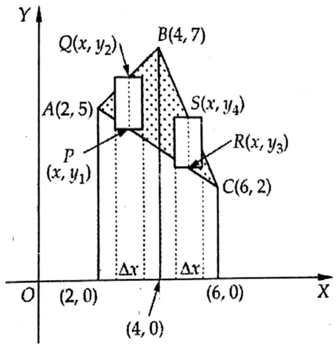
OR

Find the area of the region lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.

Ans. First we find the equations of the sides of triangle ABC by using $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

The equation of AB is $y - 5 = \frac{7-5}{4-2}(x - 2)$

⇒ x – y + 3 = 0 ...(i)



The equation of BC is $y - 7 = \frac{2-7}{6-4}(x - 4)$

⇒ 5x – 2y + 34 = 0 ...(ii)

The equation of side AC is $y - 5 = \frac{2-5}{6-2}(x - 2)$

⇒ 3x + 4y – 26 = 0 ...(iii)

Clearly, Area of ΔABC = Area ADB + Area BDC

Area ADB: To find area ADB, we slice it into vertical strips. We observe that each vertical strip has its lower end on side AC and the upper end on AB. So, the approximating rectangle has

Length = $(y_2 - y_1)$, Width = Δx and Area = $(y_2 - y_1)$ Δx. Since the approximating rectangle can move from x = 2 to x = 4.

∴ Area ADB $\int_2^4 (y_2 - y_1)dx$

⇒ Area ADB = $\int_2^4 \left\{(x + 3) - \left(\frac{26 - 3x}{4}\right)\right\}dx$ [∵ $P(x, y_1)$ and $Q(x, y_2)$ lie on (iii) and (i) respec. ∵ $3x + 4y_1 - 26 = 0$ and $y_2 = x + 3$]

Similarly, we have

⇒ Area BDC $= \int_4^6 (y_4 - y_3)dx$

⇒ Area BDC $= \int_4^6 \left\{\left(\frac{34 - 5x}{2}\right) - \left(\frac{26 - 3x}{4}\right)\right\}dx$ [∵ $R(x, y_3)$ and $S(x, y_4)$ lie on (iii) and (ii) respec. ∵ $3x + 4y_3 - 26 = 0$ and $5x + 2y_4 - 34 = 0$]

$\therefore$ Area of $\Delta ABC = \int_2^4 \left\{(x+3) - \left(\frac{26-3x}{4}\right)\right\} dx + \int_4^6 \left\{\left(\frac{34-5x}{2}\right) - \left(\frac{26-3x}{4}\right)\right\} dx$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{4}\left[\frac{7x^2}{2} - 14x\right]_2^4 + \left[42x - \frac{7x^2}{2}\right]_4^6$

= 7 square units.

OR

The given equation of the circle $x^2 + y^2 = 8x$ can be expressed as $(x - 4)^2 + y^2 = 16$. Thus, the centre of the circle is (4, 0) and radius is 4. Its intersection with the parabola $y^2 = 4x$ gives

$x^2 + 4x = 8x$

or $x^2 - 4x = 0$

or $x(x - 4) = 0$

or $x = 0, x = 4$

Thus, the points of intersection of these two curves are O(0, 0) and P(4,4) above the x-axis.

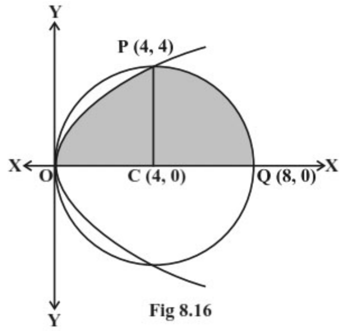


Fig 8.16

From the Fig 8.16, the required area of the region OPQCO included between these two curves above x-axis is

= (area of the region OCPO) + (area of the region PCQP)

$= \int_0^4 y dx + \int_4^8 y dx$

$= 2\int_0^4 \sqrt{x}\, dx + = \int_4^8 \sqrt{4^2 - (x-4)^2} dx$

$= 2 \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4 + \int_0^4 \sqrt{4^2 - t^2} dt,$ where, x – 4 = t

$= \frac{32}{3} + \left[\frac{t}{2}\sqrt{4^2 - t^2} + \frac{1}{2} \times 4^2 \sin^{-1}\frac{t}{4}\right]_0^4$

$= \frac{32}{3} + \left[\frac{4}{2} \times 0 + \frac{1}{2} \times 4^2 \sin^{-1} 1\right] = \frac{32}{3} + \left[0 + 8 \times \frac{\pi}{2}\right] = \frac{32}{3} + 4\pi = \frac{4}{3}(8 + 3\pi)$ square units

27. Find the vector and Cartesian equations of the plane passing through the points (2, 2, –1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

OR

Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and the point (–1, 3, –4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane, thus obtained.

Ans. Given A(2, 2, –1), B(3, 4, 2) and C(7, 0, 6) equation of plane:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 2 & y - 2 & z + 1 \\ 3 - 2 & 4 - 2 & 2 + 1 \\ 7 - 2 & 0 - 2 & 6 + 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 2 & y - 2 & z + 1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$$

Expending along R_1,

(x – 2) (20) – (y – 2) (–8) + (z + 1) (–12) = 0

5(x – 2) + 2(y – 2) – 3(z + 1) = 0

$5x - 10 + 2y - 4 - 3z - 3 = 0$

$5x + 2y - 3z - 17 = 0$ Cartesian form

$\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) - 17 = 0$ vector form

The equation of the plan parallel to the plane $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) - 17 = 0$ is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) - d = 0$.

Since it passes through (4, 3, 1). Therefore, $(4\hat{i} + 3\hat{j} + \hat{k}).(5\hat{i} + 2\hat{j} - 3\hat{k}) = d$

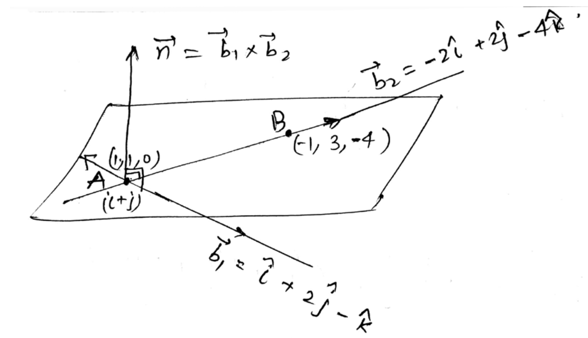
$20 + 6 - 3 = d$

$d = 23.$

$\therefore$ the required equation of plane is $\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 23$

OR

Given: line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$



$\overrightarrow{b_1} = \hat{i} + 2\hat{j} - \hat{k}$

A vector joining point A(1, 1, 0) and B(–1, 3, –4)

$\overrightarrow{AB} = \overrightarrow{b_2} = -2\hat{i} + 2\hat{j} - 4\hat{k}$

$\therefore$ normal vector to the plane is given by

$\vec{n} = \vec{b_1} \times \vec{b_2}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 2 & -4 \end{vmatrix}$$

$= \hat{i}(-8 + 2) - \hat{j}(-4 - 2) + \hat{k}(2 + 4)$

$\vec{n} = -6\hat{i} + 6\hat{j} + 6\hat{k}.$

Equation of plane passing through (–1, 3, –4)

$(\vec{r} - \vec{a}).\vec{n} = 0$

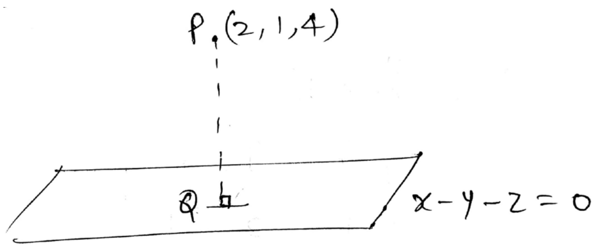
$(\vec{r} - (-\hat{i} + 3\hat{j} - 4\hat{k})).(-6\hat{i} + 6\hat{j} + 6\hat{k}) = 0$

$\vec{r}.(-\hat{i} + \hat{j} + \hat{k}) - (1 + 3 - 4) = 0$

$\vec{r}.(-\hat{i} + \hat{j} + \hat{k}) = 0$

$-x + y + z = 0$

or $x - y - z = 0.$



$$PQ = \frac{|2 - 1 - 4|}{\sqrt{1^2 + (-1)^2 + (-1)^2}}$$

$$PQ = \frac{3}{\sqrt{3}} = \sqrt{3} \text{ units.}$$

28. A manufacturer has three machine operators A, B and C. The first operator A produces 50% of defective items, whereas the other two operators B and C produces 5% and 7% detective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A?

Ans. Let E_1 = Machine operator A

E_2 = Machine operator B

E_3 = Machine operator C

and F = Item is defective

$\therefore$

$$P(E_1) = \frac{50}{100} = \frac{5}{10} \quad \Bigg| \quad P(F/E_1) = \frac{1}{100}$$

$$P(E_2) = \frac{30}{100} = \frac{3}{10} \quad \Bigg| \quad P(F/E_2) = \frac{5}{100}$$

$$P(E_3) = \frac{20}{100} = \frac{2}{10} \quad \Bigg| \quad P(F/E_3) = \frac{7}{100}$$

Then, $P(E_1/F) = \dfrac{P(E_1).P(F/E_1)}{P(E_1).P(F/E_1) + P(E_2).P(F/E_2) + P(E_3).P(F/E_3)}$

$$= \frac{\frac{5}{10}.\frac{1}{100}}{\frac{5}{10}.\frac{1}{100} + \frac{3}{10}.\frac{5}{100} + \frac{2}{10}.\frac{7}{100}} = \frac{5}{5+15+14} = \frac{5}{34}$$

29. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹15 and on an item of model B is ₹10. How many of items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.

Ans. Let the number of items of model A and B be x and y respectively.

Then maximize profit Z = 15x + 10y ...(i)

Subject to contraints

2x + y ≤ 40 ...(ii)

2x + 3y ≤ 80 ...(iii)

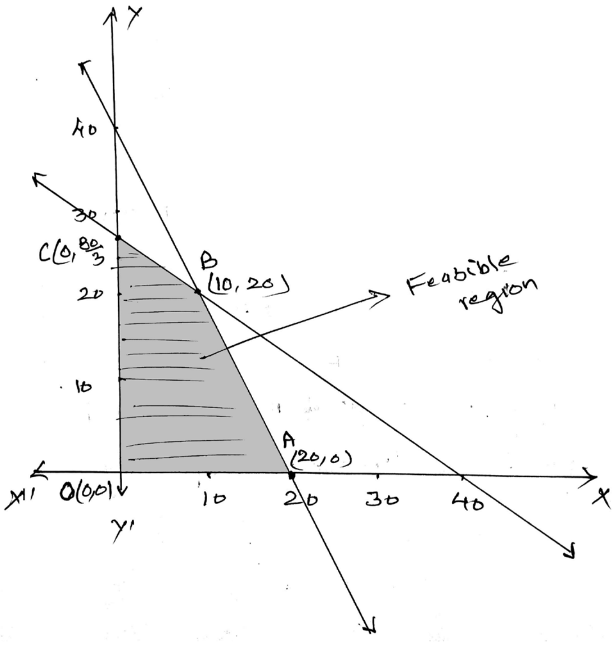
and x ≥ 0, y ≥ 0

Let 2x + y = 40

x	0	20
y	40	0

and 2x + 3y = 80

x	10	40
y	20	0



Corner Point	***Optimum Value (15x + 10y)***
(20, 0)	₹ 300
(10, 20)	₹350 (Maximum)
$\left(0, \frac{80}{3}\right)$	₹ $\frac{800}{3}$

No. of item of model A and B should be manufactured 10 and 20 respectively and maximum profit = ₹350.

By corner point theorem optimum value will be maximum at corner point (10, 20).

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