SOLVED NCERT QUESTIONS

1. Choose the correct alternative from the clues given at the end of each statement:

(a) The size of the atom in Thomson's model is _____________ the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

(b) In the ground state of an atom, electrons are in stable equilibrium, while in _____________ electrons always experience a net force. (Thomson's model/Rutherford's model.)

(c) A classical atom based on _____________ is doomed to collapse. (Thomson's model/Rutherford's model.)

(d) An atom has a nearly continuous mass distribution in a _____________ but has a highly non-uniform mass distribution in _____________ (Thomson's model/Rutherford's model.)

(e) The positively charged part of the atom possesses most of the mass in _____________ (Rutherford's model/both the models.)

Sol. (a) No different from (b) Rutherford's model

(c) Rutherford's model (d) Thomson's model; Rutherford's model

(e) Both the models.

2. Suppose you are given a chance to repeat the α-particle scattering experiment using a thin sheet solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K). What results do you expect?

Sol. The α-particle scattering for large impact parameters would be the same. For smaller impact parameters the scattering would reduce considerably as hydrogen being a small atom would not be able to exert much force on the bombarding α-particle. Even for head-on collision there would be hardly any scattering for the same reason.

3. What is the shortest wavelength present in the Paschen series of spectral lines?

Sol. For shortest wavelength, photon has maximum energy

$\therefore\ E = 0$

$\therefore\ n_f = 3 \qquad n_i = \infty$

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{1}{\lambda} = \frac{R}{9}$$

$$\lambda = \frac{9}{R} = \frac{9}{1097 \times 10^{7}} = 8.204 \times 10^{-7} \text{ m}$$

4. A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

Sol. Given $E_f - E_i = 2.3$ eV

Also, $h\nu = E_f - E_i = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19}$ J

$$\Rightarrow \nu = \frac{2.3 \times 1.6 \times 10^{-19}}{h} = \frac{2.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 0.557 \times 10^{15}$$

$\therefore \ \nu \simeq 5.6 \times 10^{14}$ Hz

5. The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

Sol. $E_1 = -13.6$ eV

We know, for ground state of H-atom,

K.E. $= -E_1 = -(-13.6)$ eV $= 13.6$ eV

P. E. $= 2E_1 = 2 \times (-13.6)$ eV $= -27.2$ eV.

6. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon.

Sol. Given $n_i = 1$, $n_f = 4$

For H-atom, $E_1 = -13.6$ eV and $E_4 = \dfrac{E_1}{n^2} = \dfrac{E_1}{4^2} = \dfrac{-13.6}{16} = -0.85$ eV

$\therefore \quad E_f - E_i = -0.85 - (-13.6) = 12.75 \text{ eV} = 12.75 \times 1.6 \times 10^{-19}$ J
$= 20.4 \times 10^{-19}$ J.

Frequency of photon absorbed is, $\nu = \dfrac{E_f - E_i}{h} = \dfrac{20.4 \times 10^{-19}}{6.6 \times 10^{-34}}$
$= 3.09 \times 10^{15}$ Hz.

Wavelength of photon is $\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{8}}{3.09 \times 10^{15}} = 0.97 \times 10^{-7}$ m
$= 97 \times 10^{-9}$ m $= 97$ nm.

7. (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $n = 1$, 2 and 3 levels.

(b) Calculate the orbital period in each of these levels.

Sol. (a) Velocity of electron in n-th orbit of H-atom is, $v_n = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{2\pi e^2}{nh}\right)$

For $n = 1$, $v_1 = 9\times10^9 \times \dfrac{2\times3.14\times(1.6\times10^{-19})^2}{1\times6.6\times10^{-34}}$

(We know, $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2})

$\Rightarrow v_1 = 21.9 \times 10^5$ m/s $\simeq 2.2 \times 10^6$ m/s

For $n = 2$, by formula, $v_2 = \dfrac{v_1}{2} = 1.1 \times 10^6$ m/s

For $n = 3$, $v_3 = \dfrac{v_1}{3} = 0.73 \times 10^6$ m/s

Size of the n-th orbit is given by, $r_n = \dfrac{n^2}{m}\left(\dfrac{h}{2\pi}\right)^2 \dfrac{4\pi\varepsilon_0}{e^2}$

(b) Thus orbital velocity, $v_n = \dfrac{\text{Orbital Circumference}}{\text{Time Period}} = \dfrac{2\pi r_n}{T}$

$\Rightarrow \quad T = \dfrac{2\pi r_n}{v_n}$

$\Rightarrow T = 2\pi\times4\pi\varepsilon_0\dfrac{n^2h^2}{4\pi^2me^2}\times4\pi\varepsilon_0.\dfrac{nh}{2\pi e^2} = \dfrac{4\varepsilon_0^2h^3n^3}{me^4}$

Substitute values of $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

$h = 6.6 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C

We get, $T = \dfrac{4\times(8.85\times10^{-12})^2\times(6.6\times10^{-34})^3}{9.1\times10^{-31}\times(1.6\times10^{-19})^4}\times n^3$

$= \dfrac{4\times(8.85)^2\times(6.6)^3}{9.1\times(1.6)^4}\times10^{-24-102+31+76}\times n^3$

$T = 1505 \times 10^{-19}$ s $\times n^3 = 1.5 \times 10^{-16}$ s $\times n^3$

For $n = 1$; $T_1 = 1.5 \times 10^{-16}$ s

For $n = 2$; $T_2 = 1.5 \times 10^{-16} \times 2^3 = 12 \times 10^{-16}$ s $= 1.2 \times 10^{-15}$ s

For $n = 3$; $T_3 = 1.5 \times 10^{-16} \times 3^3 = 40.5 \times 10^{-16} = 4.05 \times 10^{-15}$ s.

8. The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits?

Sol. The radius of n-th orbit is given by, $r_n = 4\pi\varepsilon_0\dfrac{n^2h^2}{4\pi^2me^2} \quad \Rightarrow \quad r_n \propto n^2$

$\Rightarrow \quad \dfrac{r_1}{r_2} = \dfrac{n_1^2}{n_2^2}$ (for 1st and 2nd orbit)

$\Rightarrow \quad r_2 = r_1 \times \frac{n_2^2}{n_1^2} = r_1 \times \frac{2^2}{1^2} = 4r_1 = 4 \times 5.3 \times 10^{-11}$ m

$\Rightarrow \quad r_2 = 21.2 \times 10^{-11} = 2.12 \times 10^{-10}$ m

And $\quad r_3 = r_1 \times \frac{n_3^2}{n_1^2} = r_1 \times \frac{3^2}{1^2}$

$\Rightarrow \quad r_3 = 9 \times 5.3 \times 10^{-11}$ m $= 47.7 \times 10^{-11}$ m

$r_3 = 4.77 \times 10^{-10}$ m.

9. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Sol. Energy of incident beam = 12.5 eV

At room temperature, H will be in ground state whose energy is $E_1 =$ – 13.6 eV

$\therefore$ Total energy of electron after absorption will be $E_1 + E_{incident} = -13.6 + 12.5 = -1.1$ eV

$$n^2 = \frac{-13.6}{E_n} = \frac{-13.6}{-1.1}$$

$n = 3.5$

Taking integral value this corresponds to a level $n = 3$

The possibilities for the electron is to jump from n-th level are

(a) For $n_2 = 3$ to $n_1 = 1$ within Lyman series

(b) For $n_2 = 2$ to $n_1 = 1$ in Lyman series.

(c) For $n_2 = 3$ to $n_1 = 2$ in Balmer series.

The wave length for each series would be

(a) For $n_2 = 3$ to $n_1 = 1$ (Lyman series)

$E_{31} = E_3 - E_1 = -1.5 - (-13.6) = 12.1$ eV

It has wavelength, $\lambda_{31} = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{12.1 \times 1.6 \times 10^{-19}} = 1.02 \times 10^{-7}$ m $= 102$ nm

(b) For $n_2 = 2$ to $n_1 = 1$ (Lyman Series)

$E_{21} = E_2 - E_1 = -3.4 - (-13.6) = +10.2$ eV

$\therefore$ Wavelength emitted, $\lambda_{21} = \frac{hc}{E_{21}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{10.2 \times 1.6 \times 10^{-19}} = 121$ nm

(c) Also for Balmer Series, $n_2 = 3$, $n_1 = 2$

$E_{32} = E_3 - E_2 = -1.5 - (-3.4) = 1.9$ eV

$\therefore$ Wavelength emitted, $\lambda_{32} = \frac{hc}{E_{32}} = \frac{6.6\times10^{-34}\times3\times10^{8}}{1.9\times1.6\times10^{-19}}$

$= 6.5 \times 10^{-7}$ m = 651 nm

$\therefore$ The excited electron can either jump from $n = 3$ level to $n = 1$ directly giving wavelength 102 nm or first jump from $n = 3$ to $n = 2$ level emitting wavelength of 651 nm and then come to ground state emitting wavelength of 121 nm.

10. In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)

Sol. According to Bohr's model the angular momentum in the n-th orbit is given by $mvr = \frac{nh}{2\pi}$

Putting $m = 6 \times 10^{24}$ kg, $v = 3 \times 10^4$ m/s, $r = 1.5 \times 10^{11}$ m

For earth, the quantum number is

$$n = \frac{mvr}{h}\times 2\pi = \frac{6\times10^{24}\times3\times10^{4}\times1.5\times10^{11}\times2\times3.14}{6.6\times10^{-34}}$$

$= 25.5 \times 10^{24+4+11+34} \simeq 2.55 \times 10^{73+1} \simeq 2.6 \times 10^{74}$

11. Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e. scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Sol. (a) About the same as we are talking about average angle of deflection.

(b) Much less as in Thomson's model there is no such thing as a massive central core called nucleus.

(c) This implies that scattering of α-particles is due to single collision only. If thickness increases then chances of single collision increases because the number of target atoms would increase. Thus, moderately scattered α-particles would increase in number.

(d) In Thomson's model, positive charge is uniformly spread out, hence there would be hardly any noticeable deflection due to single collision. Hence, multiple scattering has to be considered for average scattering angle.

In Rutherford's model most of the scattering is due to single collision hence multiple scattering can be ignored.

12. The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Sol. In Bohr's model, the Coulombic force of attraction, $F_C = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{r^2}$

yield the first orbit of H-atom having radius r_0 as

$$r_0 = \frac{4\pi\varepsilon_0 (h/2\pi)^2}{m_e e^2} \qquad \text{(put } Z = 1\text{)}$$

If atoms were bound by gravitational force, $F_G = \dfrac{G m_p m_e}{r^2}$

then Orbital radius for $n = 1$ would be $r_{OG} = \dfrac{(h/2\pi)^2}{G m_p m_e^2}$

Substituting standard values, we get $r_{OG} = 1.2 \times 10^{29}$ m

This is much greater than the universe itself.

13. Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level $(n - 1)$. For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Sol. The frequency ν of the emitted radiation when H-atom de-excites from n to $(n - 1)$ level is

$$E = h\nu = E_{n-1} - E_n$$

$$\Rightarrow \qquad \nu = \frac{E_{n-1} - E_n}{h} = \frac{1}{2}\frac{mc^2\alpha^2}{h}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

where $K = 1/4\pi\varepsilon_0$ and $\alpha = \dfrac{2\pi Ke^2}{ch}$ = fine structure constant.

$$\Rightarrow \qquad \nu = \frac{1}{2}\frac{mc^2\alpha^2}{h}\left[\frac{n^2-(n-1)^2}{n^2(n-1)^2}\right] = \frac{mc^2\alpha^2}{2h}\left[\frac{(n+n-1)(n-n+1)}{n^2(n-1)^2}\right]$$

$$= \frac{mc^2\alpha^2}{2hn^2}\frac{2n-1}{(n-1)^2}$$

For large n, $(n-1) \approx n$ and $2n-1 \approx 2n \Rightarrow \nu = \dfrac{mc^2\alpha^2}{2h}.\dfrac{2n}{n^2} = \dfrac{mc^2\alpha^2}{hn^3}$

Put value of $\alpha = \dfrac{2\pi Ke^2}{ch}$

$$\nu = \frac{mc^2}{hn^3}\times\frac{4\pi^2K^2e^4}{c^2h^2} \qquad \Rightarrow \quad \nu = \frac{4\pi^2mK^2e^4}{n^3h^3} \qquad ...(1)$$

In Bohr's atom, velocity of electron is $v = \dfrac{nh}{2\pi mr}$, radius $r = \dfrac{n^2h^2}{4\pi^2mKe^2}$

$\therefore$ Frequency of revolution of electron, $\nu = \dfrac{1}{T} = \dfrac{v}{2\pi r} = \dfrac{nh}{2\pi mr}.\dfrac{4\pi^2mKe^2}{2\pi n^2h^2}$

$$\Rightarrow \qquad \nu = \frac{Ke^2}{nhr}$$

Substitute for r, $\nu = \dfrac{Ke^2}{nh}\left(\dfrac{4\pi^2mKe^2}{n^2h^2}\right) \Rightarrow v = \dfrac{4\pi^2mK^2e^4}{n^3h^3}$ ...(2)

From (1) and (2), we can say that for large values of n, classical frequency of revolution of electron in n-th orbit is same as frequency of radiation emitted when H-atom de excites from level n to level $(n-1)$.

14. Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom ($\sim 10^{-10}$ m).

(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e and c. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves

c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e and e and confirm that its numerical value has indeed the correct order of magnitude.

Sol. (a) The quantity $\left(\frac{e^2}{\varepsilon_0 mc^2}\right)$ has dimensions of length. Its value is $\simeq 10^{-15}$ m is much smaller than atomic size.

(b) The quantity $\left(\frac{\varepsilon_0 (h)^2}{me^2}\right)$ has dimensions of length. Its value is $\approx 10^{-10}$ m of the order of atomic size. However constants like 2π or 4π have to be added to arrive at correct formula.

15. The total energy of an electron in the first excited state of the hydrogen is about –3.4 eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Sol. We know, Kinetic energy of electron = $\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$

Potential energy = –2(K. E.)

Total energy = P. E. + K. E. = – K. E. where electric potential would be zero at infinity.

(a) Total energy = – 3.4 eV ⇒ K. E. = – (– 3. 4) = + 3.4 eV.

(b) P. E. = – 2 K. E. = – 6.8 eV.

(c) If zero of potential is changed then K. E. which is independent of this choice remains same, whereas P. E. and total energy of the state would change.

16. If Bohr's quantisation postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun.

Sol. Bohr's quantisation is proportional to h (Planck's constant). But angular momenta associated with planetary motion are of the order of 10^{70} h. This would mean $n \simeq 10^{70}$. For such large values of n, the differences in successive energies and angular momenta are so small that they are continuous and not discrete.

17. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom (i.e. an atom in which a negatively charged muon (μ^-) of mass about 207 m_e orbits around a proton).

Sol. A muonic H-atom in which negatively charged muon of mass 207 m_e revolves around a proton is given. ($m_\mu = 207\ m_e$)

Bohr's model gives $r \propto \dfrac{1}{m}$

$$\therefore \quad \frac{r_\mu}{r_e} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e} = \frac{1}{207}$$

r_e = radius of 1st Bohr orbit in H-atom = 0.5 Å = 0.5×10^{-10} m.

$$\Rightarrow \quad r_\mu = \frac{1}{207} \times 0.5 \times 10^{-10}\, m \qquad \Rightarrow \quad r_\mu \simeq 2.5 \times 10^{-13} \text{ m}$$

Also, $E \propto m \Rightarrow \dfrac{E_\mu}{E_e} = \dfrac{m_\mu}{m_e} = \dfrac{207 m_e}{m_e} = 207$

For H-atom at ground state, E = – 13.6 eV

$\Rightarrow \quad E_\mu = 207 \times (-13.6) = -2815.2$ eV

$\Rightarrow \quad E_\mu = -2.82$ keV.