NCERT TEXTUAL EXERCISE (SOLVED)

1. Give the magnitude and direction of the net force acting on

(a) a drop of rain falling down a constant speed,

(b) a cork of mass 10 g floating on water,

(c) a kite skillfully held stationary in the sky,

(d) a car moving with a constant velocity of 30 km/h on rough road

(e) a high speed electron in space far from all gravitational objects, and free of electric and magnetic fields.

Sol. (a) As the raindrop is falling with a constant speed, its acceleration $a = 0$, hence net force $F = ma = 0$.

(b) As the cork is floating on water, its weight is being balanced by the upthrust (= weight of water displaced), hence net force on the cork = 0.

(c) As the kite is held stationary, net force on the kite is zero, in accordance with Newton's 1st law.

(d) As car moving with constant velocity, $a = 0$. So, $F = ma = 0$.

(e) As no fields (gravitational, electrical or magnetic) act on the electron, net force $F = 0$.

2. A pebble of mass 0.05 kg is thrown vertically upwards. Give the magnitude and direction of net force on the pebble,

(a) during its upward motion,

(b) during its downward motion,

(c) at the highest point, where it is momentarily at rest.

Do your answers change if the pebble were thrown at an angle, say, 45° with the horizontal direction?

Sol. If a body is thrown vertically upwards or downwards, the acceleration acting on the body is uniform i.e. acceleration due to gravity = g, in the downward direction. Therefore, the net force on the pebble in all the three cases is vertically downwards.

As $m = 0.05$ kg and $a = 9.8$ m/s^2.

$\therefore$ In all three cases, $F = ma = 0.05 \times 9.8 = 0.49$ N, vertically downwards.

If the pebble were thrown at an angle of 45° with the horizontal direction, it will have horizontal and vertical components of velocity. These components do not affect the force on the pebble. Hence, our answers do not alter any case. However, in case (c), the pebble will not be at rest. It will have horizontal component of velocity at the heightest point.

3. Give the magnitude and direction of the net force acting on.

(a) A stone of mass 0.1 kg just after it is dropped from the window of a stationary train,

(b) The same stone as above just it is dropped from the window of a train running at a constant velocity of 36km/h.

(c) The same stone as above just after it is dropped from the window of train accelerating with 1 m/s^2

(d) The same stone as above just after it is dropped from the window of a train accelerating with 1 m/s^2 the stone being at rest relative to the train. Neglect the resistance of air throughout and take $g = 9.8$ m/s^2.

Sol. (a) Given, $m = 0.1$ kg, $a = + g = 9.8$ m/s^2.

Net force, $F = ma = 0.1 \times 9.8 = 0.98$ N, which acts vertically downwards.

(b) When the train is running at a constant velocity, its acceleration = 0, i.e. no force acts on the stone due to this motion.

$\therefore$ Force on the same F = Weight of stone = $mg = 0.1 \times 9.8 = 0.98$ N. Which acts vertically downwards.

(c) When the train is running with a acceleration, 1m/s^2, additional force, $F' = ma = 0.1 \times 1 = 0.1$ N acts on the stone in the horizontal direction. But once the stone is dropped from the train, F' becomes zero and the net force on the stone, $F = ma = 0.1 \times 9.8 = 0.98$ N, acting vertically downwards.

(e) As the stone is lying on the floor of the train, its acceleration is same as that of the train.

$\therefore$ Force acting on the stone, $F = ma = 0.1 \times 1 = 0.1$ N. This force is along the horizontal direction of motion of the train. Note that, weight of the stone in this case is being balanced by the normal reaction.

4. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1}. How long does the body take to stop?

Sol. Here, $F = -50$ N, $m = 20$ kg, $u = 15$ ms^{-1} and $v = 0$;

Now, F = ma, a = F/m = –50/20 = –2.5 ms^{-2}

From relation, $v = u + at$, $0 = 15 - 2.6t$, $t = 15/2.5 = 6$ s

5. A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 ms^{-1} to 3.5 m/s in 25 s. The direction of motion of the body remains unchanged. What is the magnitude and direction of the force?

Sol. Here, $m = 3.0$ kg, $u = 2.0$ m/s and $v = 3.5$ m/s, $t = 25$ s;

Now, $F = ma = m\ (v - u)/t = 3.0\ (3.5 - 2.0)/\ 25 = 0.18$ N

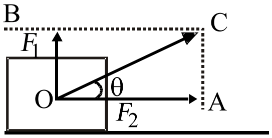
$\therefore$ The force is along the direction of motion.

6. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.

Sol. Here, $m = 5$ kg, F_1 = OA = 8 N and F_2 = OB = 6 N

From the adjoining figure,

Resultant force, $F = \sqrt{F_1^2 + F_2^2} = \sqrt{8^2 + 6^2} = 10$ N



tan COA = tanθ = AC/OA = OB/OA = 6/8 = 0.75

$\therefore$ θ = 36°52′

which is the direction of the resultant force, and hence the direction of the acceleration.

$\therefore$ $a = F/m = 10/5 = 2$ m/s

7. The driver of a three wheeler moving with speed of 36 km/h see a child standing in the middle of the road and brings his vehicle to rest in 4 s just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

Sol. Here, u = 36 km/h = 10 m/s, t = 4s, m = 400 + 65 = 465 kg

Now, Retarding force, $F = ma = m\ (v - u)/t = 465\ (0 - 10)/\ 4 = -1162.5$ N

8. A rocket with lift off mass 20,000 kg is blasted upwards with net initial acceleration of 5 m/s^2. Calculate the initial thrust of the blast.

Sol. Here, m = 20,000 kg, a = 5 m/s^2

The thrust should be such that it overcomes the force of gravity besides giving it an upward acceleration of 5 m/s^2. Thus, the force should produce a net acceleration of 9.8 + 5.0 = 14.8 m/s^2.

As thrust = force = mass × acceleration.

$\therefore$ Force, $F = 20{,}000 \times 14.8 = 2.96 \times 10^5$ N.

9. A particle of mass 0.40 kg moving initially with a constant speed of 10 m/s to the north is subject to a constant force of 8.0 N directed towards the south for 30 s. Take the instant the force is applied to be $t = 0$, and the position of the particle at that time to be $x = 0$, predict its position at $t = -5$ s, 25 s, 100 s?

Sol. Here, $m = 0.4$ kg, $u = 10$ m/s due north

F = – 8 N, (negative sign shows the force directed opposite).

Therefore, $a = F/m = -8/0.4 = -20$ m/s $(0 \leq t \leq 30$ s)

o When $t = -5$ s, $x = ut = 10(-5) = -50$ m

o When $t = 25$ s, $x = ut + \frac{1}{2} at^2 = 10 \times 25 + \frac{1}{2}(-20)(25)^2 = -6000$ m

Upto $t = 30$ s, motion is under acceleration,

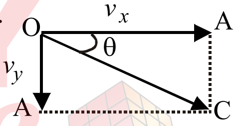
i.e. $x_1 = ut + \frac{1}{2} at^2 = 10 \times 30 + \frac{1}{2}(-20)(30)^2 = -8700$ m

At $t = 30$ s, $v = u + at = 10 - 20 \times 30 = -590$ m/s

During $t = 30$ to 100 s, $x_2 = vt = -590 \times 70 = -41{,}300$ m

Total distance, $x_1 + x_2 = -(8{,}700 + 41{,}300)$ m $= -50$ km

10. A truck starts from rest and accelerates uniformly with 2 m/s^2. At t =10 s, a stone is dropped by a person standing on the top of the truck (6 m high from ground). What are the (a) velocity and (b) acceleration of the stone at $t = 11$ s? Neglect air resistance, and take $g = 9.8$ m/s^2.



Sol. Given, $u = 0$, $t = 10$ s, $a = 2$ m/s^2

The velocity of the truck when the stone is dropped,

$v = u + at = 0 + 2 \times 10 = 20$ m/s.

(a) When the stone is dropped, horizontal velocity,

$v_x = v = 20$ m/s; v_x remains constant.

In vertical direction, $a = g = 9.8$ m/s^2, $u = 0$, $t = 11 - 10 = 1$ s.

$v_y = v = u + at = 0 + 9.8 \times 1 = 9.8$ m/s

Resultant velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 9.8^2} = 22.3$ m/s

If θ-angle which v_x makes with resultant velocity,

$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(9.8/20) = \tan^{-1}(0.49) = 29°$

(b) When the stone is dropped from the car, horizontal force on the stone is zero. Acceleration due to gravity, i.e., 9.8 m/s^2 in the vertically downward direction. The path of the stone would be parabolic.

11. A bob of mass 0.1 kg hung from the ceiling of room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 m/s. What is the trajectory of the bob, if the string is cut when the bob is (a) at one of its extreme positions (b) at its mean position?

Sol. (a) At each extreme position, velocity of the bob is zero. If the extreme position, it is only under the action of 'g'. Hence the bob will fall vertically downwards.

(b) At each mean position, velocity of the bob is 1 m/s along the tangent to the arc, which is in the horizontal direction. If the string is cut at mean position, the bob will behave as horizontal projectile. Hence, it will follow a parabolic path.

12. A man of mass 70 kg stands on a weighing machine in lift, which is moving

(a) upwards with a uniform speed of 10 m/s,

(b) downwards with a uniform acceleration of 5 m/s^2,

(c) upwards with a uniform acceleration of 5 m/s^2. What would be the reading on the scale in each case?

What would be the reading if the lift mechanism failed and it came down freely under the gravity? Take g = 9.8 m/s^2.

Sol. Given m = 70 kg, a = 9.8 m/s^2

The weighing machine gives the reading of the reaction force, R, which is apparent weight.

(a) When lift is moving upwards with a uniform speed, acceleration = 0,

$\therefore$ $R = mg = 70 \times 9.8 = 686$ N

(b) When lift is moving downwards with acceleration = 5 m/s^2,

$\therefore$ $R = m(g - a) = 70 \times (9.8 - 5) = 336$ N

(c) When lift is moving upwards with acceleration = 5 m/s^2,

$\therefore$ $R = m(g + a) = 70 \times (9.8 + 5) = 1036$ N

When lift is coming down freely under gravity, acceleration, $a = g$,

$\therefore$ $R = m\,(g - g) = 0$

13. Two masses 8 kg and 12 kg are connected at the two ends of light inextensible string that passes over a frictionless pulley. Find the acceleration of the masses and the tension in the string, when the masses are released.

Sol. Given, m_1 = 8 kg, m_2 = 12 kg

Force equations,

$a = (m_2 - m_1)g/\,(m_2 + m_1)$

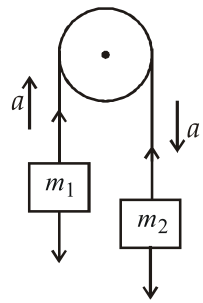
$= (12 - 8) \times 9.8/\,(12 + 8)$

$= 1.96$ m/s^2

and, $T = 2\,m_1 m_2 g\,/\,(m_2 + m_1)$

$= (2 \times 8 \times 12 \times 9.8)/(8 + 120)$

$= 94.1$ N



14. Two billiard balls each of mass 0.05 kg moving in opposite directions with speed 6 m/s collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?

Sol. Given, Initial momentum of A = 0.05 (6) = 0.3 kg m/s,

When ball is reversed after collision, speed would be reversed, final momentum = 0.05 (–6) = –0.3 kg m/s,

Impulse at A = change in momentum = final momentum – initial momentum

= (–0.3 –0.3) = –0.6 m/s

15. A shell of mass 0.02 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m/s what is the recoil speed of the gun?

Sol. Given, mass of shell, m = 0.02 kg, Mass of gun, M = 100 kg and Muzzle speed of shell = 80 m/s

According to the principle of conservation of linear momentum,

$mV + Mv = 0$

$\therefore \quad v = -mV/M = -0.02 \times (80/100) = 0.016$ m/s

16. A horizontal force of 500 N pulls two masses 10 kg and 20 kg (lying on a frictionless horizontal table) connected by a light string. What is the tension in the string? Does your answer depend of the end at which the pull is applied ?

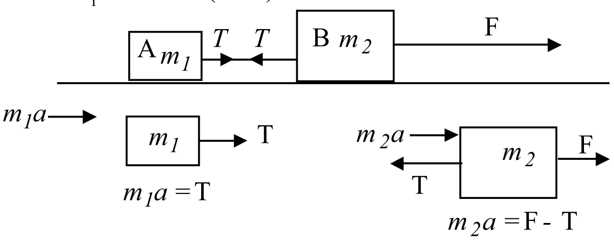
Sol. Given, F = 500 N, m_1 = 10 kg and m_2 = 20 kg

If T-tension of the string, a-acceleration of the system along the direction of the applied force

$\therefore \quad a = F/(m_1 + m_2) = 500/(10 + 20) = 50/3$ m/s

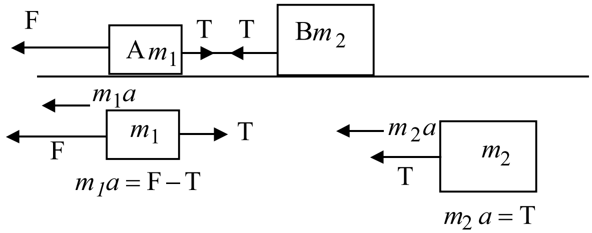
(a) When force is applied of heavier block,

$T = m_1 a$ = 10 × (50/3) N = 166.67 N



(b) When force is applied of lighter block, $T = m_2 a$ = 20 × (50/3) = 333.33 N

Which is different from case (a), that shows answer depends on which the force is applied.



17. A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse to the ball? Mass of the ball is 0.15 kg.

Sol. Let, XY – direction of the bat, AO-direction of the ball before the strike,

OB – direction of the ball after the strike,

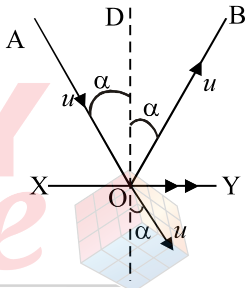
OD – normal to the bat XY,

$\therefore$ $\alpha = \angle DOA = 45/2 = 22.5°$

Let, u – initial velocity of the ball along AO,

$u\cos\alpha$ – component of the velocity along DO

$u\sin\alpha$ – component of the velocity along XY



From the figure, it is clear that velocity along horizontal direction is just changing its direction not the magnitude.

$\therefore$ Impulse on the ball $= mu\cos\alpha - (-mu\cos\alpha) =$

$2\,mu\cos\alpha = 2 \times 0.15 \times 15 \cos 22.5° = 4.16$ kg/ms.

18. A stone of mass 0.25 kg tied to the end of string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min. in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N?

Sol. Given, $m = 0.25$ kg, $r = 1.5$ m, ν = rpm = 40/60 rps

$T = mr\omega^2 = mr(2\pi\nu)^2 = 4\pi^2 mr\nu^2 = 4 \times (22/7)^2 \times (0.25) \times 1.5 \times (2/3)^2$

$= 6.6$N $\quad T_{max} = mv_{max}^2/r$

$v_{max} = \sqrt{(T_{max} r/m)} = \sqrt{(200 \times 1.5/0.25)} = \sqrt{1200} = 34.6$ m/s

19. Explain why

(a) A horse cannot pull a cart and run the empty space,

(b) Passengers are thrown forward from their seats when a speeding bus stops suddenly,

(c) It is easier to pull a lawn mower than to push it?

(d) A cricketer moves his hands backwards when holding ?

Sol. (a) When a cart is pulled, the horse pushes the ground with a force at an angle. The ground offers an equal reaction in opposite direction, through the feet of the horse. The horizontal component of this reaction helps the cart to move in forward direction. In empty space, reaction force would be zero, so horse cannot pull the cart.

(b) If the speeding bus stops suddenly, the lower part of the passengers, in contact with bus, would be at rest. However, upper part of the body would still be in motion. Therefore, passengers would move forward. This occurs due to inertia of motion.

(c) When a lawn mower is pulled, the force acts along the handle, which has two components. The upwardly directed vertical component reduced the weight of mower, however, horizontal component of the force helps to push forward the mower. Whereas, when a lawn mower is pushed, force is applied downwards and thereby, vertical component, directed downward, increased the weight of mower, which creates difficulties in pushing the mower. Therefore, pushing of mower is difficult than pulling the same.

(d) When a cricketer holds a catch, the impulse received at hand = $F \times t$ = change in linear momentum of the ball = constant. By moving the hand backward, the cricketer increases the time of impact and reduces the force, so as the reaction, thereby reduces the chances of hurting severely.

20. A helicopter of mass 1000 kg rises with vertical acceleration of 15 m/s^2. The crew and the passengers weigh 300 kg. Give the magnitude and direction of (a) force on the floor by the crew and passengers, (b) action of the rotor of the helicopter on surrounding air, (c) force on the helicopter due to the surrounding air. Take g = 9.8 m/s^2.

Sol. Given, mass of the helicopter, m_1 = 1000 kg, mass of the crew and passengers, m_2 = 300 kg, upwards acceleration a = 15 m/s^2 and g = 9.8 m/s^2.

(a) Force on the floor of helicopter by the crew and passengers = apparent weight of crew and passengers = $m_2 (g + a) = 300\ (10 + 15) = 7500$ N

(b) Action of the rotor of helicopter is downwards, when helicopter rising up, therefore, force of action $= (m_2 + m_1)(g + a) = (1000 + 300)(9.8 + 15) = 32,500$ N (vertically downwards)

(c) Force on the helicopter for the surrounding air would equal to the force of action (since action = reaction), therefore, force of reaction

= 32,500 N (vertically upwards).

21. A steam of water flowing horizontally with a speed of 15 m/s pushes out of tube of cross-sectional area 10^{-2} m^2 and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming that it does not rebound?

Sol. Given, v = 15 m/s; area of cross section, $a = 10^{-2}$ m^2 and

volume of water coming out per second = $a \times v = 10^{-2} \times 15 = 0.15$ m^2/s

∴ Mass of the water per second = m = volume × (density of water)

= (0.15) × (10^3) = 150 kg^{-1}.

∴ Force excreted on the wall = $mv/t = 150 \times 15/1 = 2250$ N

22. Ten one-rupee coins are put on top of one another on a table. Each coin has a mass m kg. Give the magnitude and direction of

(a) The force on the 7th coin (counted from the bottom) due to all coins above it,

(b) The force on the 7th coin by the 8th coin and

(c) The reaction of the 6th coin on the 7th coin.

Sol. (a) Force on the 7th coin = weight of the 3 coins lying above 7th coin, F = 3 mg N (acts vertically).

(b) Eighth coin is under the weight of 2 coins above it and its own weight, force on 7th coin due to 8th coin = sum of the 3 coins = 2 mg + mg = 3 mg N

(c) Sixth coin is under the weight of 4 coins above it, therefore, reaction = – F = – 4 mg N (negative sign shows the reaction is vertically upwards)

23. An aircraft executes a horizontal loop at speed of 720 km/h with its wings banked at 15°. What is the radius of the loop?

Sol. Given, $\alpha = 15°$, v = 720 km/h = 200 m/s, g = 9.8 m/s.

∴ $\tan \alpha = v^2/rg$, $r = v^2/g \tan \alpha = (200)^2/(9.8 \times \tan 15°) = 15.23$ m

24. A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km/h. The mass of the train is 106 kg. What provides the centripetal force required for this purpose? The engine or the rails? The outer of the inner rails? Which rail will wear out faster, the outer or the inner rail? What is the angle of banking required to prevent wearing out the rails?

Sol. Here, lateral thrust exerted by the rails to wheel is proving the necessary centripetal force and the train would exert an equal and opposite thrust (by Newton's 3rd law) on the rails causing its wear and tear.

Therefore, outer rail's wear and tear would faster due to the larger force applied on it by train.

∴ $\tan\alpha = v^2/rg$,

Given, $r = 30$ m, $v = 54$ km/h = 15 m/s, $g = 9.8$ m/s^2.

$\alpha = \tan^{-1} v^2/rg \tan^{-1} (15)^2/(30 \times 9.8) = 37.4°$.

25. Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid partition. The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally at A. What are

(a) The reaction of the partition

(b) The action, reaction forces between A and B?

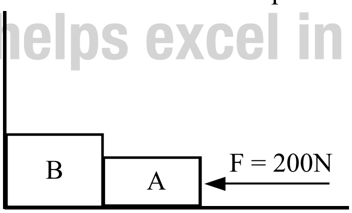
What happens when the partition is removed? Does the answer to (b) change, when the bodies are in motion? Ignore difference between μ_s and μ_k.

Sol. Here, Mass of A, $m_A = 5$ kg and Mass of B, $m_B = 10$ kg; coefficient of friction, $\mu = 0.15$; force acting on A, $F = 200$ N

(a) Limiting friction acts from right

$= f = \mu (m_A + m_B) g = 0.15 (5 + 10) \times 9.8 = 22.05$ N

∴ net force exerted from left on the partition,



$F' = F - f = 200 - 22.05 = 177.95$ N

∴ Reaction of the partition = 177.95 N, acting towards right.

(b) Limiting frictional force acting on A, $f_A = \mu m_A g = 0.15 \times 5 \times 9.8 =$ 7.35 N

∴ Net force exerted by body A on body B, $F'' = F - F_x = 200 - 7.35$ = 192.65 N (acting towards left).

∴ Reaction of body B on body A = 192.65 N (acting towards right)

If the partition is removed, both the bodies would move under the net force, $F' = 177.95$ N

The acceleration, $a = F'/(m_A + m_B) = 177.95/(5+10) = 11.86$ m/s^2.

∴ The force which produces the motion of A, $F_A = m_A a = 5 \times 11.86 =$

59.3 N

∴ net force exerted by body A on body $B = F'' - F_A = 192.65 - 59.3 = 133.35$ N

∴ Reaction of body B on body A = 133.35 N (acting towards right).

Therefore, answer of (b) would differ in this case.

26. A block of mass 15 kg is placed on a long trolley. The coefficient of friction between the block and the trolley is 0.18. The trolley acceleration from rest with 0.5 m/s^2 for 20 s and then moves with a uniform velocity. Discuss the motion of the block as viewed by

(a) a stationary observer on the ground,

(b) an observer moving with the trolley.

Sol. Given, $m = 15$ kg, $\mu = 0.18$, $a = 0.5$ m/s^2, $t = 20$ s,

Force acting on the block due to the motion of the trolley, $F' = ma = 15 \times 0.5 = 7.5$ N, which acts on the forward direction.

Force due to the liming friction on the block, $F = \mu R = \mu mg = 0.18 \times 15 \times 9.8 = 26.46$ N, which opposes the motion of the block and the block would be stationary. Static frictional force would adjust itself to equal and opposite to counter balance F'.

(a) ∴ For stationary observer, the block would be at rest with respect to the trolley.

If the trolley moves with uniform velocity, the block would also be at stationary as forward force is zero and frictional force is the only force action on it.

(b) When the observer moves with an acceleration with the trolley, it would be non-inertial frame, where law of inertia would not be valid.

27. A disc revolves with a speed of $33\frac{1}{3}$ rev/min and has radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the center of the record. If the coefficient of friction between the coins and the record is 0.15, which of the two coins will revolve with the record?

Sol. When the frictional force is sufficient to provide the centripetal force the coil revolves with the coin. Coin would slip the record, if this force is not sufficient.

Force of friction, $f = \mu R = \mu mg$, Centripetal force, $mv^2/r = m\, r\, \omega^2$

In order to prevent slipping, $\mu mg \geq m\, r\, \omega^2$, i.e. $\mu g \geq r\, \omega^2$

Case 1: Given, $r = 4$ cm $= 0.04$ m, $\nu = 33\frac{1}{3}$ rev/min

$\omega = 2\pi\nu = 2 \times (22/7) \times (33\frac{1}{3\times 60}$ rev/min$) = 3.49\ s^{-1}$.

$\therefore$ $r\,\omega^2 = (0.04)\,(3.49)^2 = 0.49\ ms^{-2}$ and $\mu g = 0.15 \times 9.8 = 1.47\ m/s^2$

$\therefore$ $mg \geq r\,\omega^2$, hence first coin will rotate with the record.

Case 2: Given, $r = 14$ cm $= 0.14$ m, and $\omega = 3.49\ s^{-1}$.

$\therefore$ $r\,\omega^2 = (0.14)\,(3.49)^2 = 1.7\ m/s^2$ and $\mu g = 0.15 \times 9.8 = 1.47\ m/s^2$

$\therefore$ μg is less than $r\omega^2$, hence second coin will not rotate with the record.

28. A 70 kg man stands in contact against the inner wall of a hollow cylinder drum of radius 3 m rotating about its vertical axis with 200 rpm. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?

Sol. Given, $m = 70$ kg, $r = 3$ m, $\nu = 200$ rpm $= 200/60$ rps, and $\mu = 0.15$

Horizontal force provided by the wall on the man = centripetal force = m r ω^2. Frictional force, acting vertically upwards, opposes the weight of the man.

If floor is removed, man would stuck the wall, when $mg = f < \mu N$ i.e. $g < \mu r\omega^2$.

$\therefore$ Minimum angular speed of rotation of the cylinder is $\omega = \sqrt{(g/\mu r)} = \sqrt{[10/(0.15 \times 3)]} = 4.7$ rad/s.

29. A thin circular wire of radius R rotates about its vertical diameter with an angular frequency ω, Show that a small bead on the wire remains at its lowermost point for $\omega \leq \sqrt{(g/R)}$. What is the angle made by the radius vector joining the center to the bead with the vertical downward direction for $\omega \leq \sqrt{(2g/R)}$? Neglect friction.

Sol. θ – angle between radius vector joining the bead to the center and vertical downward direction.

N – normal reaction.

$mg = N \cos\theta$ …(i)

and $m\,r\,\omega^2 = N \sin\theta$

or, $m\,(R\sin\theta)\,\omega^2 = N\sin\theta$ …(ii)

From (i) and (ii), $mg = mR\omega^2 \cos\theta$ $\Rightarrow$ $\cos\theta = g/R\omega^2$

Since $|\cos\theta| \leq 1$, bead will emain its lower most point.

$\Rightarrow g/R\omega^2 \leq 1$ or, $\omega \leq \sqrt{(g/R)}$

If, $\omega = \sqrt{(2g/R)}$, $\cos\theta = g/R\,(2g/R) = ½$ i.e. $\theta = 60°$