EXERCISE 8.1

1. Expand the expression $(1-2x)^5$.

Sol. By using binomial theorem, the expression $(1-2x)^5$ can be expanded as

$$(1-2x)^5 = {}^5C_0\,(1)^5 - {}^5C_1\,(1)^4(2x) + {}^5C_2\,(1)^3(2x)^2 - {}^5C_3\,(1)^2(2x)^3 + {}^5C_4\,(1)^1(2x)^4 - {}^5C_5\,(2x)^5$$

$$= 1 - 5\,(2x) + 10\,(4x^2) - 10(8x^3) + 5(16x^4) - (32x^5)$$

$$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

2. Expand the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Sol. By using binomial theorem, the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ can be expanded as

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2 - {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5$$

$$= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32}$$

$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$$

3. Expand the expression $(2x-3)^6$

Sol. By using binomial theorem, the expression $(2x-3)^6$ can be expanded as

$$(2x-3)^6 = {}^6C_0\,(2x)^6 - {}^6C_1\,(2x)^5(3) + {}^6C_2\,(2x)^4(3)^2 - {}^6C_3\,(2x)^3(3)^3 + {}^6C_4\,(2x)^2(3)^4 - {}^6C_5\,(2x)(3)^5 + {}^6C_6\,(3)^6$$

$$= 64x^6 - 6\,(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) + 15(4x^2)(81) - 6(2x)(243) + 729$$

$$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729.$$

4. Expand the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$.

Sol. By using binomial theorem, the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$ can be expanded as

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3$$

$$+ {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5$$

$$= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$$

5. Expand $\left(x + \frac{1}{x}\right)^6$.

Sol. By using binomial theorem, the expression $\left(x + \frac{1}{x}\right)^6$ can be expanded as

$$\left(x + \frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2$$

$$+ {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6(x)^5\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x^2}\right) + 20(x)^3\left(\frac{1}{x^3}\right) + 15(x)^2\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

6. Using binomial theorem, evaluate $(96)^3$

Sol. $\therefore$ $(96)^3 = (100 - 4)^3$

$= {}^3C_0(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3$

$= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3$

$= 1000000 - 120000 + 4800 - 64$

$= 884736$

7. Using binomial theorem, evaluate $(102)^5$

Sol. $\therefore$ $(102)^5 = (100 + 2)^5$

$= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5$

$= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5$

$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$

$= 11040808032$

8. Using binomial theorem, evaluate $(101)^4$

Sol. $\therefore \quad (101)^4 = (100 + 1)^4$

$= {}^4C_0(100)^4 + {}^4C_1(100)^3\,(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4$

$= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + (1)^4$

$= 100000000 + 4000000 + 60000 + 400 + 1$

$= 104060401$

9. Using binomial theorem, evaluate $(99)^5$

Sol. $\therefore \quad (99)^5 = (100 - 1)^5$

$= {}^5C_0(100)^5 - {}^5C_1(100)^4\,(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3$
$+ {}^5C_4(100)(1)^4 - {}^5C_5(1)^5$

$= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1$

$= 10010000500 - 500100001$

$= 9509900499$

10. Using binomial theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Sol. $(1.1)^{10000}$ can be obtained as

$(1.1)^{10000} = (1 + 0.1)^{10000}$

$= {}^{10000}C_0 + {}^{10000}C_1(1)^{10000-1}\,(0.1)^1 +$ Positive terms

$= 1 + 10000 \times 0.1 +$ Positive terms

Hence. $(1.1)^{10000} > 1000$

11. Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$.

Sol. Using binomial theorem, the expressions, $(a + b)^4$ and $(a - b)^4$, can be expanded as

$(a + b)^4 = {}^4C_0\, a^4 + {}^4C_1\, a^3b + {}^4C_2\, a^2b^2 + {}^4C_3\, ab^3 + {}^4C_4\, b^4$

$(a - b)^4 = {}^4C_0\, a^4 - {}^4C_1\, a^3b + {}^4C_2\, a^2b^2 - {}^4C_3\, ab^3 + {}^4C_4\, b^4$

$\therefore \quad (a + b)^4 - (a - b)^4 = {}^4C_0 + {}^4C_1\, a^3b + {}^4C_2\, a^2b^2 + {}^4C_3\, ab^3 + {}^4C_4\, b^4$
$- [{}^4C_0\, a^4 - {}^4C_1\, a^3b + {}^4C_2\, a^2b^2 - {}^4C_3\, ab^3 + {}^4C_4\, b^4]$

$= 2({}^4C_1\, a^3b + {}^4C_3\, ab^3) = 2\,(4a^3b + 4ab^3)$

$= 8ab\,(a^2 + b^2)$

By putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\left(\sqrt{3}\right)\left(\sqrt{2}\right)\left\{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right\}$$

$$= 8\left(\sqrt{6}\right)\{3+2\} = 40\sqrt{6}$$

12. Find $(x+1)^6+(x-1)^6$. Hence or otherwise evaluate $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$.

Sol. Using binomial theorem, the expressions, $(x+1)^6$ and $(x-1)^6$, can be expanded as

$(x+1)^6 = {}^6C_0\, x^6 + {}^6C_1\, x^5 + {}^6C_2\, x^4 + {}^6C_3\, x^3 + {}^6C_4\, x^2 + {}^6C_5\, x + {}^6C_6$

$(x-1)^6 = {}^6C_0\, x^6 - {}^6C_1\, x^5 + {}^6C_2\, x^4 - {}^6C_3\, x^3 + {}^6C_4\, x^2 - {}^6C_5\, x + {}^6C_6$

$\therefore$ $(x+1)^6 + (x-1)^6 = 2[{}^6C_0\, x^6 + {}^6C_2\, x^4 + {}^6C_4\, x^2 + {}^6C_6]$

$= 2[x^6 + 15x^4 + 15x^2 + 1]$

By putting $x=\sqrt{2}$, we obtain

$$\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6 = 2\left[\left(\sqrt{2}\right)^6+15\left(\sqrt{2}\right)^4+15\left(\sqrt{2}\right)^2+1\right]$$

$= 2\,(8 + 15 \times 4 + 15 \times 2 + 1)$

$= 2(8 + 60 + 30 + 1)$

$= 2\,(99) = 198$

13. Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

Sol. In order to show that $9^{n+1} - 8n - 9$ is divisible by 64, it has to be proved that, $9^{n+1} - 8n - 9 = 64\,k$, where k is some natural number

By binomial theorem,

$(1+a)^m = {}^mC_0 + {}^mC_1\, a + {}^mC_2\, a^2 + \ldots + {}^mC_m\, a^m$

For $a = 8$ and $m = n + 1$, we obtain

$(1+8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2\,(8)^2 + \ldots + {}^{n+1}C_{n+1}\,(8)^{n+1}$

$\Rightarrow$ $9^{n+1} = 1 + (n+1)(8) + 8^2\,[\,{}^{n+1}C_2 + {}^{n+1}C_3\, 8 + \ldots + {}^{n+1}C_{n+1}\,(8)^{n+1}]$

$\Rightarrow$ $9^{n+1} = 9 + 8n + 64[\,{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \ldots + {}^{n+1}C_{n+1}\,(8)^{n+1}]$

$\Rightarrow$ $9^{n+1} - 8n - 9 = 64k$, where $k = {}^{n+1}C_2 + {}^{n+1}C_3\, 8 + \ldots + {}^{n+1}C_{n+1}\,(8)^{n+1}$ is a natural number

Thus, $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

14. Prove that $\sum_{r=0}^{n} 3^r\ {}^nC_r = 4^n$.

Sol. By binomial theorem

$\text{LHS} = \sum_{r=0}^{n} 3^r\ {}^nC_r$

$\text{LHS} = 3^0\, {}^nC_0 + 3^1\, {}^nC_1 + 3^2\, {}^nC_2 + \ldots + 3^n\, {}^nC_n$

$\text{RHS} = 4^n \Rightarrow (1+3)^n$

$= {}^nC_0\, 3^0 + {}^nC_1\, 3^1 + {}^nC_2\, 3^2 + \ldots + 3^n\, {}^nC_n$

Hence proved.

EXERCISE 8.2

1. Find the coefficient of x^5 in $(x + 3)^8$.

Sol. Assuming that x^5 occurs in the $(r + 1)$th term of the expansion $(x + 3)^8$, we obtain $T_{r+1} = {}^8C_r (x)^{8-r}(3)^r$

Comparing the indices of x in x^5 and in T_{r+1}, we obtain $r = 3$

Thus, the coefficient of x^5 is ${}^8C_3 (3)^3 = \frac{8!}{3!5!} \times 3^3 = 1512$

2. Find the coefficient of a^5b^7 in $(a - 2b)^{12}$

Sol. Assuming that a^5b^7 occurs in the $(r + 1)$th term of the expansion $(a - 2b)^{12}$, we obtain $T_{r+1} = {}^{12}C_r (a)^{12-r}(-2b)^r = {}^{12}C_r (-2)^r (a)^{12-r}(b)^r$

Comparing the indices of a and b in $a^5 b^7$ and in T_{r+1}, we obtain $r = 7$.

Thus, the coefficient of a^5b^7 is

$${}^{12}C_7 (-2)^7 = -\frac{12!}{7!5!} \cdot 2^7 = -101376$$

3. Write the general term in the expansion of $(x^2 - y)^6$.

Sol. The general term in the expansion of $(x^2 - y^6)$ is

$T_{r+1} = {}^6C_r (x^2)^{6-r}(-y)^r = (-1)^r\, {}^6C_r\, x^{12-2r} y^r$

4. Write the general term in the expansion of $(x^2 - yx)^{12}$, $x \neq 0$

Sol. The general term in the expansion of$(x^2 - yx)^{12}$ is

$T_{r+1} = {}^{12}C_r (x^2)^{12-r}(-yx)^r = (-1)^r\, {}^{12}C_r\, x^{24-2r}.y^r.x^r = (-1)^r\, {}^{12}C_r\, x^{24-r}.y^r$

5. Find the 4th term in the expansion of $(x - 2y)^{12}$.

Sol. The 4th term in the expansion of $(x - 2y)^{12}$ is

$$T_4 = T_{3+1} = {}^{12}C_3 (x)^{12-3} (-2y)^3 = (-1)^3 \cdot \frac{12!}{3!9!} \cdot x^9 \cdot (2)^3 \cdot y^3$$

$$= -1760\, x^9 y^3$$

6. Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$.

Sol. Thus, 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, is

$$T_{13} = T_{12+1} = {}^{18}C_{12} (9x)^{18-12} \left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= (-1)^{12} \frac{18!}{12!6!} (9)^6 (x)^6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{\sqrt{x}}\right)^{12}$$

$$= 18564$$

7. Find the middle terms in the expansions of $\left(3-\frac{x^3}{6}\right)^7$.

Sol. It is known that in the expansion of $(a + b)^n$, if n is odd, then there are two middle terms, $\left(\frac{n+1}{2}\right)^{th}$, $\left(\frac{n+1}{2}+1\right)^{th}$ term.

Therefore, the middle terms in the expansion of $\left(3-\frac{x^3}{6}\right)^7$ are $\left(\frac{7+1}{2}\right)^{\text{th}} = 4^{\text{th}}$ term and $\left(\frac{7+1}{2}+1\right)^{\text{th}} = 5^{\text{th}}$ term

$$T_4 = T_{3+1} = {}^7C_3\,(3)^{7-3}\left(-\frac{x^3}{6}\right)^3$$

$$= -\frac{105}{8}x^9$$

$$T_5 = T_{4+1} = {}^7C_4\,(3)^{7-4}\left(-\frac{x^3}{6}\right)^4 = (-1)^4\,\frac{7!}{4!3!}(3)^3\cdot\frac{x^{12}}{6^4}$$

$$= \frac{35}{48}x^{12}$$

Thus, the middle terms in the expansion of are $\left(3-\frac{x^3}{6}\right)^7$ are $-\frac{105}{8}x^9$ and $\frac{35}{48}x^{12}$.

8. Find the middle terms in the expansions of $\left(\frac{x}{3}+9y\right)^{10}$

Sol. It is known that in the expansion $(a + b)^n$, if n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{\text{th}}$ term.

Therefore, the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is $\left(\frac{10}{2}+1\right)^{\text{th}} = 6^{\text{th}}$ term

$$T_6 = T_{5+1} = {}^{10}C_5\left(\frac{x}{3}\right)^{10-5}(9y)^5 = \frac{10!}{5!5!}\cdot\frac{x^5}{3^5}\cdot 9^5\cdot y^5$$

$= 61236x^5y^5$

Thus, the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is $61236\,x^5y^5$.

9. In the expansion of $(1 + a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

Sol. Assuming that a^m occurs in the $(r + 1)$th term of the expansion $(1 + a)^{m+n}$, we obtain $T_{r+1} = {}^{m+n}C_r\,(1)^{m+n-r}(a)^r = {}^{m+n}C_r a^r$

Comparing the indices of a in a^m and in T_{r+1}, we obtain $r = m$

Therefore, the coefficient of a^m is

$$ {}^{m+n}C_m = \frac{(m+n)!}{m!(m+n-m)!} = \frac{(m+n)!}{m!n!} \quad ...(i)$$

Assuming that a^n occurs in the $(r + 1)$th term of the expansion $(1 + a)^{m+n}$, we obtain $T_{r+1} = {}^{m+n}C_r\,(1)^{m+n-r}(a)^r = {}^{m+n}C_r a^r$

Comparing the indices of a in a^n and in T_{r+1}, we obtain $r = n$

Therefore, the coefficient of a^n is

$$ {}^{m+n}C_n = \frac{(m+n)!}{n!(m+n-n)!} = \frac{(m+n)!}{n!m!} \quad ...(ii)$$

Thus, from (i) and (ii), it can be observed that the coefficients of a^m and a^n in the expansion of $(1 + a)^{m+n}$ are equal.

10. The coefficients of the $(r - 1)$th, rth and $(r + 1)$th terms in the expansion of $(x + 1)^n$ are in the ratio 1 : 3 : 5. Find n and r.

Sol. It is known that $(r + 1)$th term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r\, a^{n-r}b^r$.

Therefore, $(r - 1)$th term in the expansion of $(x + 1)^n$ is

$T_{r-1} = {}^nC_{r-2}\,(x)^{n-(r-2)}(1)^{(r-2)} = {}^nC_{r-2}x^{n-r+2}$

r th term in the expansion of $(x + 1)^n$ is

$T_r = {}^nC_{r-1}\,(x)^{n-(r-1)}(1)^{(r-1)} = {}^nC_{r-1}x^{n-r+1}$

Therefore, the coefficients of the $(r - 1)$th, rth, and $(r + 1)$th terms in the expansion of $(x + 1)^n$ are ${}^nC_{r-2}$, ${}^nC_{r-1}$, and ${}^nC_r$ respectively. Since these coefficients are in the ratio 1 : 3 : 5, we obtain

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3} \text{ and } \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$$

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!}$$

$$= \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)(n-r+1)!} = \frac{r-1}{n-r+2}$$

$$\therefore \quad \frac{r-1}{n-r+2} = \frac{1}{3}$$

$\Rightarrow \quad n - 4r + 5 = 0$ ....(i)

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!}$$

$$= \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)(n-r)!} = \frac{r}{n-r+1}$$

$\therefore \quad \frac{r}{n-r+1} = \frac{3}{5}$

$\Rightarrow \quad 3n - 8r + 3 = 0$ ....(ii)

Solve (i) and (ii)

$4r - 12 = 0$

$\therefore$ r = 3

Putting the value of r in (i), we obtain

$n - 12 + 5 = 0$

$\therefore \quad n = 7$

Thus, $n = 7$ and $r = 3$

11. Prove that the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

Sol. Assuming that x^n occurs in the $(r + 1)$th term of the expansion of $(1 + x)^{2n}$, we obtain $T_{r+1} = {}^{2n}C_r\,(1)^{2n-r}(x)^r = {}^{2n}C_r x^r$

Comparing the indices of x in x^n and in T_{r+1}, we obtain $r = n$.

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is

$${}^{2n}C_n = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!} = \frac{(2n)!}{(n!)^2} \quad \text{...(i)}$$

Assuming that x^n occurs in the $(r+1)$th term of the expansion $(1 + x)^{2n-1}$, we obtain

$T_{r+1} = {}^{2n-1}C_r\,(1)^{2n-1-r}(x)^r = {}^{2n-1}C_r(x)^r$

Comparing the indices of x in x^n and T_{r+1}, we obtain $r = n$.

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$ is

$${}^{2n-1}C_n = \frac{(2n-1)!}{n!(2n-1-n)!} = \frac{(2n-1)!}{n!(n-1)!}$$

$$= \frac{2n \cdot (2n-1)!}{2n \cdot n!(n-1)!} = \frac{(2n)!}{2 \cdot n!n!} = \frac{1}{2}\left[\frac{(2n)!}{(n!)^2}\right] \quad \text{...(ii)}$$

From (i) and (ii), it is observed that

$$\frac{1}{2}\left({}^{2n}C_n\right) = {}^{2n-1}C_n$$

$\Rightarrow \quad {}^{2n}C_n = 2({}^{2n-1}C_n)$

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

Hence, proved.

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

Sol. Assuming that x^2 occurs in the $(r + 1)$th term of the expansion $(1 + x)^m$, we obtain $T_{r+1} = {}^mC_r\,(1)^{m-r}(x)^r = {}^mC_r(x)^r$

Comparing the indices of x in x^2 and in T_{r+1}, we obtain $r = 2$.

Therefore, the coefficient of x^2 is ${}^mC_2$.

It is given that the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

$\therefore \quad {}^mC_2 = 6$

$\Rightarrow \quad \dfrac{m!}{2!(m-2)!} = 6$

$\Rightarrow \quad \dfrac{m(m-1)(m-2)!}{2\times(m-2)!} = 6$

$\Rightarrow \quad m(m - 1) = 12$

$\Rightarrow \quad m^2 - m - 12 = 0$

$\Rightarrow \quad (m - 4)(m + 3) = 0$

$\Rightarrow \quad m = 4$ or $m = -3$

Thus, the positive value of m, for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6, is 4.