NCERT TEXTUAL EXERCISE (SOLVED)

1. State, for each of the following physical quantities, if it is a scalar or a vector. Volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Sol. **Scalars:** Volume, mass, speed, density, number of moles, angular frequency.

Vectors: Acceleration, velocity, displacement, angular velocity.

2. Pick out the two scalar quantities in the following lists: force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, reaction as per Newton's third law, relative velocity.

Sol. Work and current are the scalar qualities in the given list.

3. Pick out the only vector quantity in the following lists: temperature, pressure, impulse, time, power, total path-length, energy, gravitational potential, coefficient of friction, charge.

Sol. Since, Impulse = Change in momentum = Force × Time. As momentum and force are vector quantities, hence impulse is a vector quality.

4. State with reasons, whether the following algebraic operations with scalars and vectors are meaningful.

(a) Adding any two scalars

(b) Adding a scalar to a vector of the same dimension

(c) Multiplying any two scalars (d) Adding any two vectors

(e) Adding a component of vector to the same vector.

Sol. (a) **No,** because only the scalars of the same dimensions can be added.

(b) **No,** because of scalar cannot be added to a vector.

(c) **Yes,** when power P is multiplied by time t, we get work done = Pt, which is a useful operation.

(d) **No,** because the two vectors of same dimensions can be added.

(e) **Yes,** because both are vectors of the same dimensions.

5. Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar

(b) Each component of a vector is always a scalar.

(c) The total path length is always equal to the magnitude of the displacement vector of a particle.

(d) The average speed of a particle (defined as total path divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.

(e) Three vectors not lying in plane can never add up to give a null vector.

Sol. (a) True, because magnitude is pure number.

(b) False, because each component is a vector.

(c) True only, if the particle moves along a straight line in the same direction, otherwise false.

(d) True, because the total path length is either greater than or equal to the magnitude of the displacement vector.

(e) True, as they cannot represent the three sides of a triangle taken in the same order.

6. Establish the following inequalities geometrically or otherwise:

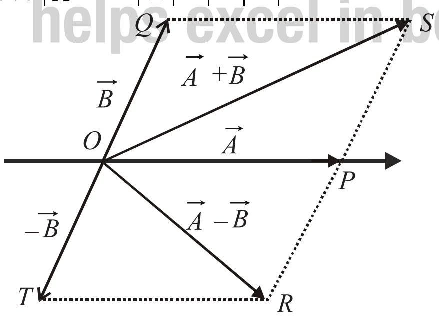
(a) $|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$ (b) $|\vec{A}+\vec{B}| \ge ||\vec{A}|-|\vec{B}||$

(c) $|\vec{A}-\vec{B}| \le |\vec{A}|+|\vec{B}|$ (d) $|\vec{A}-\vec{B}| \ge ||\vec{A}|-|\vec{B}||$

When does the equality sign apply?

Sol. Consider two vectors $\vec{A}$ and $\vec{B}$ be represented by the sides $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of a parallelogram OPSQ. According to parallelogram law of vector addition, ($\vec{A} + \vec{B}$) will be represented by $\overrightarrow{OS}$ as shown in the adjoining figure. Thus, $OP = |\vec{A}|$, $OQ = PS = |\vec{A}|$ and $OS = |\vec{A} + \vec{B}|$.

(a) To prove $|\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$



We know that the length of one side of triangle is always less than the sum of the lengths of the other two sides. Hence from ΔOPS, we have

$OS < OP + PS$ or, $OS < OP + OQ$ or, $|\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$ ...(1)

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same straight line and in the same direction then,

$|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$ ...(2)

(b) $|\vec{A} + \vec{B}| \geq ||\vec{A}| - |\vec{B}||$

From ΔOPS, we have,

$OS + PS > OP$ or, $OS > |OP - PS|$ or, $OS > |OP - OQ|$ ...(3)

(since $PS = OQ$)

The modulus of $(\overrightarrow{OS} - \overrightarrow{PS})$ has been taken because the LHS is always positive but the RHS may be negative if $OP < PS$. Thus from (3) we have,

$|\vec{A} + \vec{B}| > ||\vec{A}| - |\vec{B}||$ ...(4)

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along a straight line in opposite directions, then

$|\vec{A} + \vec{B}| = ||\vec{A}| - |\vec{B}||$ ...(5)

Considering (4) and (5) together, we get,

$|\vec{A} + \vec{B}| \geq ||\vec{A}| - |\vec{B}||$

(c) $|\vec{A} - \vec{B}| \leq |\vec{A}| + |\vec{B}|$

In figure, $|\vec{A}| = OP$ and $|-\vec{B}| = OT = PR$ and $|(\vec{A} - \vec{B})| = OR$

From ΔOPS we note that $OR < OP + PR$.

or $|\vec{A} - \vec{B}| < ||\vec{A}| + |-\vec{B}||$

or, $|\vec{A} - \vec{B}| < ||\vec{A}| + |\vec{B}||$ ...(6)

If the two vectors are acting along the straight line but in opposite direction, then,

$|\vec{A} - \vec{B}| = ||\vec{A}| + |\vec{B}||$ ...(7)

Considering (6) and (7) together, we get,

$|\vec{A} - \vec{B}| \leq |\vec{A}| + |\vec{B}|$

(d) $|\vec{A} - \vec{B}| \geq ||\vec{A}| - |\vec{B}||$

In figure, from ΔOPS we have,

$OR + PR > OP$ or, $OR > |OP - PR|$

or, $OR > |OP - OT|$ ...(8) (since $OT = PR$)

The modulus of $(\overrightarrow{OP} - \overrightarrow{OT})$ has been taken because LHS is positive and RHS may be negative if OP < OT.

From (8),

$|\vec{A} - \vec{B}| > |\vec{A}| - |\vec{B}|$ ...(9)

If the two vectors $\vec{A}$ and $\vec{B}$ are along the same straight line in the same direction then,

$|\vec{A} - \vec{B}| = |\vec{A}| - |\vec{B}|$ ...(10)

Considering (9) and (10) together, we get,

$|\vec{A} - \vec{B}| \geq ||\vec{A}| - |\vec{B}||$

7. Given $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, which of the following statements are correct?

(a) *A, B, C* and *D* must be null vector

(b) The magnitude of $\vec{A}+\vec{C}$ equals to the magnitude of $\vec{B}+\vec{D}$.

(c) The magnitude of $\vec{A}$ can never be greater than the sum of the magnitude of $\vec{B}$, $\vec{C}$ and $\vec{D}$.

(d) $\vec{B}+\vec{C}$ must lie in the plane of $\vec{A}+\vec{D}$, if *A* and $\vec{D}$ are not collinear and in the line of $\vec{A}$ and $\vec{D}$, if they are collinear.

Sol. (a) False, because $\vec{A}+\vec{B}+\vec{C}+\vec{D}$ can be zero in many ways other than $\vec{A},\vec{B},\vec{C}$ and $\vec{D}$ must each be null vector.

(b) True, since $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, therefore, $\vec{A}+\vec{C} = -(\vec{B}+\vec{D})$ or, $|\vec{A}+\vec{C}| = |\vec{B}+\vec{D}|$

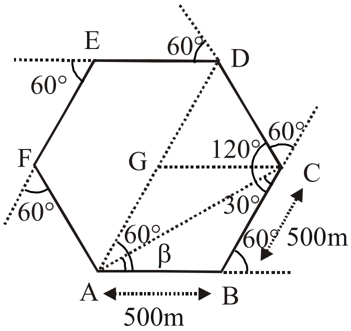
(c) True, since $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, therefore, $\vec{A} = -(\vec{B}+\vec{C}+\vec{D})$

It means the magnitude of $\vec{A}$ is equal to the magnitude of vector $(\vec{B}+\vec{C}+\vec{D})$. Since the sum of the magnitude of $\vec{B}$, $\vec{C}$ and $\vec{D}$ may be equal or greater than the magnitude of $\vec{A}$, hence the magnitude of $\vec{A}$ can never be greater than the sum of the magnitude of $\vec{B}$, $\vec{C}$ and $\vec{D}$.

(d) True, since $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, therefore, $\vec{A}+(\vec{B}+\vec{C})+\vec{D} = 0$. The resultant sum of the three vectors $\vec{A}$, $(\vec{B}+\vec{C})$, $\vec{D}$ can be zero only if $(\vec{B}+\vec{C})$ lies in the plane of $\vec{A}$ and $\vec{D}$ and these three vectors are represented by the three sides of the triangle taken in one order. If $\vec{A}$ and $\vec{D}$ are collinear, then $(\vec{B}+\vec{D})$ must be in the line of $\vec{A}$ and $\vec{D}$, only then vector sum of all the vectors will be zero.

8. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eight turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Sol. In this question, the path is a regular hexagon ABCDEF of side length 500 m. In figure,



Let the motorist start from A.

Third Turn

The motorist will take the 3rd turn at D. Displacement vector at D = $\overrightarrow{AC}$

Magnitude of this displacement = 500 + 500 = 1000 m

Total path length from A to $D = AB + BC + CD = 500 + 500 + 500 = 1500$ m

Sixth Turn

The motorist will take the 6th turn at A. ∴ Displacement vector is null vector.

Magnitude of this displacement = 500 + 500 = 1000 m

Total path length = $AB + BC + CD + DE + EF = 500 + 500 + 500 + 500 + 500 + 500 = 3000$ m

Eighth Turn

The motorist takes the 8th turn at C.

∴ Displacement vector = $\overrightarrow{AC}$, which is represented by the diagonal of the parallelogram ABCG.

$$\therefore \sqrt{[(500)^2 + (500)^2 + 2 \times (500) \times (500) \cos 60°]}$$

$$= \sqrt{[(500)^2 + (500)^2 + 250000]} = 866.03 \text{ m}$$

Total path length = 500 × 8 = 4000 m = 4 km

tan β = 500 sin 60°/{500 + 500 cos 60°]

$$= (500\sqrt{3}/2)/\{500(1+1/2)\} = 1/\sqrt{3} = \tan 30° \text{ or, } \beta = 30°$$

9. A passenger arriving in new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cab man takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of the average velocity? Are the two equal?

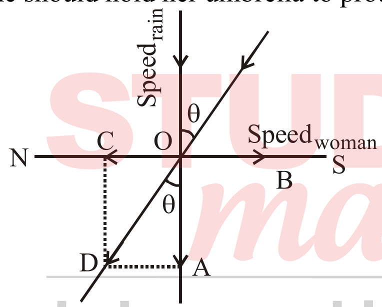
Sol. Here, actual path length travelled, s = 23 km, Displacement = 10km, Time taken, t = 28 min = 26/60 h.

(a) Average speed of the taxi = (actual path length)/(time taken) = 23/(28/60) = 49.3 km/h

(b) Magnitude of average velocity = displacement/time taken = 10/(28/60) = 21.4 km/h

The average speed is not equal to the magnitude of average velocity. The two are equal for the motion of taxi along a straight path in one direction.

10. Rain is falling vertically with a speed of 30 m/s. *A* woman rides a bicycle with a speed of 10 m/s in the north to south direction. What is the direction in which she should hold her umbrella to protect herself from the rain?



Sol. The rain is falling along OA with speed 30 m/s and woman rider is moving along OS with 10 m/s i.e. OA = 30 m/s and OB = 10 m/s. The woman rider can protect herself from the rain if she holds her umbrella in the direction of relative velocity of rain w.r.t. woman. To do so apply equal and opposite velocity of woman on the rain i.e. impress the velocity 10 m/s due north on rain which is represented by OC. Now the relative velocity of rain w.r.t. woman will be represented by diagonal OD of parallelogram $OADC$.

If $\angle AOD = \theta$, then in ΔOAD, $\tan\theta = AD/OA = OC/OA = 10/30 = 0.3333 = \tan 18°26'$

$\theta = 18°26'$ with vertical in forward direction, i.e., towards south.

11. A man can swim with a speed of 4 km/h in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Sol. Time to cross the river, t = (width of river)/(speed of man) = 1 km/(4 km/h)

$= \frac{1}{4}\ h = 15$ min.

Distance moved along the river in time $t = v_r \times t = 3$ km/h $\times \frac{1}{4} h = 750$ m

12. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m/s can go without hitting the ceiling of the ball?

Sol. Given, $u = 40$ m/s, $H = 25$ m

Let, θ = the angle of projection with the horizontal direction to have the maximum range, with maximum height = 25 m

Maximum height, $H = 25 = u^2\sin^2\theta/ 2g = (40)^2\sin^2\theta/(2 \times 9.8)$

$\sin\theta = (25 \times 2 \times 9.8/40^2)^{1/2} = 0.5534 = \sin 33.6°$ or, $\theta = 33.6°$

Horizontal range, $R = u^2\sin 2\theta/ g = (40)^2\sin(2 \times 33.6°)/9.8 = 150.5$ m

13. A cricketer can throw a ball to a maximum horizontal distance of 100 m. With the same speed how high above the ground can the cricketer throw the same ball?

Sol. Let u = velocity of projection of the ball. The ball will cover maximum horizontal distance when angle of projection with horizontal, $\theta = 45°$. Then $R_{max} = u^2/g$

Now, $100 = u^2/g$

In order to study the motion of the ball along vertical direction, consider a point on the surface of earth as the origin and vertical upward direction as the positive direction of Y axis. Taking motion of the ball along vertical upward direction, we get, $u_y = u$, $a_y = -g$, $v_y = 0$,

we know, $v_y = u_y + a_y t \Rightarrow 0 = u + (-g)t$ or, $t = u/g$

Again, $y = u_y t + \frac{1}{2} a_y t^2$

$\therefore\ y = u(u/g) + \frac{1}{2}(-g)u^2/g^2 = u^2/g - \frac{1}{2}u^2/g = u^2/g = 100/2 = 50$ m

14. A stone tied to the end of a string 80 cm long is whirled in horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s. What is the magnitude and direction of acceleration of the stone?

Sol. Given, $r = 80$ cm $= 0.8$ m, $\nu = 14/25\ s^{-1}$, $\omega = 2\pi\nu = 2 \times 22/7 \times 14/25 = 88/25$ rad/s

Centripetal acceleration, $a = r\omega^2 = 80 \times \left(\frac{88}{25}\right)^2 = 991.2\ \text{cm/s}^2$

This acceleration is directed along the radius of the cirular path towards the centre of the circle.

15. The position of a particle is given by $\vec{r} = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}$ m, where t is in seconds and the coefficient have the power units for $\vec{r}$ to be in meters.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2 s?

Sol. (a) Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0 + \hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}) = 3.0\hat{i} - 4.0t\hat{j}$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.0\hat{i} - 4.0t\hat{j}) = -4.0\hat{j}$

(b) At time t = 2s, $\vec{v} = 3.0\hat{i} - 4.0 \times 2\hat{j} = 3.0\hat{i} - 8.0\hat{j}$

$v = \sqrt{(3)^2 + (-8)^2}$ m/s $= \sqrt{73} = 8.54$ m/s

If θ is the angle which v makes with x-axis, then tan θ $= v_y/v_x = -8/3$ = –2.667 = tan 69.5°.

∴ θ = 69.5° below the x-axis.

16. A particle starts from the origin at $t = 0$ with a velocity of $10.0\hat{j}$ m/s and moves in the X–Y plane with a constant acceleration of $8.0\hat{i} + 2.0\hat{j}$ m/s^2

(a) at what time is the X-coordinate of the particle 16 m? What is the Y-coordinate of the particle at that time?

(b) What is the speed of the particle at that time?

Ans. Given, $\vec{u} = 10.0\hat{j}$ m/s at $t = 0; \vec{a} = \frac{d}{dt}(\vec{v}) = (8.0\hat{i} + 2.0\hat{j})$ m/s^2

Integrating the above relation with the change of time, from 0 to t, velocity changes from u to v, we get,

$\vec{v} - \vec{u} = (8.0\hat{i} + 2.0\hat{j})t$ *or*, $\vec{v} = \vec{u} + (8.0\hat{i} + 2.0\hat{j})t$

$\vec{v} = \frac{d}{dt}(\vec{r})$ *or*, $d\vec{r} = \vec{v}\, dt$

so, $d\vec{r} = (u + 8.0t\hat{i} + 2.0t\hat{j})\, dt$

Integrating the above relation with the change of time, from 0 to t, displacement changes from 0 to r, we get, $\vec{r} = (\vec{u}t + \tfrac{1}{2}8.0t^2\hat{i} + \tfrac{1}{2}2.0t^2\hat{j})$ or,

$x\hat{i} + y\hat{j} = 10t\hat{j} + 4t^2\hat{i} + t^2\hat{j} = 4t^2\hat{i} + (10t + t^2)\hat{j}$

Here, it is given, $x = 4t^2$ and $y = 10t + t^2$

$\Rightarrow \quad t = \left(\frac{x}{4}\right)^{\frac{1}{2}} = 2$ s.

(a) At $x = 16$ m, $t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2$ s. and, $y = 10 \times 2 + 2^2 = 24$ m

(b) Velocity of the particle at time t is $v = 10\hat{j} + 8t\hat{i} + 2t\hat{j}$,

when, $t = 2$ s, then, $v = 10\hat{j} + 8 \times 2t\hat{i} + 2 \times 2t\hat{j} = 16\hat{i} + 14\hat{j}$

Speed = $|\vec{v}| = \sqrt{16^2 + 14^2} = 21.26$ m/s

17. $\hat{i}$ and $\hat{j}$ are unit vectors along X and Y axis, repectively.

(a) What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

(b) What are the components of a vector $A = 2\hat{i} + 3\hat{j}$ along the direction of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$?

Sol. (a) Magnitude of $(\hat{i} + \hat{j}) = |\hat{i} + \hat{j}| = \sqrt{(1)^2 + (1)^2)} = \sqrt{2}$

Let the vector $(\hat{i} + \hat{j})$ = make an angle θ with the direction of $\hat{i}$, then cos θ=

$(\hat{i} + \hat{j}) \cdot \hat{i} / |\hat{i} + \hat{j}|.|\hat{i}| = 1/(\sqrt{2})(1) = 1/\sqrt{2} = \cos 45°$ or, $\theta = 45°$

Magnitude of $(\hat{i} - \hat{j}) = |\hat{i} - \hat{j}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

Similarly if vector $(\hat{i} - \hat{j})$ makes an angle θ with the direction of $\hat{i}$ then

$\cos\theta = (\hat{i} - \hat{j}) \cdot \hat{i} / |\hat{i} - \hat{j}| \cdot |\hat{i}| = 1/(\sqrt{2})(1) = 1/\sqrt{2} = \cos 45°$ or, $\theta = 45°$

Here $\theta = -45°$ with $\hat{j}$.

(b) Now, $A = 2\hat{i} + 3\hat{j}$

To find the component of $\vec{A}$ along the vector $(\hat{i} + \hat{j})$, we have to find out the unit vector $(\hat{i} + \hat{j})$.

then, $\hat{a} = \frac{(\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|} = \frac{\hat{i} + \hat{j}}{\sqrt{1^2 + 1^2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Magnitude of the component of $\vec{A}$ along

$(\hat{i} + \hat{j}) = (\vec{A}.\ \hat{a}) = (2\hat{i} + 3\hat{j}).\frac{1}{\sqrt{2}}(\hat{i} + \hat{j}) = \frac{1}{\sqrt{2}}(2 + 3) = \frac{5}{\sqrt{2}}$

Therefore, component of $\vec{A}$ along

$$(\hat{i}+\hat{j}) = (\overrightarrow{A}.a)\,\hat{a} = (5/\sqrt{2})\left[(\hat{i}+\hat{j})/\sqrt{2}\right] = (5/2)(\hat{i}+\hat{j}) = \frac{5}{2}(\hat{i}+\hat{j})$$

Let $\hat{b}$ is unit vector along the direction

$$(\hat{i}-\hat{j}) = (\hat{i}-\hat{j})/\left|\hat{i}-\hat{j}\right| = (\hat{i}-\hat{j})/\sqrt{(1^2+(-1)^2)} = (\hat{i}-\hat{j})/\sqrt{2}$$

Therefore, component of $\vec{A}$ along $(\hat{i}+\hat{j})$ will be = $(\overrightarrow{A}.\hat{b})\,\hat{b}$

$$= \left\{2\hat{i}+3\hat{j}\,.\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\right\}\frac{(\hat{i}-\hat{j})}{\sqrt{2}} = \frac{(2-3)}{\sqrt{2}}\,\frac{(\hat{i}-\hat{j})}{\sqrt{2}} = \frac{-1}{2}(\hat{i}-\hat{j})$$