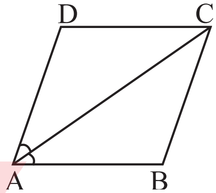
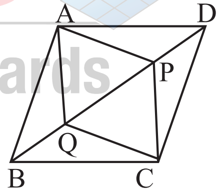
EXERCISE – 1.1

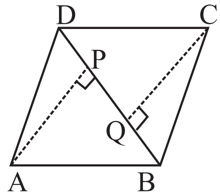
1. If the angles of a quadrilateral be in the ratio 3 : 5 : 9 : 13, find all the angles of the quadrilateral.
2. If the diagonals of a parallelogram are equal, show that it is a rectangle.
3. Show that if the diagonals of a quadrilateral bisect each other at right angles, it is a rhombus.
4. Show that the diagonals of a square are equal and bisect each other at right angles.
5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, it is a square.
6. Diagonal AC of a parallelogram ABCD bisects ∠A (see figure). Show that
   (i) it bisects ∠C also,
   (ii) ABCD is a rhombus.



7. If ABCD be a rhombus, show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.
8. If ABCD be a rectangle in which diagonal AC bisects ∠A as well as ∠C, show that (i) ABCD is a square, (ii) diagonal BD bisects ∠B as well as ∠D.
9. In a parallelogram ABCD, two points P and Q are taken on diagonal BD, such that DP = BQ (see figure). Show that :

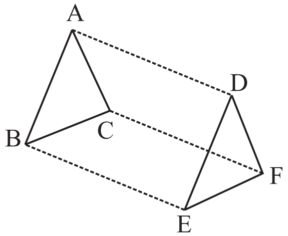


   (i) ΔAPD ≅ ΔCQB (ii) AP = CQ
   (iii) ΔAQB ≅ ΔCPD (iv) AQ = CP
   (v) APCQ is a parallelogram.
10. In a parallelogram ABCD, AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively (see figure). Show that :



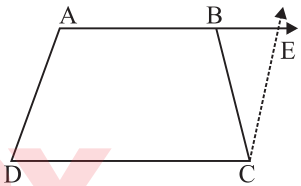
   (i) ΔAPB ≅ ΔCQD (ii) AP = CQ
11. In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that

(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(iv) AC = DF
(vi) $\Delta ABC \cong \Delta DEF$.



12. ABCD is a trapezium in which AB || CD and AD = BC (see figure). Show that

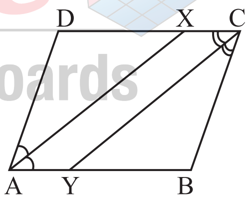
(i) $\angle A = \angle B$
(ii) $\angle C = \angle D$
(iii) $\Delta ABC \cong \Delta BAD$
(iv) diagonal AC = diagonal BD



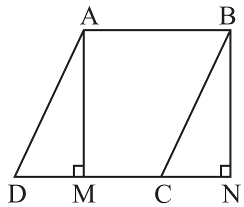
TEST YOURSELF – QR 1

1. Prove that the bisectors of any two consecutive angles of a parallelogram intersect at right angles.

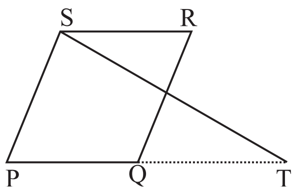
2. In the figure, ABCD is a parallelogram and the line segments AX and CY bisect the angles A and C respectively. Show that AX || CY.



3. In the figure, ABCD is a parallelogram, AM and BN are respectively perpendiculars from A and B to DC and DC produced. Prove that AM = BN.

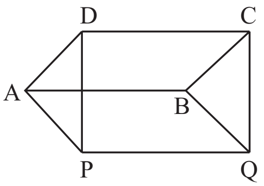


4. In the figure, PQRS is a parallelogram. PQ is produced to T so that QT = PQ. Prove that ST and QR bisect each other.



5. The median AD of a ΔABC is produced to M such that AD = DM. Prove that ABMC is a parallelogram.

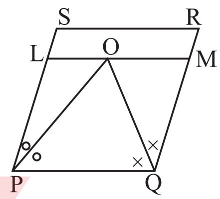
6. In the figure, ABCD and PQBA are two parallelograms. Prove that



(i) DPQC is a parallelogram

(ii) ΔDAP ≅ ΔCBQ

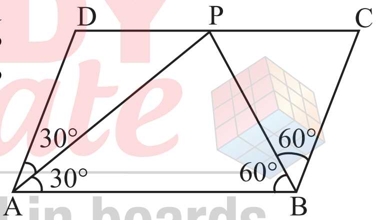
7. In the figure. PQRS is a parallelogram, PQ and QO are the angle bisectors of ∠P and ∠Q respectively. A line LOM is drawn parallel to PQ. Prove that



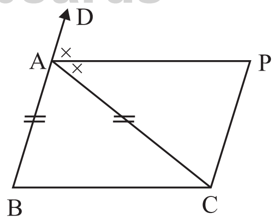
(i) PL = QM

(ii) LO = OM

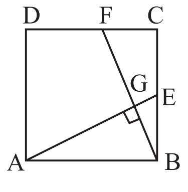
8. In the figure, ABCD is a parallelogram and ∠DAB = 60º. If the bisectors AP and BP of angles A and B respectively, meet at P on CD, prove that P is the midpoint of CD.



9. In the figure, ABC is an isosceles triangle in which AB = AC. CP ∥ AB and AP is the bisector of exterior ∠CAD of ΔABC. Prove that ∠PAC = ∠BCA and ABCP is a parallelogram.

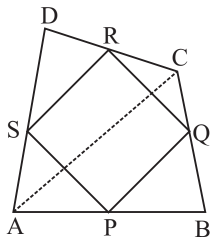


10. In the figure, ABCD is a square. E is any point on BC and BF is perpendicular to AE. Prove that BF = AE.



EXERCISE – 1.2

1. In a quadrilateral ABCD in which P, Q, R and S are the midpoints of the sides AB, BC, CD and DA respectively (see figure) and AC is a diagonal. Show that



(i) SR || AC and SR = $\frac{1}{2}$AC

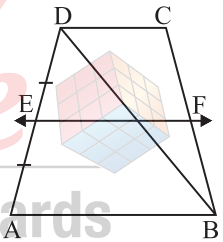
(ii) PQ = SR

(iii) PQRS is a parallelogram.

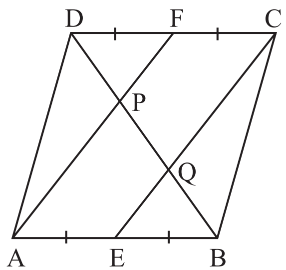
2. In ABCD a rhombus, P, Q, R and S are respectively the midpoints of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

3. In a rectangle ABCD and P, Q, R and S are the midpoints of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

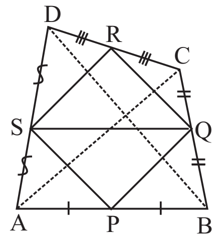
4. In a trapezium ABCD in which AB || DC, BD is a diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the midpoint of BC.



5. In a parallelogram ABCD, E and F are the midpoints of sides AB and CD respectively (see figure). Show that the line segments AF and EC trisect the diagonal BD.



6. Show that the line segments joining the midpoints of the opposite sides of a quadrilateral bisect each other.



7. In a triangle ABC right angled at C, a line through the midpoint M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the midpoint of AC (ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2}AB$

TEST YOURSELF – QR 2

1. Prove that the four triangles, formed by joining in pairs the midpoints of three sides of a triangle, are congruent to each other.

2. If ABC be an isosceles triangle with AB = BC and let D, E and F the midpoints of BC, CA and AB respectively. Show that $AD \perp FE$ and AD is bisected by FE.

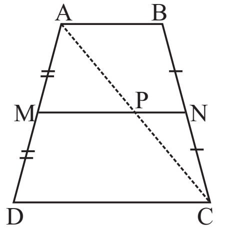
3. P, Q and R are the midpoints of sides BC, CA and AB respectively of a triangle ABC such that PR and BQ meet at X and CR and PQ meet at Y. Prove that $XY = \frac{1}{4}BC$.

4. Show that the quadrilateral, formed by joining the midpoints of the sides of a square, is also a square.

5. In a parallelogram ABCD, P is a point on AD such that $AP = \frac{1}{3}AD$ and Q is a point on BC such that $CQ = \frac{1}{3}BP$. Prove that AQCP is a parallelogram.

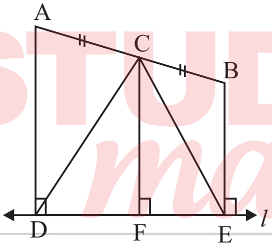
6. ABCD is a trapezium in which side AB is parallel to side DC and E is the midpoint of side AD. If F be a point on the side BC such that the line segment EF is parallel to side DC, prove that $EF = \frac{1}{2}(AB + DC)$.

7. ABCD is a trapezium in which AB || DC. M and N are the midpoints of AD and BC respectively. If AB = 12 cm and MN = 14 cm, find CD.



8. The diagonals of a ☐ABCD are perpendicular. Show that the quadrilateral formed by joining the mid points of its sides, is a rectangle.

9. Points A and B are on the same side of a line l such that AD ⊥ l and BE ⊥ l. If C be the midpoint of AB, prove that CD = CE.



10. M and N divide the side AB of a ABC into three equal parts. Line segments MP and NQ both are parallel to BC and meet AC in P and Q respectively. Prove that P and Q divide AC into three equal parts.

ANSWERS

NCERT Exercises and Assignments

Exercise – 1.1

1. Let the angles be $(3x)^o$, $(5x)^o$, $(9x)^o$ and $(13x)^o$.

Then, $3x + 5x + 9x + 13x = 360$

$\therefore$ $30x = 360$

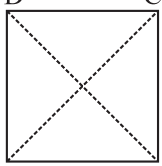
$\therefore$ $x = \dfrac{360}{30}$

$= 12$

$\therefore$ The angles are $(3 \times 12)^o$, $(5 \times 12)^o$, $(9 \times 12)^o$ and $(13 \times 12)^o$, i.e. 36^o, 60^o, 108^o and 156^o.

2. In the parallelogram ABCD, it is given that AC = BD.

To prove: ABCD is a rectangle.



Proof: In Δs ABC and DCB, we have

AB = DC [opposite sides of a ||gm]

BC = BC [common]

and, AC = DB [given]

$\therefore$ By SSS criterion of congruence.

ΔABC ≅ ΔDCB

$\therefore$ ∠ABC = ∠DCB ...(i) [CPCT]

But AB || DC and BC cuts them.

∠ABC + ∠DCB = 180^o ...(ii)

$\therefore$ 2∠DCB = 180^o [sum of consecutive interior angles]

$\therefore$ ∠ABC = 90^o

Thus,

∠ABC = ∠DCB = 90^o

$\therefore$ ABCD is a parallelogram one of whose angle is 90^o.

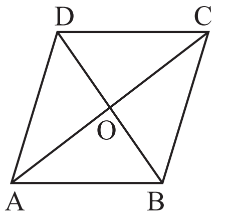
Hence, ABCD is a rectangle.

3. In the quadrilateral ABCD, it is given that the diagonals AC and BD intersect at O such that AO = OC, BO = OD and AC ⊥ BD.

To prove: ABCD is a rhombus.

Proof: First we shall prove that ABCD is a parallelogram.

In ∆s AOD and COB, we have



AO = CO [given]

OD = OB [given]

∠AOD = ∠COB [vertically opposite angles]

By SAS criterion of congruence,

∆AOD ≅ ∆COB

AD = CP [CPCT]

Similarly, by proving ∆AOB ≅ ∆COD, we get, AB = CD

Hence, ABCD is a parallelogram.

Now, we shall prove that parallelogram ABCD is a rhombus.

In ∆s AOD and COD, we have

AO = CO [given]

∠AOD = ∠COD [right angles]

OD = OD [common]

∴ By SAS criterion of congruence,

∆AOD ≅ ∆COD

AD = CD ...(i) [CPCT]

Now, ABCD is a parallelogram. [Proved above]

∴ AB = CD and AD = BC [opp. sides of a ||gm are equal]

∴ AB = CD = AD = BC [using (i)]

Hence, quadrilateral ABCD is a rhombus.

Alternate Method:

We shall show that all sides of this quadrilateral are equal.

∴ Consider ∆AOD and ∆BOC

AO = OC [given]

OD = OB [given]

∠AOD = ∠BOC [vertically opposite angles]

∴ By SAS criterian of congurence,

∆AOD ≅ ∆BOC

By CPCT AD = BC … (i)

Similarly ∆COD ≅ ∆AOD

∴ By CPCT AB = CD … (ii)

Now, consider ∆AOD and ∆COD

AO = OC [given]

OD = OD [common]

∠AOD = ∠COD [right angles]

By SAS criterion of congruence,

$\Delta AOD \cong \Delta COD$

$\therefore$ By CPCT AD = DC … (iii)

From (i), (ii) and (iii), we get

AB = BC = CA = DA

$\Rightarrow$ ABCD is a rhombus.

4. Given: A square ABCD.

To prove: AC = BD, $AC \perp BD$ and OA = OC, OB = OD.

Proof: Consider ΔABD and ΔABC.

AD = BC [sides of a square]

AB = AB [common]

$\angle DAB = \angle CBA$ [right angles]

$\therefore$ $\Delta ABD \cong \Delta ABC$ [by SAS]

$\therefore$ AC = BD [by CPCT]

Now consider ΔAOD and ΔCOD.

OD = OD [Common]

AD = DC [sides of squares]

$\angle ADO = \angle CDO$ [diagonals of square bisect the opposite angles]

$\therefore$ $\Delta AOD \cong \Delta COD$ [by SAS]

$\Rightarrow$ AO = OC … (i) [by CPCT]

and $\angle AOD = \angle COD$ … (ii)

But $\angle AOD + \angle COD = 180°$ [Linear pair Axiom]

$\Rightarrow$ $\angle AOD = \angle COD = 90°$ [by (ii)]

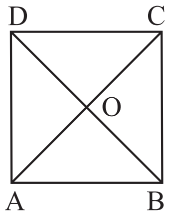
Similarly by congurence of triangles COD and COB, we get OB = OD … (iii)

$\therefore$ By (i), (ii) and (iii), we have

Diagonals bisect each other at 90°.

5. Given: A quadrilateral ABCD in which the diagonals AC = BD, AO = OC, BO = OD and $AC \perp BD$.

To prove: Quadrilateral ABCD is a square.



Proof:

AO = OC [given]

OD = OB [given]

and the diagonals AC and BD are bisected at O, ABCD is a parallelogram

Now, we shall prove that it is a square.

In Δs AOB and AOD, we have

AO = BO [common]

$\angle AOB = \angle AOD$ [each = 90º, given] and, OB = OD [given]

By SAS criterion of congruence,

$\Delta AOB \cong \Delta AOD$

$\therefore$ AB = AD [CPCT]

But AB = CD and AD = BC [Opposite sides of a parallelogram]

AB = BC = CD = AD ...(ii)

Now, in triangles ABD and BAC, we have

AB = BA [common]

AD = BC [Opposite sides of a parallelogram are equal]

and, BD = AC [given]

$\therefore$ By SSS criterion of congruence,

$\Delta ABD \cong \Delta BAC$

$\therefore$ $\angle DAB = \angle CBA$ [CPCT]

$\therefore$ $\angle DAB + \angle CBA = 180^\circ$ [Sum of consecutive interior angles]

$\therefore$ $2\angle DAB = 180^\circ$

$\angle DAB = 90^\circ$ ...(iii)

From (ii) and (iii), ABCD is a square.

Alternate Method:

Since in a quadrilateral diagonals bisect each other, hence $\Rightarrow$ ABCD is a parallelogram.

Now, ABCD is a parallelogram with diagonals AC and BD are equal.

$\therefore$ ABCD is a rectangle.

i.e. each angle is 90° … (i)

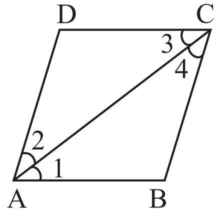
Now, ABCD is a rectangle with diagonals bisect each other at 90°

$\therefore$ ABCD is a rhombus

i.e. all sides equal … (ii)

From (i) and (ii), $\Rightarrow$ ABCD is a square.

6. (i) **Given:** A parallelogram ABCD in which diagonal AC bisects $\angle A$.



To prove: AC bisects $\angle C$.

Proof: ABCD is a parallelogram, $\therefore$ AB || DC.

Now, AB || DC and AC intersects $\angle A$.

$\angle 1 = \angle 3$ ...(i) [alternate interior angles]

Again, AD || BC and AC intersects them

$\therefore$ $\angle 2 = \angle 4$ ...(ii) [alternate interior angles]

But it is given that AC is the bisector of $\angle A$

$\therefore$ $\angle 1 = \angle 2$ ...(iii)

From (1), (2) and (3), we have

$\angle 3 = \angle 4$

Hence, AC bisects ∠C.

(ii) **To prove :** That ABCD is a rhombus.

From part (i) : (1). (2) and (3) give ∠1 = ∠2 = ∠3 = ∠4

Now in Δ ABC, ∠1 = ∠4

∴ AB = BC[Sides opp. to equal angles in a Δ are equal]

Similarly, in ADC, we have

AD = DC

Also, ABCD is a ||gm

AB = CD, [Opposite sides of a parallelogram are equal]

AD = BC

Combining theses, we get AB = BC = CD = DA

Hence, ABCD is a rhombus.

7. **Given :** A rhombus ABCD.

To prove that (i) Diagonal AC bisects ∠A as well ∠C.

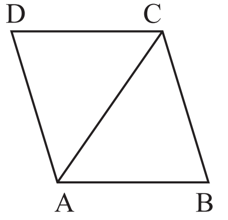
(ii) Diagonal BD bisects ∠B as well as ∠D.

Proof : In Δ ADC,

AD = DC [Sides of a rhombus are equal]

∴ ∠DAC = ∠DCA ...(1)

[Angles opp. to equal sides of a triangle are equal]



Now AB || DC and AC intersects them

∠BCA = ∠DAC ...(2) [Alternate angles]

From (1) and (2), we have

∠DCA = ∠BCA

∴ AC bisects ∠C

In ΔABC,

AB = BC [Sides of a rhombus are equal]

∴ ∠BCA = ∠BAC ...(3)

[Angles opp. to equal sides of a triangle are equal]

From (2) and (3), we have

∠BAC = ∠DAC

∴ AC bisects ∠A

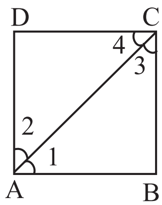
Hence, diagonal AC bisects ∠A as well as ∠C.

Similarly, diagonal BD bisects ∠B as well as ∠D.

8. **Given:** ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C.

To prove: (i) ABCD is a square and (ii) diagonal BD bisects ∠B as well as ∠D.

Proof: (i) Since AC bisects ∠A as well as ∠C in the rectangle ABCD.



$\therefore \quad \angle 1 = \angle 2 = \angle 3 = \angle 4$ $\left[\because \text{ Each angle} = \frac{90^\circ}{2} = 45^\circ\right]$

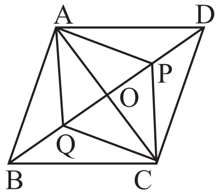
$\therefore$ In ΔADC, $\angle 2 = \angle 4$

AD = CD [sides opposite to equal angles]

Thus, the rectangle ABCD is a square.

(ii) In a square, diagonals bisect the opposite angles. So, BD bisects $\angle B$ as well as $\angle D$.

9. **Given:** ABCD is a parallelogram. P and Q are points on the diagonal BD such that DP = BQ. (see figure). **To prove:**



(i) $\Delta APD \cong \Delta CQB$

(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

Construction: Join AC to meet BD in O.

Proof

(i) In Δs APD and CQB, we have

AD = CB [opposite sides of the parallelogram ABCD]

$\angle ADP = \angle CBQ$ [alternate interior angles]

DP = BQ [given]

$\therefore$ By SAS criterion of congruence,

$\Delta APD \cong \Delta CQB$

(ii) AP = CQ [opposite sides of the parallelogram APCQ]

(iii) In Δs AQB and CPD, we have

AB = CD [opposite sides of the parallelogram ABCD]

AQ = CP [opposite sides of the parallelogram APCQ]

BQ = DP [given]

$\therefore$ By SSS criterion of congruence,

$\Delta AQB \cong \Delta CPD$

(iv) AQ = CP [opposite sides of the parallelogram APCQ]

(v) Since AP = CQ and AQ = AP, so APCQ is a parallelogram.

10. (i) Since ABCD is a parallelogram, therefore, DC || AB.

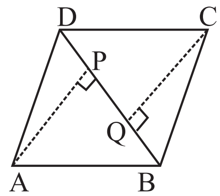
Now, DC || AB and transversal BD intersects them at B and D.

$\angle ABD = \angle BDC$ [alternate interior angles]

Now, in Δs APB and CQD, we have

$\angle ABP = \angle QDC$ [$\because \angle ABD = \angle BDC$]

$\angle APB = \angle CQD$ [Each angle = 90°]



and, AB = CD [opposite sides of a parallelogram]

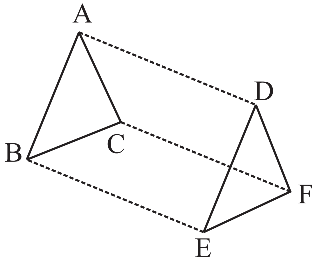
$\therefore$ By AAS criterion of congruence,

$\Delta APB \cong \Delta CQD$

(ii) Since $\Delta APB \cong \Delta CQD$

AP = CQ [CPCT]

11. **Given:** Two Δs ABC and DEF are such that AB = DE and AB || DE. Also BC = EF and BC || EF.



To Prove

(i) quadrilateral ABED is a parallelogram.

(ii) quadrilateral BEFC is a parallelogram.

(iii) AD || CF and AD = CF.

(iv) quadrilateral ACFD is a parallelogram.

(v) AC || DF and AC = DF

(vi) $\Delta ABC \cong \Delta DEF$.

Proof

(i) Consider the quadrilateral ABED. We have AB = DE and AB || DE

$\Rightarrow$ One pair of opposite sides are equal and parallel.

$\Rightarrow$ ABED is a parallelogram.

(ii) Now, consider quadrilateral BEFC. We have BC = EF and BC || EF

$\Rightarrow$ One pair of opposite sides are equal and parallel.

$\Rightarrow$ BEFC is a parallelogram

(iii) Now, AD = BE and AD || BE ...(I) [$\because$ ABED is a parallelogram]

and CF = BE and CF || BE ...(II) [$\because$ BEFC is a parallelogram]

From (I) and (II), we have

AD = CF and AD || CF.

(iv) Since AD = CF and AD || CF.

$\Rightarrow$ One pair of opposite sides are equal and parallel.

$\Rightarrow$ ACFD is a parallelogram.

(v) Since ACFD is parallelogram,

$\therefore$ AC = DF and AD || DF. [Opposite sides of the parallelogram ACFD]

(vi) In Δs ABC and DEF, we have

AB = DE [Opposite sides of the parallelogram ABED]

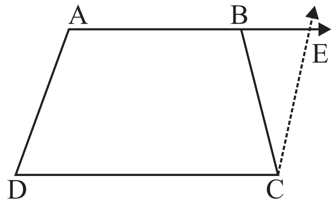
BC = EF [Opposite sides of the parallelogram BEFC]

and CA = FD [Opposite sides of the parallelogram ACFD]

$\therefore$ By SSS criterion of congruence,

$\Delta ABC \cong \Delta DEF$

12. Given: ABCD is a trapezium in which AB || CD and AD = BC.



To Prove

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) ΔABC ≅ ΔBAD

(iv) diagonal AC = diagonal BD

Construction: Let us extend AB. Then, draw a line through C, which is parallel to AD, intersecting AE at point E.

Proof: AECD is a parallelogram.

(i) AD = CE [opposite sides of the parallelogram AECD]

AD = BC [given]

∴ BC = CE

∠CEB = ∠CBE ...(I) [angle opposite to equal sides]

Consider parallel lines AD and CE. AE is the transversal line for them.

∠A + ∠CEB = 180º [Angles on the same side of transversal]

∠A + ∠CBE = 180º ...(II) [from (I)]

∠B + ∠CBE = 180º ...(III) [linear pair angles]

From relations (II) and (III), we obtain

∠A = ∠B

(ii) Since AB || CD

∠A + ∠D = 180º [angles on the same side of the transversal]

Also, ∠C + ∠B = 180° [angles on the same side of the transversal]

∴ ∠A + ∠D = ∠C + ∠B

but, ∠A = ∠B [From Result (I)]

∴ ∠C = ∠D

(iii) In ΔABC and ΔBAD, we have

AB = BA [common side]

BC = AD [given]

∠B = ∠A [Proved in (I)]

∴ ΔABC ≅ ΔBAD [SAS congruence rule]

(iv) We know that,

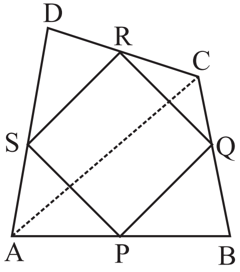
ΔABC ≅ ΔBAD

∴ AC = BD [By CPCT]

Exercise – 1.2

1. **Given:** A quadrilateral ABCD is such that P, Q, R and S are the midpoints of the sides AB, BC, CD and DA respectively. Also AC is its diagonal.

To Prove



(i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram

Proof

(i) In ΔACD, we have S as the midpoint of AD and R the mid-point of CD.

Then SR || AC and SR = $\frac{1}{2}$AC [Midpoint theorem]

(ii) In Δ ABC, we have P as the midpoint of the side AB and Q the midpoint of the side BC.

Then PQ || AC and PQ = $\frac{1}{2}$AC [Midpoint theorem]

Thus, we have proved that

$$\left.\begin{array}{l} PQ \parallel AC \\ SR \parallel AC \end{array}\right\} \Rightarrow PQ \parallel SR$$

Also $$\left.\begin{array}{l} PQ = \frac{1}{2}AC \\ SR = \frac{1}{2}AC \end{array}\right\} \Rightarrow PQ = SR$$

(iii) Since PQ = SR and PQ || SR

∴ One pair of opposite sides are equal and parallel.

∴ PQRS is a parallelogram.

2. **Given:** ABCD is a rhombus in which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined to obtain a quadrilateral PQRS.

To Prove: PQRS is a rectangle.

Construction: Join AC.

Proof: In ABC, P and Q are the midpoints of AB and BC respectively.

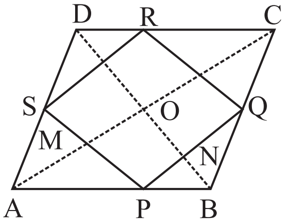
∴ PQ || AC and PQ = $\frac{1}{2}$AC ...(i)

∴ Similarly, in Δ ADC, R and S are the midpoints of CD and PD respectively.

Hence, SR || AC and SR = $\frac{1}{2}$AC ...(ii)

From (i) and (ii), we get PQ || RS and PQ = SR

Now, in quadrilateral PQRS, one pair of opposite sides PQ and SR are equal and parallel.



∴ PQRS is a parallelogram.

Join DB so that AC and BD intersects at O.

Now By midpoint theorem

SP || DB and PQ || AC

We can say that PN || MO and ON || MP

∴ MONP is a parallelogram with ∠MON = 90° (as diagonals bisect at 90° in rhombus)

∴ ∠MPN = 90° (∵ opposite angles of a parallelogram)

∴ PQRS is a rectangle, as a parallelogram with one angle is 90° is known as rectangle.

3. **Given:** ABCD is a rectangle in which P, Q, R and S are the midpoints of AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined to obtain a quadrilateral PQRS.

To Prove: PQRS is a rhombus.

Construction: Join AC.

Proof: In ABC, P and Q are the midpoints of sides AB and BC respectively.

PQ || AC and PQ = $\frac{1}{2}$ AC ...(i)

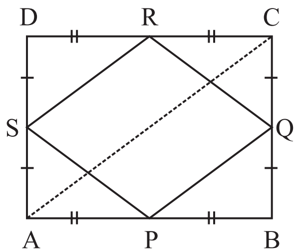
Similarly, in Δ ADC, R and S are the midpoints of sides CD and AD respectively.

∴ SR || AC and SR = $\frac{1}{2}$ AC ...(ii)

From (i) and (ii), we get

PQ || SR and PQ = SR ...(iii)

Now, in quadrilateral PQRS, one pair of opposite sides PQ and SR are parallel and equal. [From (iii)]



∴ PQRS is a parallelogram ...(iv)

Now, AD = BC [opposite sides of the rectangle ABCD]

$\frac{1}{2}$ AD = BC

∴ AS = BQ

In, Δs APS and BPQ, we have

AP = BP [∵ P is the midpoint of AB]

∠PAS = ∠PBQ [each angle = 90º]

AS = BQ [Proved above]

∴ ΔAPS ≅ ΔBPQ [SAS congruence rule]

∴ PS = PQ ... (v) [CPCT]

From (iv) and (v), we get

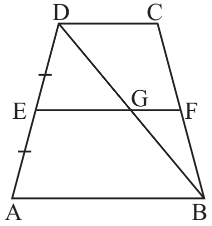
PQRS is a rhombus.

4. Given: In trapezium ABCD, AB || DC

E is the midpoint of AD and EF || AB.

To Prove: F is the midpoint of BC.

Construction: Join DB. Let it intersects EF at G.



Proof: In ΔDAB, E is the midpoint of AD [given]

$\therefore$ EG || AB [$\because$ EF || AB]

$\therefore$ By converse of midpoint theorem G is the midpoint of DB.

In Δ BCD, G is the midpoint of BD [Proved]

GF || DC [$\because$ AB || DC, EF || AB $\Rightarrow$ DC || EF]

$\therefore$ By converse of midpoint theorem, F is the midpoint of BC.

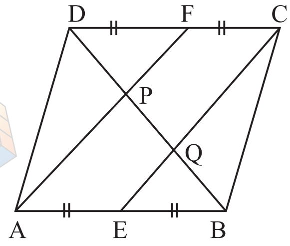
5. Given: E and F are the midpoints of sides AB and CD respectively of the parallelogram ABCD whose diagonal is BD.

To prove: BQ = QP = PD

Proof: Since, ABCD is parallelogram, [given]

$\therefore$ AB || DC and AB = DC [opposite sides of a parallelogram]

Also as E is the midpoint of AB,



$\therefore$ $AE = \frac{1}{2}AB$ ...(i)

Similarly, F is the midpoint of CD,

$\therefore$ $CF = \frac{1}{2}CD$

$\therefore$ $CF = \frac{1}{2}AB$ ... (ii) [$\because$ CD = AB]

From (i) and (ii), AE = CF

Also AE || CF

Thus, a pair of opposite sides of a quadrilateral AECF are parallel and equal.

Since quadrilateral AECF is a parallelogram,

EC || AF

EQ || AP and QC || PF

In Δ BMA, E is the midpoint of BA [given]

EQ || AP [proved]

$\therefore$ BQ = PQ ...(iii) [converse of midpoint theorem]

Similarly by taking Δ CLD, we can prove that

DP = QP ...(iv)

From (iii) and (iv), we get

BQ = PQ = DP

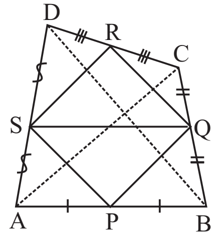
Hence, AF and CE trisect the diagonal AC.

6. **Given:** In a quadrilateral ABCD, P, Q, R and S are the midpoints of AB, BC, CD and DA respectively. PR and QS intersect each other at O.

To Prove: OP = OR, OQ = OS

Construction: Join PQ, QR, RS, SP, AC and BD.

Proof: In Δ ABC, P and Q are the midpoints of AB and BC respectively.



$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC$$

Similarly, we can prove that

$$RS \parallel AC \text{ and } RS = \frac{1}{2} AC$$

$\therefore$ PQ || SR and PQ = SR

Thus, a pair of opposite sides of a quadrilateral PQRS are parallel and equal.

$\therefore$ The quadrilateral PQRS is a parallelogram.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Diagonals PR and QS of a parallelogram PQRS, i.e. the line segments joining the midpoints of opposite sides of the quadrilateral ABCD bisect each other.

7. **Given:** The triangle ABC is right angled at C, M is the midpoint of its hypotenuse AB. Also MD || BC.

To prove

(i) D is the midpoints of AC (ii) MD ⊥ AC

(iii) $CM = MA = \frac{1}{2} AB$

Proof

(i) In Δ ABC, M is the midpoint of AB and MD || BC. Therefore, D is the midpoint of AC. (By converse of midpoint theorem)

i.e. AD = DC ...(I)

(ii) Since MD || BC. Therefore,

$\angle ADM = \angle ACB$ [corresponding angles]

$\therefore \angle ADM = 90^\circ$ [$\because \angle ACB = 90^\circ$ (given)]

But, $\angle ADM + \angle CDM = 180^\circ$ [$\because \angle ADM$ and $\angle CDM$ are angles of a linear pair]

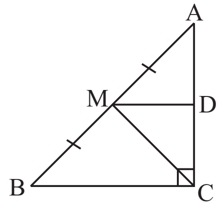
$90^\circ + \angle CDM = 180^\circ$

$\Rightarrow \angle CDM = 90^\circ$

Thus, $\angle ADM = \angle CDM = 90^\circ$ ...(II)

$\therefore$ MD ⊥ AC

(iii) In Δs AMD and CMD, we have



	AD	= CD	[From (I)]
	$\angle$ADM	= $\angle$CDM	[From (II)]
and,	MD	= MD	[common]

By SAS criterion of congruence

$\Delta AMD \cong \Delta CMD$

$\therefore$ MA = MC [$\because$CPCT]

Since M is the midpoint of AB, MA = $\frac{1}{2}$ AB

Hence, CM = MA = $\frac{1}{2}$ AB