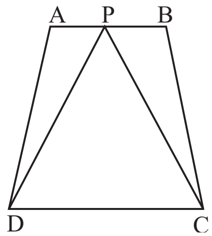
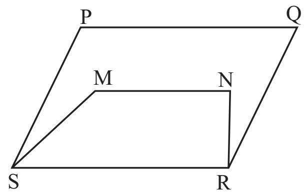
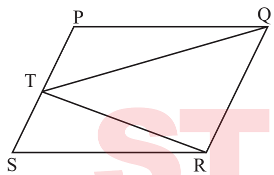
NCERT TEXTUAL EXERCISE (SOLVED)

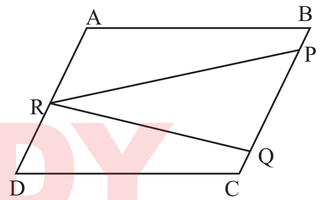
EXERCISE – 2.1

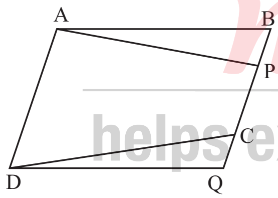
1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

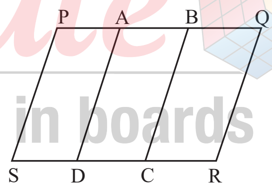
(i)


(ii)


(iii)


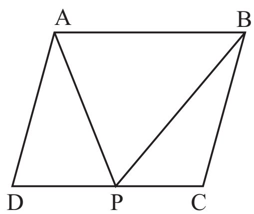
(iv)


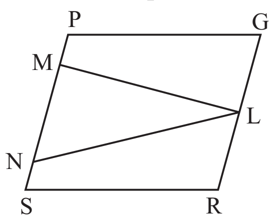
(v)


(vi)


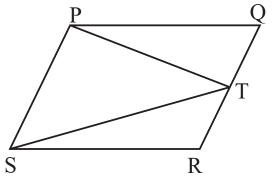
TEST YOURSELF – AR 1

1. Which of the following figures lie on the same base and in between the same parallels. In such case, write the common base and two parallels.

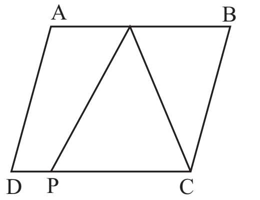
(i)


(ii)


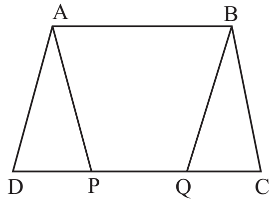
(iii)



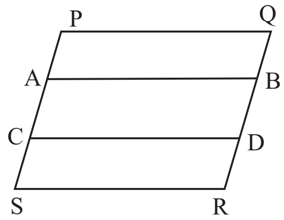
(iv)



(v)

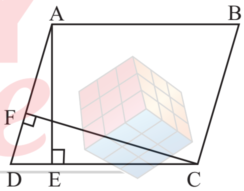


(vi)



EXERCISE – 2.2

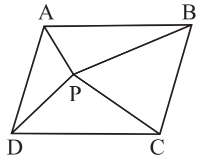
1. In the given figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



2. If E, F, G and H are respectively the midpoints of the sides of a parallelogram ABCD. Show that ar(EFGH) = $\frac{1}{2}$ ar(ABCD).

3. Let P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

4. In the given figure, P is a point in the interior of a parallelogram ABCD. Show that



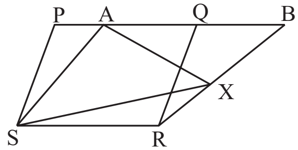
(i) ar(ΔAPB) + ar(ΔPCD) = ar $\frac{1}{2}$ (||gm ABCD)

(ii) ar(ΔAPD) + ar(ΔPBC) = ar(APB) + ar(PCD)

5. In figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that :

(i) ar (|| gm PQRS) = ar (|| gm ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (|| gm PQRS)



6. A farmer has a field in the form of a parallelogram PQRS. He took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should he do it?

TEST YOURSELF – AR 2

1. The adjacent sides of a parallelogram are 8 m and 10 m. If the distance between the longer sides is 4 m, find the distance between the shorter sides.

2. Show that the line segment joining the midpoints of a pair of opposite sides of a parallelogram divides it into two equal parts.

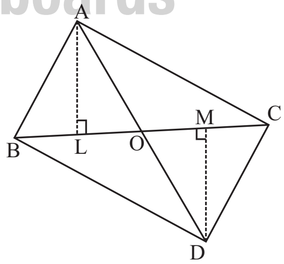
3. The diagonals of a parallelogram ABCD intersect at O. A line through O meets AB at X and CD at Y. Show that ar (quad. AXYD) = $\frac{1}{2}$ ar (||gm ABCD).

4. If the triangle and a parallelogram are on the same base and between the same parallels, prove that the area of the triangle is equal to the half the area of the parallelogram.

5. Prove that the area of a trapezium is half the product of its height and the sum of the parallel sides.

6. If each diagonal of a quadrilateral separates it into two triangles of equal areas, show that the quadrilateral is a parallelogram.

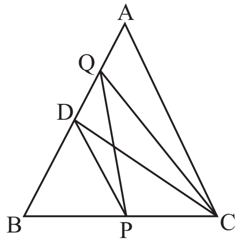
7. Show that a median of a triangle divides it into two triangles of equal area.



8. In triangles ABC and DBC are on the same base BC with vertices A, and D on opposite sides of the line BC, such that ar (ΔABC) = ar (ΔDBC), show that BC bisects AD.

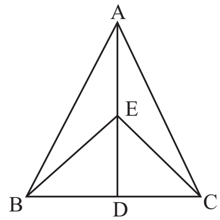
9. In ΔABC, D is the midpoint of AB and P is any point on BC. CQ || PD meets AB at Q.

Show that ar (ΔBPQ) = $\frac{1}{2}$ ar (ΔABC).



EXERCISE – 2.3

NCERT TEXTUAL EXERCISE (SOLVED)

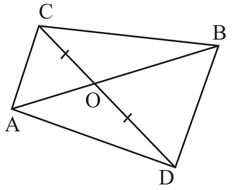


1. In the given figure, E is any point on median AD of a ΔABC. Show that ar (ΔABE) = ar (ΔACE).

2. In a triangle ABC, E is the midpoint of median AD. Show that ar (ΔBED) = $\frac{1}{4}$ ar(ΔABC).

3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.

4. In the given figure, ABC and ABD are two triangles on the same base AB. If a line segment CD is bisected by AB at O, show that ar (ΔABC) = ar (ΔABD).



5. If D, E and F be the midpoints of the sides BC, CA and AB respectively of ΔABC, show that

(i) BDEF is a parallelogram

(ii) ar (ΔDEF) = $\frac{1}{4}$ ar (ΔABC)

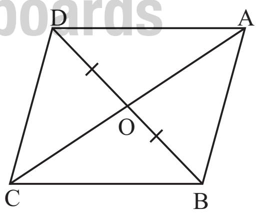
(iii) ar (||gm BDEF) = $\frac{1}{2}$ ar (ΔABC)

6. In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, show that

(i) ar (ΔDOC) = ar (ΔAOB)

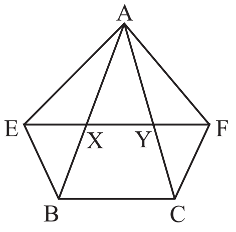
(ii) ar (ΔDCB) = ar (ΔACB)

(iii) DA || CB or ABCD is a parallelogram.

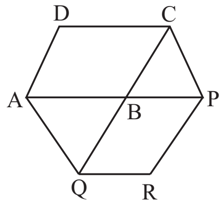


7. If D and E be points on sides AB and AC respectively of ΔABC such that ar (ΔDBC) = ar (ΔEBC), prove that DE || BC.

8. Let XY is a line parallel to side BC of triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that ar (ΔABE) = ar (ΔACF).



9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see figure). Show that ar (||gm ABCD) = ar (||gm PBQR).

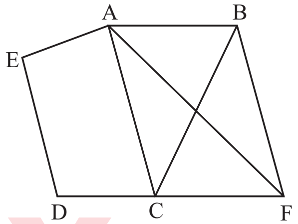


10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (ΔAOD) = ar (ΔBOC).

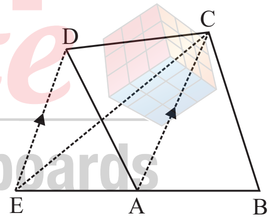
11. In a pentagon ABCDE, a line through B parallel to AC meets DC produced at F, show that

(i) ar (ΔACB) = ar (ΔACF)

(ii) ar (□AEDF) = ar (□ABCDE)

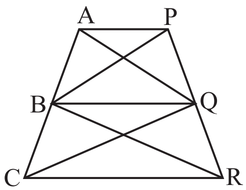


12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.



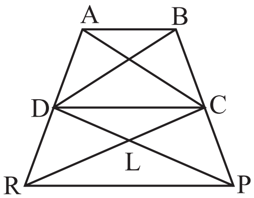
13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y respectively. Prove that ar (ΔADX) = ar (ΔACY).

14. In figure, AP || BQ || CR. Prove that ar (ΔAQC) = ar (ΔPBR).



15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (ΔAOD) = ar (ΔBOC). Prove that ABCD is a trapezium.

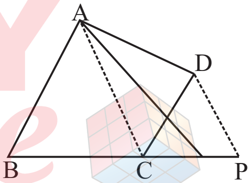
16. In figure, ar (ΔDRC) = ar (ΔDPC) and ar (ΔBDP) = ar (ΔARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



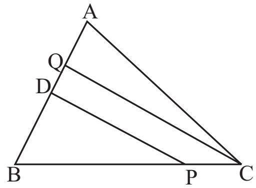
TEST YOURSELF – AR 3

1. The diagonals AC and BD of a quadrilateral ABCD, intersect at O. Prove that if BO = OD, the Δs ABC and ADC are equal in area.
2. If the diagonals AC and BD of a quadrilateral ABCD intersect at O and divide the quadrilateral into four triangles of equal area, show that the quadrilateral ABCD is a parallelogram.
3. A quadrilateral ABCD is such that diagonal BD divides its area into two equal parts. Prove that BD bisects AC.
4. If O is any point on the diagonal BD of a parallelogram ABCD. Prove that area (ΔOAB) = area (ΔOBC).
5. If the medians of a ΔABC intersect at G. Show that

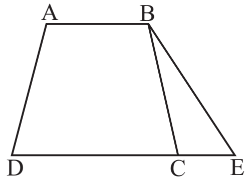
   ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC).
6. ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced at P as shown in the figure. Prove that ar (ΔABP) = ar (□ABCD).



7. In ΔABC, D is the midpoint of AB. P is any point on BC. CQ || PD meets AB in Q. Show that ar (ΔBPQ) = $\frac{1}{2}$ ar (ΔABC), where AD = DB.

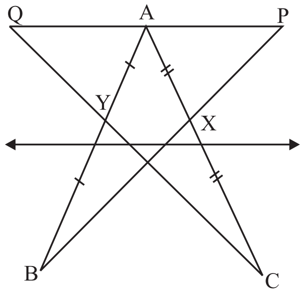


8. In the figure, ABCD is a trapezium in which AB || DC, DC is produced to E such that CE = AB, prove that ar (ΔABD) = ar (ΔBCE).



9. Let ABC is a triangle in which D is the midpoint of BC and E is the midpoint of AD. Prove that area of ΔBED = $\frac{1}{4}$ area of ΔABC.

10. In the figure, X and Y are the midpoints of AC and AB respectively. QP || BC and CYQ and BXP are straight lines. Prove that ar (ΔABP) = ar (ΔACQ).



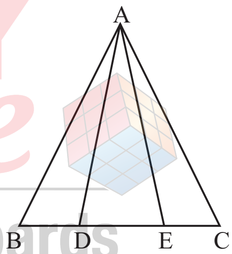
EXERCISE – 3.4

OPTIONAL

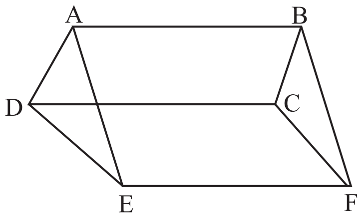
1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal area. Show that the perimeter of the parallelogram is greater than that of the rectangle.

2. In the figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ΔABD) = ar (ΔADE) = ar (ΔAEC).

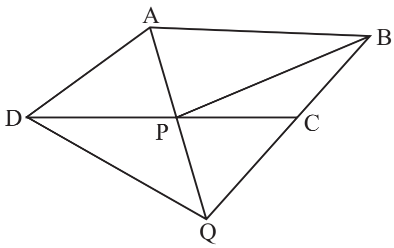
   Can you now answer the question that you have left in the 'Introduction', of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



3. In the figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ΔADE) = ar (ΔBCF).



4. In the figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (ΔBPC) = ar (ΔDPQ).



5. In the figure, ABC and BDE are two equilateral triangles such that D is the midpoint of BC. If AE intersects BC at F, show that

(i) ar (ΔBDE) = $\frac{1}{4}$ ar (ΔABC)

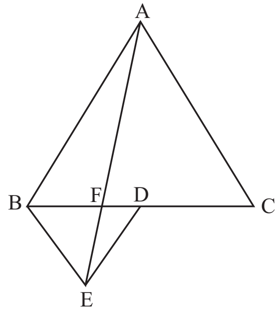
(ii) ar (ΔBDE) = $\frac{1}{2}$ ar (ΔBAE)

(iii) ar (ΔABC) = 2 ar (ΔBEC)

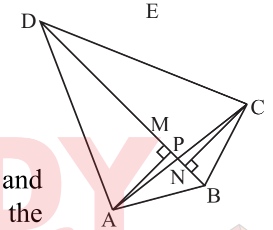
(iv) ar (ΔBFE) = ar (ΔAFD)

(v) ar (ΔBFE) = 2 ar (ΔFED)

(vi) ar (ΔFED) = $\frac{1}{8}$ ar (ΔAFC)



6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar (ΔAPB) × ar (ΔCPD) = ar (ΔAPD) × ar (ΔBPC)

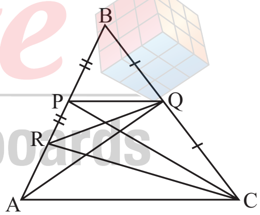


7. If P and Q be the midpoints of sides AB and BC respectively of a triangle ABC and R is the midpoint of AP, show that

(i) ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔARC)

(ii) ar (ΔRQC) = $\frac{3}{8}$ ar (ΔABC)

(iii) ar (ΔPBQ) = ar (ΔARC).



8. In the figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that

(i) ΔMBC ≅ ΔABD

(ii) ar (BYXD) = 2 ar (ΔMBC)

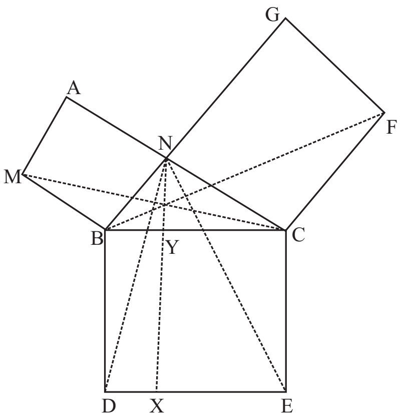
(iii) ar (BYXD) = ar (□AMBN)

(iv) ΔFCB ≅ ΔACE

(v) ar (□CYXE) = 2 ar (ΔFCV)

(vi) ar (□CYXE) = 2 ar (□ACFG)

(vii) ar (□BCED) = ar (□ABMN) + ar (□ACFG)



ANSWERS

NCERT Exercises and Assignments

Exercise – 2.1

1. The figures mentioned in the question lie on the same base and between the same parallels as indicated against them.

(i) base DC, parallels DC and AB.

(ii) base QR, parallels QR and PS.

(iii) base AD, parallels AD and BQ.

Exercise – 2.2

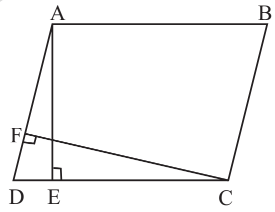
1. We know that

Area of a parallelogram = Base × Height

So, the area of parallelogram ABCD = AB × AE

$= (16 \times 8)\ \text{cm}^2$

$= 128\ \text{cm}^2$ ...(i)



Also, Area of parallelogram ABCD = AD × CF

$= (AD \times 10)\ \text{cm}^2$ ...(ii)

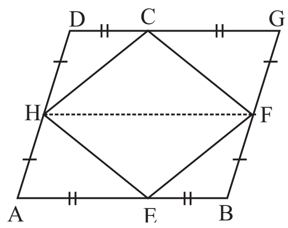
From (i) and (ii), we get

$$128 = AD \times 10$$

$$AD = \frac{128}{10}\ \text{cm}$$

$$= 12.8\ \text{cm}$$

2. The triangle HGF and the parallelogram HDCF stand on the same base HF and lie in between the same parallels HF and DC.



$\therefore$ ar (ΔHGF) = $\frac{1}{2}$ ar (||gm HDCF) ...(i)

Similarly, the triangle HEF and the parallelogram ABFH stand on the same base HF and lie between the same parallels HF and AB.

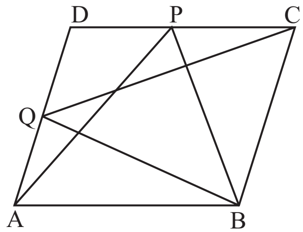
$\therefore$ ar (ΔHEF) = $\frac{1}{2}$ ar (||gm ABFH) ...(ii)

∴ Adding (i) and (ii), we get

$$ar(\Delta HGF) + ar(\Delta HEF) = \frac{1}{2}[ar(||gm\ HDCF) + ar(||gm\ ABFH)]$$

$$ar(||gm\ EFGH) = \frac{1}{2}\ ar(||\ gmABCD)$$

3. The triangle APB and the parallelogram ABCD stand on the same base AB and lie between the same parallels AB and DC.



∴ $ar(\Delta APB) = \frac{1}{2}\ ar(||gm\ ABCD)$ ...(i)

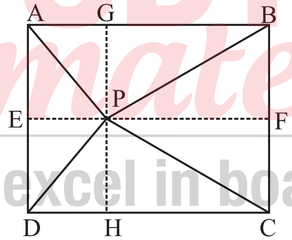
Similarly, the triangle BQC and the parallelogram ABCD stand on the same base BC and lie between the same parallels BC and AD.

∴ $ar(\Delta BQC) = \frac{1}{2}\ ar(||gm\ ABCD)$ ...(ii)

From (i) and (ii), we have

∴ $ar(\Delta APB) = ar(\Delta BQC)$

4. Draw EPF parallel to AB or DC and GPH parallel to AD or BC.



Now AGHD is a parallelogram [∵ GH || DA and AG || DH]

Similarly, HCBG, EFCD and ABFE are parallelograms.

(i) The triangle APB and the parallelogram ABEF stand on the same base AB and lie between the same parallels AB and EF.

$ar(\Delta APB) = \frac{1}{2}\ ar(||gm\ ABFE)$ ...(I)

Similarly, $ar(\Delta PCD) = \frac{1}{2}\ ar(||gm\ EFCD)$ ... (II)

Adding (I) and (II), we get

$$ar(\Delta APB) + ar(\Delta PCD) = \frac{1}{2}[ar(||gm\ ABFE) + ar(||gm\ EFCD)]$$

$$= \frac{1}{2}\ ar(ABCD) \quad ...(III)$$

(ii) The triangle APD and the parallelogram AGHD are on the same base AD and lie between the same parallels AD and HG.

$$\text{ar}(\Delta APD) = \frac{1}{2}\ \text{ar}(\|\text{gm } AGHD) \qquad \text{...(IV)}$$

Similarly, $\text{ar}(\Delta PCB) = \frac{1}{2}\ \text{ar}(\|\text{gm } GBCH) \qquad \text{...(V)}$

Adding (IV) and (V), we get

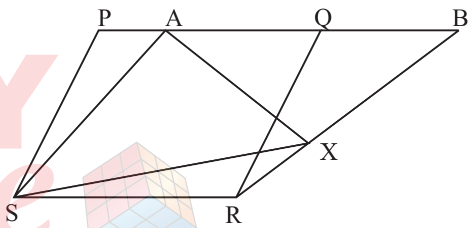
$$\text{ar}(\Delta APD) + \text{ar}(\Delta PCB) = \frac{1}{2}\ [\text{ar}(\|\text{gm } AGHD) + \text{ar}(\|\text{gm } GBCH)]$$

$$= \frac{1}{2}\ \text{ar}(\|\text{gm } ABCD) \qquad \text{... (6)}$$

From (III) and (VI), we get

$$\text{ar}(\Delta APD) + \text{ar}(\Delta PBC) = \text{ar}(\Delta APB) + \text{ar}(\Delta PCD)$$

5. (i) The parallelograms PQRS and ABRS stand on the same base RS and lie between the same parallels SR and PB.



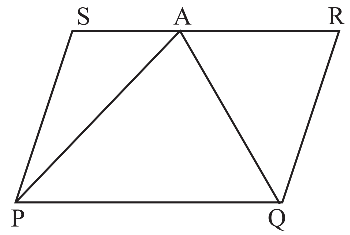
$\therefore$ ar (||gm PQRS) = ar (||gm ABRS)

(ii) The triangle AXS and parallelogram ABRS stand on the same base AS and lie between the same parallels AS and RB.

$$\therefore\ \text{ar}(\Delta AXS) = \frac{1}{2}\ \text{ar}(\|\text{gm } ABRS)$$

$$\therefore\ \text{ar}(\Delta AXS) = \frac{1}{2}\ \text{ar}(\|\text{gm } PQRS) \qquad \text{[Using (i)]}$$

6. Clearly, the field, i.e. the parallelogram PQRS is divided into 3 parts. Each part is of the shape of a triangle.



Since the triangle APQ and the parallelogram PQRS stand on the same base PQ and lie between the same parallels PQ and SR.

$$\therefore\quad \text{ar}(\Delta APQ) = \frac{1}{2}\ \text{ar}(\|\text{gm } PQRS) \qquad \text{... (i)}$$

Clearly,

$$\text{ar}(\Delta APS) + \text{ar}(\Delta AQR) = \text{ar}(\|\text{gm } PQRS) - \text{ar}(\Delta APQ)$$

$$= \text{ar}(\|\text{gm } PQRS) - \frac{1}{2}\ \text{ar}(\|\text{gm } PQRS) \qquad \text{[using (i)]}$$

$$= \frac{1}{2} \text{ ar } (||\text{gm PQRS}) \qquad ...(ii)$$

From (i) and (ii), we get

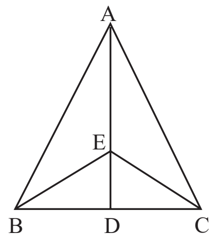
ar (ΔAPS) + ar (ΔAQR) = ar (ΔAPQ)

Thus the farmer should sow wheat and pulses either in [(ΔAPS and ΔAQR) or Δ APQ] or as [ΔAPQ or (Δs APS and AQR)].

Exercise – 2.3

1. **Given:** AD is a median of ΔABC and E is any point on AD.

To prove: ar (ΔABE) = ar (ΔACE)



Proof: Since AD is the median of ΔABC,

ar (ΔABD) = ar (ΔACD)

Also, ED is the median of ΔEBC ...(i)

∴ ar (ΔBED) = ar (ΔCED) ...(ii)

Subtracting (ii) from (i), we get

ar (ΔABD) – ar (ΔBED) = ar (ΔACD) – ar (ΔCED)

∴ ar (ABE) = ar (ACE)

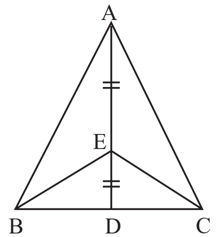
2. **Given:** A ΔABC, E is the midpoint of the median AD.

To prove: ar (ΔBED) = $\frac{1}{4}$ ar (ΔABC)

Proof : Since AD is a median of ΔABC and we know that median divides a triangle into two triangles of equal area.

∴ ar (ABD) = ar (ADC)

∴ ar (ABD) = $\frac{1}{2}$ ar (ABC) ...(i)



In ΔABD, BE is the median,

∴ ar (BED) = ar (BAE) ... (ii)

∴ ar (BED) = $\frac{1}{2}$ ar (ABD)

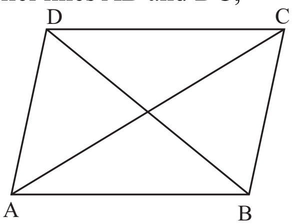
∴ ar (BED) = $\frac{1}{2} \times \frac{1}{2}$ ar (ABC) [using (i)]

∴ ar (BED) = $\frac{1}{4}$ ar (ABC)

3. **Given:** A parallelogram ABCD.

To prove: The diagonals AC and BD divide that the parallelogram ABCD into four triangles of equal area.

Proof: ΔABD and ΔABC lie on the base AB and between same parallel lines AB and DC,



∴ ar(ΔABD) = ar(ΔABC)

Subtract ar(ΔAOB) from both sides, we get

ar(ΔAOD) = ar(ΔBOC)

Similarily we can prove that

ar(ΔCOD) = ar(ΔAOD)

Now, By congurence of Δ(BOC) and Δ(DOC)

ar(ΔBOC) = ar(ΔDOC)

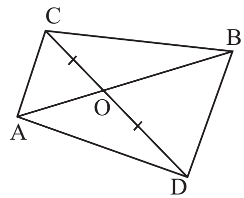
Similarily we can prove that

ar(ΔCOD) = ar(ΔAOD) [by congurence of ΔOCD and ΔOAD]

Thus ar(ΔOAB) = ar(ΔOBC = ar(ΔOCD) = ar(ΔOAD)

4. **Given:** ABC and ABD are two triangles on the same base AB.

A line segment CD is bisected by AB at O. i.e. OC = OD.



To prove: ar (ΔABC) = ar (ΔABD)

Proof: In ΔACD, we have

OC = OD [given]

∴ AO is the median

∴ ar (ΔAOC) = ar (ΔAOD)

[∵ Median divides a triangle in two triangles of equal area]

Similarly, in ΔBCD, BO is the median

ar (ΔBOC) = ar (ΔBOD) ...(ii)

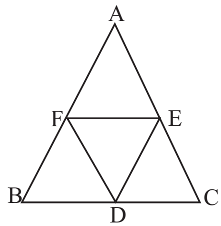
Adding (i) and (ii), we get

ar (ΔAOC) + ar (ΔBOC) = ar (ΔAOD) + ar (ΔBOD)

∴ ar (ΔABC) = ar (ΔABD)

5. (i) In ΔABC, we have

EF || BC [By midpoint theorem, since E and F are the mid points of AC and AB respectively]



EF || BD ...(I)

Also, ED || AB [By midpoint theorem, since E and D are the midpoints of AC and BC respectively]

ED || BF ...(II)

From (I) and (II), BDEF is a parallelogram.

(ii) Similarly, FDCE and AFDE are parallelograms.

ar (ΔFBD) = ar (ΔDEF) [∵ FD is a diagonal of ||gm BDEF]

ar (ΔDEC) = ar (ΔDEF) [∵ ED is a diagonal of ||gm FDCE]

and, ar (ΔAFE) = ar (ΔDEF) [∵ FE is a diagonal of ||gm AFDE]

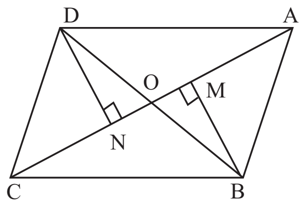
∴ ar (ΔFBD) = ar (ΔDEC) = ar (ΔAFE) = ar (ΔDEF) ...(III)

$\therefore$ ar (ΔDEF) = $\frac{1}{4}$ ar (ΔABC)

(iii) Also, ar (BDEF) = 2ar (ΔDEF)

$= 2 \times \frac{1}{4}$ ar (ABC) = $\frac{1}{2}$ ar (ABC)

6. (i) Draw DN ⊥ AC and BM ⊥ AC.



In Δs DON and BOM,

∠DNO = ∠BMO [each angle = 90º]

∠DON = ∠BOM [vertical opposite angles]

OD = OB [given]

By AAS criterion of congruent

ΔDON ≅ ΔBOM ...(I)

In Δs DCN and BAM,

∠DNC = ∠BMA [each angle = 90º]

DC = BA [given]

DN = BM [∵ ΔDON ≅ ΔBOM ⇒ DN = BM]

$\therefore$ By RHS criterion of congruence,

ΔDCN ≅ ΔBAM ...(II)

From (I) and (II), we get

$\therefore$ ar (ΔDON) + ar (ΔDCN) = ar (ΔBOM) + ar (ΔBAM)

ar (ΔDOC) = ar (ΔAOB)

(ii) Since ar (ΔDOC) = ar (ΔAOB)

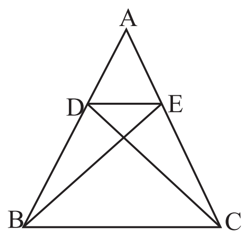
$\therefore$ ar (ΔDOC) + ar (ΔBOC) = ar (ΔAOB) + ar (ΔBOC)

$\therefore$ ar (ΔDCB) = ar (ΔACB)

Since the triangles DCB and ACB have equal areas and have the same base, so these triangles lie between the same parallels.

$\therefore$ DA || CB, i.e. ABCD is a parallelogram.

7. Since the triangles DBC and EBC are equal in area and have the same base BC.



Altitude from D of ΔDBC = Altitude from E of ΔEBC.

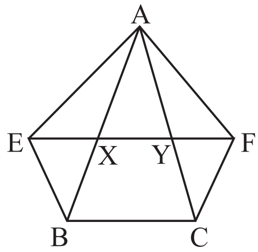
Since the triangles DBC and EBC are in between the same parallels.

$\therefore$ DE || BC

8. Since XY || BC and BE || CY,

$\therefore$ BCYE is a parallelogram.

Since the triangle, ABE and parallelogram BCYE are on the same base BE and in between the same parallels BE and AC.



ar (ΔABE) = ar (||gm BCYE) ...(i)

Now, CF || AB and XY || BC

CF || AB and XF || BC

Hence BCFX is a parallelogram.

Since the triangle ACF and parallelogram BCFX are on the same base CF and in between the same parallels AB and FC,

$\therefore$ ar (ΔACF) = $\frac{1}{2}$ ar (||gm BCFX) ...(ii)

But the parallelogram BCFX and parallelogram BCYE are on the same base BC and in between the same parallels BC and EF,

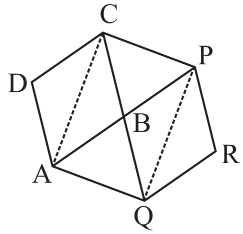
$\therefore$ ar (||gm BCFX) = ar (||gm BCYE) ...(iii)

From (i), (ii) and (iii), we get

ar (ΔABE) = ar (ΔACF)

9. Join AC and PQ.

Since AC and PQ are diagonals of the parallelogram ABCD and the parallelogram BPQR respectively.



$\therefore$ ar (ΔABC) = $\frac{1}{2}$ ar (||gm ABCD) ...(i)

and ar (ΔPBQ) = $\frac{1}{2}$ ar (||gm BPRQ) ...(ii)

Now the triangles ACQ and AQP are on the same base AQ and in between the same parallels AQ and CP.

$\therefore$ ar (ΔACQ) = ar (ΔAQP)

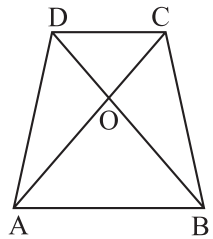
$\therefore$ ar (ΔACQ) – ar (ΔABQ) = ar (ΔAQP) – ar (ΔABQ) [subtracting ar (ABQ) from both sides]

$\therefore$ ar (ΔABC) = ar (ΔBPQ)

$\therefore$ $\frac{1}{2}$ ar (||gm ABCD) = $\frac{1}{2}$ ar (||gm BPRQ) [using (i) and (ii)]

$\therefore$ ar (||gm ABCD) = ar (||gm BPRQ)

10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.



$\therefore$ The triangles ABC and ABD are on the same base and in between the same parallels.

$\therefore$ ar (ΔABD) = ar (ΔABC)

$\therefore$ ar (ΔABD) – ar (ΔAOB) = ar (ΔABC) – ar (ΔAOB)

[subtracting ar (ΔAOB) from both sides]

$\therefore$ ar (ΔAOD) = ar (ΔBOC)

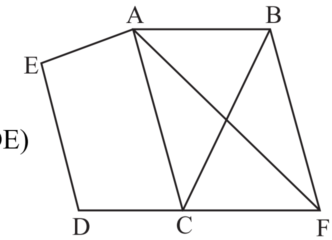
11. (i) Since the triangles ACB and ACF are on the same base AC and in between the same parallels AC and BF,

$\therefore$ ar (ΔACB) = ar (ΔACF)

(ii) Adding ar (||gm ACDE) on both sides, we get

ar (ΔACF) + ar (||gm ACDE) = ar (ΔACB) + ar (||gm ACDE)

$\therefore$ ar (AEDF) = ar (||gm ABCDE)



12. Let ABCD be the quadrilateral plot. Produce BA to meet DE, drawn parallel to CA, at E. Join EC.

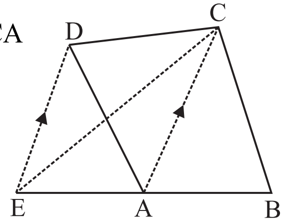
Since the triangles EAC and ADC lie on the same parallels DE and CA

$\therefore$ ar (ΔEAC) = ar (ΔADC)

Now, ar (ΔEAC) + ar (ΔABC) = ar (ΔADC) + ar (ΔABC)

i.e. quad. ABCD = ΔEBC

which is the required explanation to the suggested proposal.



13. ABCD is a trapezium in which AB || DC and XY || AC is drawn. Join XC.

ar (ΔACX) = ar (ΔACY) ...(i)

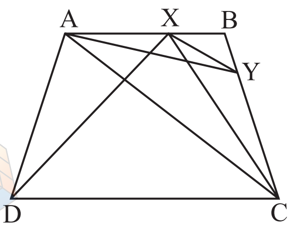
[$\because$ the triangles ACX and ACY have same base AC and are in between the same parallels AC and XY]

But ar (ΔACX) = ar (ΔADX) ...(ii)

[$\because$ the triangles ACX and ADX have same base AX and are in between the same parallels AB and DC]

From (i) and (ii), we have

ar (ΔADX) = ar (ΔACY)



14. From the figure, we have

ar (ΔAQC) = ar (ΔAQB) + ar (ΔBQC)...(i)

and ar (ΔPBR) = ar (ΔPBQ) + ar (ΔQBR) ...(ii)

But ar (ΔAQB) = ar (ΔPBQ) ...(iii)

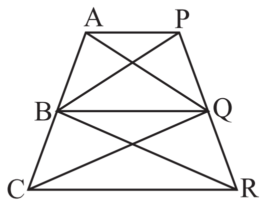
[$\because$ these triangles are on the same base BQ and in between the same parallels AP and BQ]

Also, ar (ΔBQC) = ar (ΔQBR) ...(iv)

[$\because$ These triangles are on the same base BQ and in between the same parallels BQ and CR]

Using (iii) and (iv) in (i) and (ii), we get

ar (ΔAQC) = ar (ΔPBR)

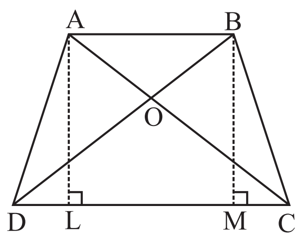


15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that

ar (ΔAOD) = ar (ΔBOC) ...(i)

Adding ar (ΔODC) on both sides, we get

ar (ΔAOD) + ar (ΔODC) = ar (ΔBOC) + ar (ΔODC)



$\therefore$ ar (ΔADC) = ar (ΔBDC)

$\therefore$ $\frac{1}{2} \times DC \times AL = \frac{1}{2} \times DC \times BM$

$\therefore$ AL = BM

$\therefore$ AB || DC

Hence, ABCD is a trapezium.

16. From the figure, we have

ar (ΔBDP) = ar (ΔARC) [given]

and ar (ΔDPC) = ar (ΔDRC) [given]

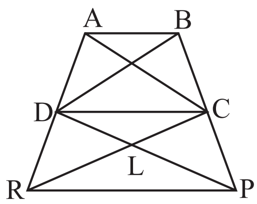
On subtracting, we get

ar (ΔBDP) – ar (ΔDPC) = ar (ΔARC) – ar (ΔDRC)

ar (ΔBDC) = ar (ΔADC)

$\therefore$ DC || AB

Hence, ABCD is a trapezium.



ar (ΔDRC) = ar (ΔDPC) [given]

On subtracting ar (ΔDLC) from both sides, we get

ar (ΔDRC) – ar (ΔDLC) = ar (ΔDPC) – ar (ΔDLC)

$\therefore$ ar (ΔDLR) = ar (ΔCLP)

On adding ar (ΔRLP) to both sides, we get

ar (ΔDLR) + ar (ΔRLP) = ar (ΔCLP) + ar (ΔRLP)

$\therefore$ ar (ΔDRP) = ar (ΔCRP)

$\therefore$ RP || DC

Hence, DCPR is a trapezium.

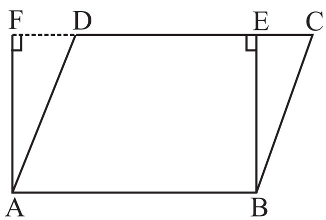
Exercise – 2.4

OPTIONAL

1. Given: A parallelogram ABCD and a rectangle ABEF with same base AB and equal areas.

To prove: Perimeter of a parallelogram ABCD > Perimeter of rectangle ABEF.

Proof: Since opposite sides of a parallelogram and a rectangle are equal.



$\therefore$ AB = DC [$\because$ ABCD is a parallelogram]

and, AB = EF [$\because$ ABEF is rectangle]

DC = EF ...(i)

AB + DC = AB + EF ...(ii)

Since, of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular segment is the shortest.

$\therefore$ BE < BC and AF < AD

$\therefore$ BC > BE and AD > AF

$\therefore$ BC + AD > BE + AF ...(iii)

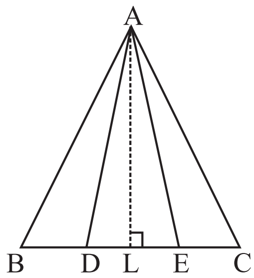
Adding (2) and (3), we get

AB + DC + BC + AD > AB + EF + BE + AF

$\therefore$ AB + BC + CD + DA > AB + BE + EF + FA

Hence, perimeter of a parallelogram ABCD > perimeter of rectangle ABEF.

2. Let AL be perpendicular to BC. So, AL is the height of triangles ABD, ADE and AEC.



$\therefore$ ar (ΔABD) = $\frac{1}{2}$ × BD × AL

ar (ΔADE) = $\frac{1}{2}$ × DE × AL

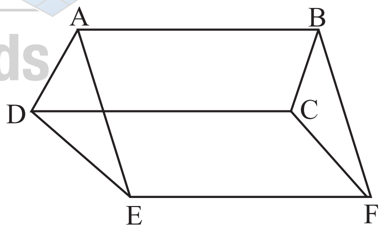
and, ar (ΔAEC) = $\frac{1}{2}$ × EC × AL

Since BD = DE = EC

$\therefore$ ar (ΔABD) = ar (ΔADE) = ar (ΔAEC)

Yes, altitudes of all triangles are the same. Budhia has use the result of this question in dividing her land in three equal parts.

3. Since opposite sides of a parallelogram are equal.



$\therefore$ AD = BC [$\because$ ABCD is a parallelogram]

DE = CF [$\because$ DCFE is a parallelogram]

and AE = BF [$\because$ ABFE is a parallelogram]

Consider the triangles ADE and BCF in which AE = BF,

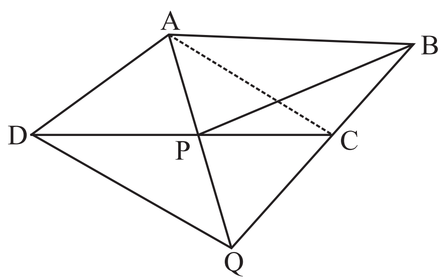
AD = BC and DE = CF

$\therefore$ By SSS criterion of congruence

$\Delta ADE \cong \Delta BCF$

$\therefore$ ar(ΔADE) = ar(ΔBCF)

4. Join AC.



Since the triangles APC and BPC are on the same base PC and in between the same parallels PC and AB.

$\therefore$ ar (ΔAPC) = ar(ΔBPC) ...(i)

Since AD = CQ and AD || CQ [given]

$\therefore$ In the quadrilateral ADQC, one pair of opposite sides is equal and parallel.

$\therefore$ ADQC is a parallelogram.

AP = PQ and CP = DP [$\because$ diagonals of a parallelogram bisect each other]

In Δs APC and DPQ, we have

AP = PQ [proved above]

$\angle APC = \angle DPQ$ [vertically opposite triangles]

and PC = PD [proved above]

By SAS criterion of congruence,

$\Delta APC = \Delta DPQ$

ar (ΔAPC) = ar (ΔDPQ)

ar (ΔBPC) = ar (ΔDPQ) ...(ii) [$\because$ congruent triangles have equal area]

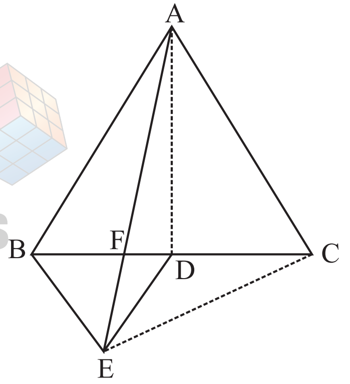
5. Join EC and AD. Let a be the side of ΔABC. Then,

$$\text{ar}(\Delta ABC) = \frac{\sqrt{3}}{4}a^2 = \Delta \quad \text{(say)}$$

(i) $$\text{ar}(\Delta BDE) = \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 \qquad \left[\because BD = \frac{1}{2}BC = \frac{a}{2}\right]$$

$$= \frac{\sqrt{3}}{16}a^2$$

$$= \frac{\Delta}{4}$$



$$\therefore \text{ar}(\Delta BDE) = \frac{1}{4}\text{ar}(\Delta ABC)$$

(ii) We have, $\text{ar}(\Delta BDE) = \frac{1}{2}\text{ar}(\Delta BEC)$ ...(i)

[$\because$ DE is a median of ΔBEC and each median divides a triangle in two other triangles of equal area]

Now, $\angle EBC = 60^\circ$

$\angle BCA = 60^\circ$

$\angle EBC = \angle BCA$

But these are alternate angles with respect to the line segments BE and CA and their transversal BC.

Hence, BE || AC.

Now, triangles BEC and BAE are on the same base BE and lie in between the same parallels BE and AC.

$\therefore$ ar (ΔBEC) = ar (ΔBAE)

$\therefore$ From (i), $\text{ar}(\Delta BDE) = \frac{1}{2}\text{ar}(\Delta BAE)$.

(iii) Since ED is a median of ΔBEC and we know that each median divides a triangle in two other triangles of equal area.

$\therefore$ ar (ΔBDE) = $\frac{1}{2}$ ar (ΔBEC)

From Part (i), ar (ΔBDE) = $\frac{1}{4}$ ar (ΔABC)

Combining these results, we get

$\frac{1}{4}$ ar (ΔABC) = $\frac{1}{2}$ ar (ΔBEC)

ar (ΔABC) = 2 ar (ΔBEC).

(iv) Now, ∠ABD = ∠BDE = 60º (given)

But ∠ABD and ∠BDE are alternate angles with respect to the line segments BA and DE and their transversal BD.

Hence BA || ED.

Now, the triangles BDE and AED are on the same base ED and lie in between the same parallels BA and DE.

ar (ΔBDE) = ar (ΔAED)

ar (ΔBDE) – ar (ΔFED) = ar (ΔAED) – ar (ΔFED)

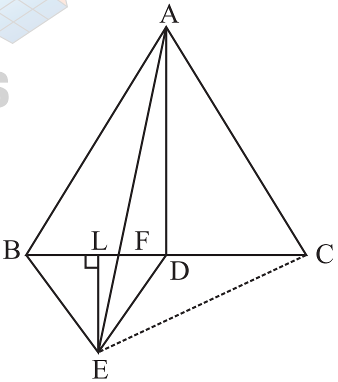
ar (ΔBEF) = ar (ΔAFD)

(v) In ΔABC, $AD^2 = AB^2 - BD^2$

$$= a^2 - \frac{a^2}{4}$$

$$= \frac{3a^2}{4}$$

$$AD = \frac{\sqrt{3}}{2}a$$



In ΔBED, $EL^2 = DE^2 - DL^2 = \left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2$

$$= \frac{a^2}{4} = \frac{a^2}{16} = \frac{3a^2}{16}$$

$$EL = \frac{\sqrt{3}a}{4}$$

$$ar\ (\Delta AFD) = \frac{1}{2} \times FD \times AD = \frac{1}{2} \times FD \times \frac{\sqrt{3}}{2}a \quad ...(I)$$

and ar (ΔEFD) = $\frac{1}{2}$ × FD × EL

$$= \frac{1}{2} \times FD = \frac{\sqrt{3}}{4} a \quad ...(II)$$

From (I) and (II), we have

ar (ΔAFD) = 2 ar (ΔEFD)

Combining this result with Part (iv) we have

ar (ΔBFE) = ar (ΔAFD) = 2 ar (ΔEFD)

(vi) From Part (i) ar (ΔBDE) = $\frac{1}{4}$ ar (ΔABC)

ar (ΔBEF) + ar (ΔFED) = $\frac{1}{4}$ × 2 ar (ΔADC)

2 ar (ΔFED) + ar (ΔFED) = $\frac{1}{2}$ [ar (ΔAFC) – ar (ΔAFD)] [using Part (v)]

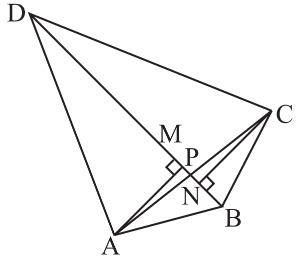
3 ar (ΔFED) = $\frac{1}{2}$ ar (ΔAFC) – $\frac{1}{2}$ × 2ar (ΔFED)

4 ar (ΔFED) = $\frac{1}{2}$ ar (ΔAFC)

ar (ΔFED) = $\frac{1}{8}$ ar (ΔAFC)

6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P.
Draw AM ⊥ BD and CN ⊥ BD.

Now, ar (ΔAPB) × ar (ΔCPD) = $\left(\frac{1}{2} \times BP \times AM\right)\left(\frac{1}{2} \times DP \times CN\right)$



$= \frac{1}{4} \times BP \times DP \times AM \times CN$ ... (i)

and, ar (ΔAPD) × ar (ΔBPC) = $\left(\frac{1}{2} \times BP \times AM\right)\left(\frac{1}{2} \times BP \times CN\right)$

$= \frac{1}{4} \times BP \times DP \times AM \times CN$ ... (ii)

From (i) and (ii), we have

ar (ΔAPB) × ar (ΔCPD) = ar (ΔAPD) × ar (ΔBPC)

7. It is given that P and Q are respectively the midpoints of sides AB and BC respectively of ΔABC and R is the midpoint of AP.

Join AQ and PC.

(i) We have ar (ΔPQR) = $\frac{1}{2}$ ar (ΔAPQ)

[∵ QR is a median of ΔAPQ and it divides the Δ into two other Δs of equal area]

$= \frac{1}{2} \times \frac{1}{2}$ ar (ΔABQ) [$\because$ QP is a median of ΔABQ]

$= \frac{1}{4}$ ar (ΔABQ) $= \frac{1}{4} \times \frac{1}{2}$ ar (ΔABC) [$\because$ AQ is a median of ΔABC]

$= \frac{1}{8}$ ar (ΔABC) ...(i)

Again, ar (ΔARC) $= \frac{1}{2}$ ar (ΔAPC) [$\because$ CR is a median of ΔAPC]

$= \frac{1}{2} \times \frac{1}{2}$ ar (ΔABC) [$\because$ CP is a median of ΔABC]

$= \frac{1}{4}$ ar (ΔABC) ...(ii)

From (i) and (ii), we get

ar (ΔPQR) $= \frac{1}{8}$ ar (ΔABC) $= \frac{1}{2} \times \frac{1}{4}$ ar (ΔABC)

$= \frac{1}{2}$ ar (ARC)

(ii) We have, ar (ΔRQC) = ar (ΔRQA) + ar (ΔAQC) – ar (ΔARC) ...(iii)

Now, ar (ΔRQA) $= \frac{1}{2}$ ar (ΔPQA) [$\because$ RQ is a median of ΔPQA]

$= \frac{1}{2} \times \frac{1}{2}$ ar (ΔAQB) [$\because$ PQ is a median of ΔAQB]

$= \frac{1}{4}$ ar (ΔAQB)

$= \frac{1}{4} \times \frac{1}{2}$ ar (ΔABC) [$\because$ AQ is a median of ΔABC]

$= \frac{1}{8}$ ar (ΔABC) ...(iv)

ar (ΔAQC) $= \frac{1}{2}$ ar (ΔABC) ...(v) [$\because$ AQ is a median of ΔABC]

ar (ΔARC) $= \frac{1}{2}$ ar (ΔAPC) [$\because$ CR is a median of ΔAPC]

$= \frac{1}{2} \times \frac{1}{2}$ ar (ΔABC) [$\because$ CP is a median of ΔABC]

$= \frac{1}{4}$ ar (ΔABC) ...(vi)

From (iii), (iv), (v) and (vi), we have

$$\text{ar}(\Delta RQC) = \frac{1}{8}\,\text{ar}(\Delta ABC) + \frac{1}{2}\,\text{ar}(\Delta APC) - \frac{1}{4}\,\text{ar}(\Delta ABC)$$

$$= \left(\frac{1}{8} + \frac{1}{2} - \frac{1}{4}\right)\text{ar}(\Delta ABC)$$

$$= \frac{1}{2}\,\text{ar}(\Delta ABC).$$

(iii) We have,

$$\text{ar}(\Delta PBQ) = \frac{1}{2}\,\text{ar}(\Delta ABQ) \qquad [\because \text{PQ is a median of } \Delta ABQ]$$

$$= \frac{1}{2}\times\frac{1}{2}\,\text{ar}(\Delta ABC) \qquad [\because \text{AQ is a median of } \Delta ABC]$$

$$= \frac{1}{4}\,\text{ar}(\Delta ABC)$$

$$= \text{ar}(\Delta ARC). \qquad [\text{using (vi)}]$$

8. (i) In triangles MBC and ABD, we have

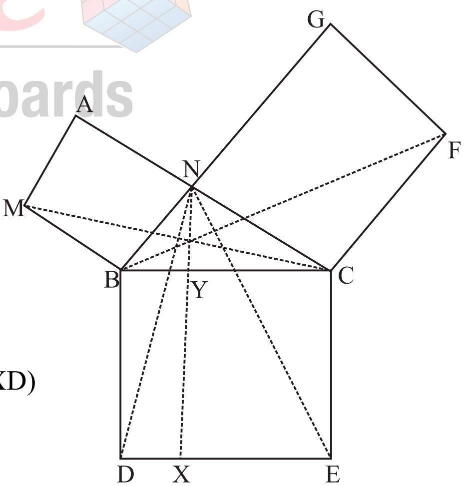
BC = BD [sides of the square BCED]

MB = AB [sides of the square ABMN]

∠MBC = ∠ABD [∵ Each angle = 90º]

∴ By SAS criterion of congruence, we have

ΔMBC ≅ ΔABD



(ii) The triangle ABD and square BYXD have the same base BD and are in between the same parallels BD and AX.

$$\therefore\ \text{ar}(\Delta ABD) = \frac{1}{2}\,\text{ar}(\|\text{gm BYXD})$$

But, ΔMBC ≅ ΔABD [proved in Part (i)]

ar (ΔMBC) = ar (ΔABD)

$$\therefore\ \text{ar}(\Delta MBC) = \text{ar}(\Delta ABD) = \frac{1}{2}\,\text{ar}(\|\text{gm BYXD})$$

ar (||gm BYXD) = 2 ar (ΔMBC)

(iii) The square AMBN and the triangle MBC have the same base MB and are in between the same parallels MB and ANC.

$$\therefore\ \text{ar}(\Delta MBC) = \frac{1}{2}\,\text{ar}(\|\text{gm AMBN})$$

ar (||gm AMBN) = 2ar (ΔMBC)

$= ar\ (||gm\ BYXD)$ [using Part (ii)]

(iv) In Δs ACE and BCF, we have

CE = BC [sides of the square BCED]

AC = CF [sides of the square ACFG]

and, ∠ACE = ∠BCF [∵ each angle = 90º]

∴ By SAS criterion of congruence,

ΔACE = ΔFCB.

(v) The triangle ACE and the square CYXE have the same base CE and are in between the same parallels CE and AYX.

$$\therefore\ ar\ (\Delta ACE) = \frac{1}{2}\ ar\ (||gm\ CYXE)$$

$$ar\ (\Delta FCB) = \frac{1}{2}\ ar\ (||gm\ CYXE) \quad [\because \Delta ACE \cong \Delta FCB, \text{ Part (iv)}]$$

$ar\ (||gm\ CYXE) = 2\ ar\ (\Delta FCB)$.

(vi) Square ACFG and ΔBCF have the same base CF and are in between the same parallels CF and BAG.

$$\therefore\ ar\ (\Delta BCF) = \frac{1}{2}\ ar\ (||gm\ ACFG)$$

$$\frac{1}{2}\ ar\ (||gm\ CYXE) = \frac{1}{2}\ ar\ (||gm\ ACFG) \quad [\text{using Part (v)}]$$

$ar\ (||gm\ CYXE) = ar\ (||gm\ ACFG)$

(vii) From Parts (iii) and (vi), we have

$ar\ (||gm\ BYXD) = ar\ (||gm\ AMBN)$

and, $ar\ (||gm\ CYXE) = ar\ (||gm\ ACFG)$

On adding we get

$ar\ (||gm\ BYXD) + ar\ (||gm\ CYXE) = ar\ (||gm\ AMBN) + ar\ (||gm\ ACFG)$

$ar\ (||gm\ BCED) = ar\ (||gm\ AMBN) + ar\ (||gm\ ACFG)$.

TEST YOURSELF – AR 1

i. base AB, parallels AB and DC.

ii. base PS, parallels PS and QR.

iii. base AB, parallels AB and DC.

TEST YOURSELF – AR 2

1. 5 cm.