1. A steel wire of length 4.7 m and cross-section 3×10^{-5} m² stretches by the same amount as a copper wire of length 3.5 m and cross-section 4×10^{-5} m² under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Ans. For steel wire, $A_1 = 3 \times 10^{-5}$ m², $\ell_1 = 4.7$ m; For copper wire, $A_2 = 4 \times 10^{-5}$ m², $\ell_2 = 3.5$ m

As $\Delta\ell_1 = \Delta\ell$ and $F_1 = F_2 = F$

$$\therefore\ Y_1 = \frac{F_1\ell_1}{A_1\Delta\ell_1} = \frac{F}{3\times10^{-5}} \times \frac{4.7}{\Delta\ell}; \qquad Y_2 = \frac{F_2\ell_2}{A_2\Delta\ell_2} = \frac{F\times3.5}{4\times10^{-5}\Delta\ell}$$

$$\therefore\ \frac{Y_1}{Y_2} = \frac{4.7\times4\times10^{-5}}{3.5\times3\times10^{-5}} = 1.8$$

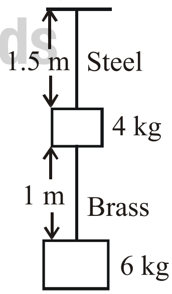
2. Calculate the elongation of the steel and brass wire in the adjacent figure. Unloaded length of steel wire is 1.5 m and of brass wire is 1m, diameter of each wire = 0.25 cm. Young's modulus of steel is 2×10^{11} Pa and that of brass is 0.91×10^{11} Pa.

Ans. For steel wire: Total force = $F_1 = 4 + 6 = 10$ kgf $= 10 \times 9.8$ N

$\ell_1 = 1.5$ m, $r_1 = \dfrac{0.25}{2}$ cm $= 0.125 \times 10^{-2}$ m; $Y_1 = 2 \times 10^{11}$ Pa, $\Delta\ell_1 = ?$

For brass wire, $F_2 = 6$ kg f $= 6 \times 9.8$ N, $r_2 = 0.125 \times 10^{-2}$ m

$Y_2 = 0.91 \times 10^{11}$ Pa, $\ell_2 = 1$ m



$$\because\quad Y = \frac{F\ell}{A\Delta\ell} \qquad \therefore \Delta\ell = \frac{F\ell}{AY} = \frac{F\ell}{\pi r^2 Y}$$

$$\frac{F_1\ell_1}{\pi r_1^2 Y_1} = \frac{10\times9.8\times1.5\times7}{22\times(0.125\times10^{-2})^2\times2\times10^{11}} = 1.49 \times 10–4 \text{ m}$$

$$\Delta\ell_2 = \frac{F_2\ell_2}{\pi r_2^2 Y_2} = \frac{6\times9.8\times1\times7}{22\times(0.125\times10^{-2})^2\times0.91\times10^{11}}$$

$= 1.3 \times 10^{-4}$ m

3. The edges of aluminium cube are 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face? (1 Pa = 1 N/m²) (g = 10 m/s²)

Ans. $A = 0.1 \times 0.1 = 10^{-2}$ m², $F = mg = 100 \times 10$ N

$$\text{Shearing strain} = \frac{\Delta L}{L} = \frac{\text{Shearing stress}}{\text{Shear modulus}}$$

$$\therefore \frac{\Delta L}{L} = \frac{F/A}{\eta} \Rightarrow \Delta L = \frac{FL}{A\eta} = \frac{100\times10\times0.1}{10^{-2}\times25\times10^{9}} \Rightarrow \Delta L = 4\times10^{-7}\text{ m}$$

4. Four identical hollow cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Assume the load distribution to be uniform, calculate the compressional strain in each column. The Young's modulus of steel is 2 × 10¹¹ Pa.

Ans. Load on each column = $F = \frac{mg}{4} = \frac{50,000\times9.8}{4}\text{N} = 12500\text{ N}$

$$A = \pi(r_2^2 - r_1^2) = \frac{22}{7}[(0.40)^2 - (0.30)^2] = 0.22\text{ m}^2$$

$$\text{Compressional strain} = \frac{\text{Stress}}{Y} = \frac{F/A}{Y} = \frac{F}{AY}$$

$$\frac{12500\times9.8}{4\times\frac{22}{7}\times[(0.40)^2-(0.30)^2]\times2\times10^{11}} = 2.8\times10^{-6}$$

5. A piece of copper having a rectangular cross section 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force producing only elastic deformation. Calculate the resulting strain. Given: $Y_c = 1.2\times10^{11}$ N/m².

Ans. $A = 15.2\times19.2\text{ mm}^2 = 15.2\times19.2\times10^{-6}\text{ m}^2; F = 44,500\text{ N}$

$$\text{Strain} = \frac{F}{AY} = \frac{44500}{15.2\times19.1\times10^{-6}\times1.2\times10^{11}} = 0.001277$$

6. A steel cable with a radius of 1.5 cm supports a chair lift at a sky area. If the maximum stress is not to exceed 108 N/m², what is the maximum load the cable can support?

Ans. Maximum load = Maximum Stress × Area of cross-section

$$= 10^8\times\frac{22}{7}(1.5\times10^{-2})^2$$

$$= 7.07\times10^4\text{ N}$$

7. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. The wires at each end are of copper and the middle one is of iron. Determine the ratio of their diameters if each has the same tension. Young's modulus of elasticity for copper and iron are 110 × 10⁹ N/m² and 190 × 10⁹ N/m², respectively.

Ans. Each wire has same tension, so each wire will have same extension. Also as they have same length, each wire will have same strain

$$Y = \frac{F\ell}{A\Delta\ell} = \frac{F\ell}{\pi(D/2)^2\Delta\ell} = \frac{4F\ell}{\pi D^2 \Delta\ell} \qquad \therefore\ D^2 \propto \frac{1}{Y}$$

$$\therefore\ \frac{D_{Cu}}{D_{Iron}} = \sqrt{\frac{Y_{Iron}}{Y_{Cu}}} = \sqrt{\frac{190\times10^9}{110\times10^9}} = \sqrt{\frac{19}{11}} = 1.31$$

8. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm². Calculate the elongation of the wire when the mass is at the lowest point of its path.

$Y_{steel} = 2 \times 10^{11}$ N/m²

Ans. $m = 14.5$ kg, $\ell = 1$ m, $\omega = 2$ rps $= 2 \times 2\pi$ rad/s $\quad A = 0.065 \times 10^{-4}$ m²

Total force on the ring at its lowest position on the vertical circle $= F = mg + mr\omega^2$

$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times \left(\frac{22}{7}\right)^2 \times 4 = 142.1 + 2291.6 = 2433.7\text{ N}$$

$$Y = \frac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \frac{F\ell}{AY} = \frac{2433.7\times1}{0.065\times10^{-4}\times2\times10^{11}} = 1.87 \times 10^{-3}\text{ m} = 1.87\text{ mm}$$

9. Compute the bulk modulus of water from the following data: Initial volume = 100 litre, pressure increase = 100 atmosphere, final volume = 100.5 litre (1 atmosphere = 1.013 × 10⁵ Pa)

Ans. $\Delta V = 100.5 - 100 = 0.5$ litre $= 0.5 \times 10^{-3}$ m³; $P = 100$ atm $= 100 \times 1.013 \times 10^5$ Pa

$V = 100$ litre $= 100 \times 10^{-3}$ m³

$$\text{Bulk modulus} = K = \frac{P}{\Delta V / V} = \frac{PV}{\Delta V} = \frac{100\times1.013\times10^5\times100\times10^{-3}}{0.5\times10^{-3}}$$

$\Rightarrow K = 2.026 \times 10^9$ Pa

10. What is the density of ocean water at a depth where the pressure is 80 atm, given that its density at the surface is 1.03 × 10³ kg/m³? Compressibility of water = 45.8 × 10⁻¹¹/Pa. Given: 1 atm = 1.013 × 10⁵ Pa.

Ans. $P = 80$ atm $= 80 \times 1.013 \times 10^5$ Pa

Compressibility = $\frac{1}{K}$ = 45.8 × 10^{-11} /pa; Density of water at the surface

= ρ = 1.03 × 10^3 kg/m^3

Let ρ′ = density of water at a given depth; V' = Vol. of water of mass M at given depth

V = Vol. of same mass of water at the surface

$$\therefore\ V = \frac{M}{\rho} \text{ and } V' = \frac{M}{\rho'}$$

Change in Vol. = $\Delta V = V - V' = M\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)$

$$\text{Vol. strain} = \frac{\Delta V}{V} = \frac{M\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)}{\frac{M}{\rho}} = 1 - \frac{\rho}{\rho'} \Rightarrow \frac{\Delta V}{V} = 1 - \frac{1.03\times10^3}{\rho'}$$

Also $K = \frac{P}{\Delta V / V}$ $\quad \therefore \frac{\Delta V}{V} = \frac{P}{K}$ = 80 × 1.013 × 10^5 × 45.8 × 10^{-11}

= 3.712 × 10^{-3}

$$\therefore\ 1 - \frac{1.03\times10^3}{\rho'} = 3.712\times10^{-3} \Rightarrow \rho' = \frac{1.03\times10^3}{1 - 3.712\times10^{-3}}$$

= 1.034 × 10^3 kg/m^3

11. Compute the fractional change in volume of a glass slab when subjected to a hydraulic pressure of 10 atmosphere. Bulk modulus of elasticity of glass = 37 × 10^9 N/m^2 and 1 atm = 1.013 × 10^5 Pa.

Ans. P = 10 atm = 10 × 1.013 × 10^5 Pa, K = 37 × 10^9 N/m^2

$$\text{Vol. strain} = \frac{\text{Bulk modulus}}{\text{Stress}} = \frac{p}{K} = \frac{10\times1.013\times10^5}{37\times10^9} = 2.74 \times 10^{-5}$$

$\therefore$ Fractional change in volume = $\frac{\Delta V}{V}$ = 2.74 × 10^{-5}

12. Determine the volume contraction of a solid copper cube, 10 cm on an edge when subjected to a hydraulic pressure of 7 × 10^6 Pa. Bulk modulus of copper = 140 GPa

Ans. L = 10 cm = 0.1 m; P = 7 × 10^6 Pa; K = 140 GPa = 140 × 10^9 Pa

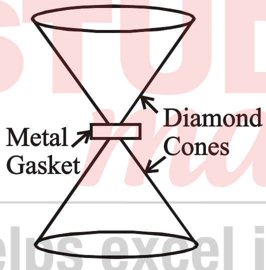
$$K = \frac{P}{\Delta V / V} = \frac{PV}{\Delta V} = \frac{PL^3}{\Delta V} \Rightarrow \Delta V = \frac{PL^3}{K} = \frac{7\times10^6\times(0.1)^3}{140\times10^9} = 5 \times 10^{-8} \text{ m}^3$$

13. How much should the pressure on a litre of water be changed to compress it by 0.1%? (Bulk modulus of elasticity of water = 2.2 × 10⁹ N/m²).

Ans. V = 1 litre = 10^{-3} m³. $\dfrac{\Delta V}{V} = \dfrac{0.1}{100} = 10^{-3}$

$$K = \frac{PV}{\Delta V} \Rightarrow P = K\frac{\Delta V}{V} = 2.2 \times 10^9 \times 10^{-3} = 2.2 \times 10^6 \text{ N/m}^2$$

14. Anvils made of single crystal of diamond, with the shape as shown in the adjacent figure, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



Ans. $D = 0.5$ mm $= 0.5 \times 10^{-3}$ m, $F = 50{,}000$ N

$$P = \frac{F}{\pi r^2} = \frac{F}{\pi D^2/4} = \frac{4\times50{,}000\times7}{22\times(5\times10^{-4})^2} = 2.5 \times 10^{11} \text{ Pa}$$

15. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9 × 10⁷ Pa? Assume that each rivet is to carry one quarter of the load.

Ans. $r = \dfrac{6}{2} = 3$ mm $= 3 \times 10^{-3}$ m; Max. stress $= 6.9 \times 10^7$ Pa

Max. load on a rivet = Max. stress × area of cross-section

$= 6.9 \times 10^7 \times \dfrac{22}{7} \times (3 \times 10^{-3})^2$

$\therefore$ Max. tension $= 4 \times$ Max. load $= 4 \times 6.9 \times 10^7 \times \dfrac{22}{7} \times 9 \times 10^{-6}$

$= 7.8 \times 10^3$ N

16. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in the adjacent figure. The cross-sectional area of wires A and B are 1 mm² and 2 mm² respectively. At what point along the rod should a mass *m* be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.

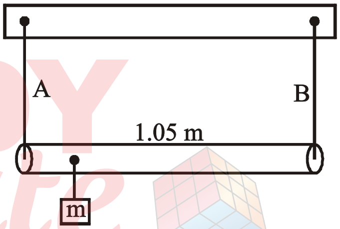
Given $Y_{steel} = 2 \times 10^{11}$ n/m² and $Y_{Aluminium} = 7 \times 10^{10}$ N/m².

Ans. For steel wire A, $A_1 = 1$ mm² $= 1 \times 10^{-6}$ m², $Y_1 = 2 \times 10^{11}$ N/m²

For aluminium wire B, $A_2 = 2$mm² $= 2 \times 10^{-6}$ m², $Y_2 = 7 \times 10^{10}$ N/m²

$\ell_1 = \ell_2 =$ (for both A and B)

(a) Let the mass m be suspended at a distance x from A. Let F_1 and F_2 are tensions in A and B that produce equal stress in both.



$$\therefore \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1\times10^{-6}}{2\times10^{-6}} = \frac{1}{2}$$

For equilibrium of the rod, the moments of forces about the point of suspension of the mass should be equal.

$$F_1 x = F_2 (1.05 - x) \quad \frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{1}{2} \Rightarrow 2.1 - 2x = x \Rightarrow x = 0.7 \text{ m}$$

(b) For equal strain in both the wires,

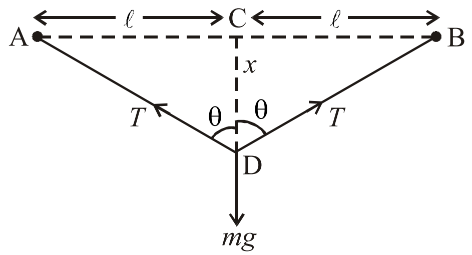
$$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{1}{2} \times \frac{2\times10^{11}}{7\times10^{10}} = \frac{10}{7}$$

Equating the moments again,

$$F_1 x = F_2 (1.05 - x) \Rightarrow \frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{10}{7}$$

$$\Rightarrow 10x = 7.35 - 7x \Rightarrow x = 0.4324 \text{ m}$$

17. A mild wire of steel of length 1 m and cross-sectional area 0.5×10^{-2} cm² is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid point of the wire. Calculate the depression of the mid-point. $g = 10$ m/s², $Y = 2 \times 10^{11}$ N/m²



Ans. Let x be the depression of the mid–point

$\therefore$ From the figure, AC = CB = ℓ = 0.5 m; m = 100 g = 0.1 kg

CD = x, AD = BD = $\sqrt{\ell^2 + x^2}$

Increase in length, $\Delta\ell$ = AD + DB – AB = 2AD – AB

$$\Delta\ell = \sqrt{\ell^2 + x^2} - 2\ell = 2\ell\left(1 + \frac{x^2}{\ell^2}\right)^{1/2} - 2\ell = 2\ell\left(1 + \frac{x^2}{2\ell^2}\right) - 2\ell$$

(expending binomially and neglecting higher powers of as x^2/ℓ^2 as $x << \ell$)

$$\Rightarrow \Delta\ell = 2\ell + \frac{x^2}{2\ell^2} \times 2\ell - 2\ell = \frac{x^2}{\ell}$$

$$\text{Strain} = \frac{\Delta\ell}{2\ell} = \frac{x^2/\ell}{2\ell} = \frac{x^2}{2\ell^2}$$

Let T be the tension in the string

$$\therefore \quad 2\,T\cos\theta = mg \Rightarrow T = \frac{mg}{2\cos\theta}$$

$$\cos\theta = \frac{x}{\sqrt{\ell^2 + x^2}} = \frac{x}{\ell\left(1 + \frac{x^2}{\ell^2}\right)^{1/2}} = \frac{x}{\ell\left(1 + \frac{x^2}{2\ell^2}\right)}$$

As $x << \ell$, $1 + \frac{x^2}{2\ell^2} \approx 1$ $\qquad \therefore \cos\theta = \frac{x}{\ell}$

$$\therefore T = \frac{mg}{2(x/\ell)} = \frac{mg\ell}{2x} \text{ and Stress} = \frac{T}{A} = \frac{mg\ell}{2Ax}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{mg\ell}{2Ax} = \frac{2\ell^2}{x^2} = \frac{mg\ell^3}{Ax^3}$$

$$\therefore \quad x = \ell\left[\frac{mg}{YA}\right]^{1/3} = 0.5\left[\frac{0.1 \times 10}{2 \times 10^{11} \times 0.5 \times 10^{-6}}\right]^{1/3} = 1.074 \times 10^{-2} \text{ m}$$

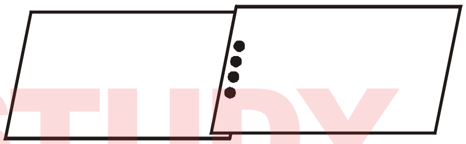
18. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 2.3 × 10^9 Pa? Assume that each rivet is to carry one quarter of the load.

Sol. Let the tension exerted by riveted strip = F

This tension would provide shearing force on the four rivets, which share it equally.

$\therefore$ Shearing force on each rivet = $\frac{F}{4}$

and shearing stress on each rivet = $\frac{F/4}{A} = \frac{F}{4A}$



As the maximum shearing stress on each rivet is given to be 2.3 × 10^9 Pa, so we have

$$\frac{F_{max}}{4A} = 2.3 \times 10^9$$

or $F_{max} = 4A \times 2.3 \times 10^9 = 4 \times \pi r^2 \times 2.3 \times 10^9$

$= 4 \times \frac{22}{7} \times (3.0 \times 10^{-3})^2 \times 2.3 \times 10^9$

$= 260.2 \times 10^3$ N = 260 kN