Hints/Solutions to Studymate Practice Boards Paper
Mathematics (Class - X)

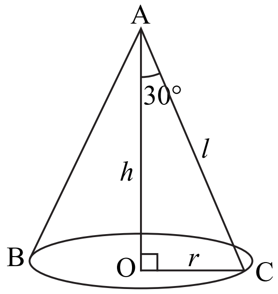
Code No. 45/1

1. In ΔAOC;

$$\sin\theta = \frac{OC}{AC}$$

$$\sin 30° = \frac{r}{l}$$

$$\frac{1}{2} = \frac{r}{l}$$



$\therefore$ Ratio is 1 : 2 **[1]**

2. Using Empirical relationship

3 Median – 2 Mean = Mode

$3 \times 27.9 - 2 \times 26.8 = \text{Mode}$

$83.7 - 53.6 = \text{Mode}$

$30.1 = \text{Mode}$ **[1]**

3. HCF $(a, b) = 2^3 . 3^2 = 8 \times 9 = 72$ **[1]**

4. Probability of getting no head = $\frac{1}{2^{20}}$

Probability of getting at least 1 head = $1 - \frac{1}{2^{20}}$ **[1]**

5. Product of zeroes = 4

$\therefore \quad \frac{-6}{a} = 4$

$\therefore \quad a = \frac{-3}{2}.$ **[1]**

6. $\cos\theta = \tan 40° \tan 50°$

$= \tan 40° \cot 40°$ $\quad [\because \tan\theta = \cot(90 - \theta)]$

$\cos\theta = 1$

$\therefore \quad \theta = 0°$ **[1]**

7. $A = (3, 4)$, $B (7, 7)$

Let $C = (x, y)$

$\therefore \quad AB = \sqrt{(7-3)^2 + (7-4)^2} = 5$ **[½]**

Since A, B and C are collinear,

and $AB = 5$ and $AC = 10$

$\therefore$ B is the mid-point of AC. **[½]**

$\therefore \quad \frac{3+x}{2} = 7$ and $\frac{3+y}{2} = 7$ **[½]**

$\therefore \quad x = 11$ and $y = 10$

$\therefore$ Coordinates of $C = (11, 10)$. **[½]**

8. $abx^2 + b^2x - acx - bc$

$bx(ax + b) - c(ax + b)$

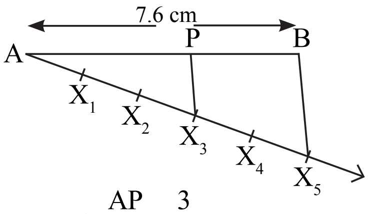
$(ax + b)(bx - c) = 0$ **[½]**

$\Rightarrow x = \frac{-b}{a}$ or $x = \frac{c}{b}$ **[½]**

$\therefore \alpha + \beta = \frac{ac - b^2}{ab} = \left[\frac{-b}{a}\right]$ **[½]**

$\& \ \alpha\beta = \frac{-bc}{ab} \Rightarrow \frac{-c}{a}$ **[½]**

9.



$\therefore \frac{AP}{BP} = \frac{3}{2}$ **[2]**

10. Let one root = α; then second root = α^2

$p(x) = x^2 - 2x - k = 0$

Let one root = α

Second root = α^2

$\alpha + \alpha^2 = 2$

$\alpha^2 + \alpha - 2 = 0$ **[½]**

$\alpha^2 + 2\alpha - \alpha - 2 = 0$

$\alpha[\alpha + 2] - 1[\alpha + 2] = 0$

$[\alpha - 1][\alpha + 2] = 0$

$\alpha = 1 \ \& \ \alpha = -2$ **[½]**

Put $x = 1$ in $p(x)$

$1 - 2 - k = 0$

$-k = 1$ or $k = -1$ **[½]**

Put $x = -2$ in $p(x)$

$4 + 4 - k = 0$

$k = 8$

$\therefore$ value of k is –1 and 8. **[½]**

11. AP is –12, –9, –6, ..., 21

$21 = -12 + (n - 1)3$

$\frac{33}{3} = n - 1 \Rightarrow 12 = n$ **[½]**

$\Rightarrow$ If 1 is added to each term; then A.P is

–11, –8, –5, ..., 22

$22 = -11 + (n - 1)(3)$

$\frac{33}{3} = n - 1 \Rightarrow 11 + 1 = n$

$12 = n$

$S_{12} = \frac{12}{2}[-22 + 11 \times 3]$

$= \frac{12}{2} \times 11 = 66$ **[½]**

12. Diagonal of cube will be the diameter of the sphere.

$\therefore$ Length of diagonal of cube = $12\sqrt{3}$ **[½]**

If edge of cube is a **[½]**

Hence, $\sqrt{3}a = 12\sqrt{3}$ **[½]**

$\Rightarrow$ $a = 12$ **[½]**

Surface area = $6a^2 = 864 \text{ cm}^2$

13. $99x + 101y = 1499$ ...(1)

$101x + 99y = 1501$ ...(2)

Adding (1) & (2)

$200x + 200y = 3000$

or $x + y = 15$ ...(3) **[½]**

Subtracting (2) from (1);

$-2x + 2y = -2$

or $-x + y = -1$ ...(4) [½]

Adding equations (3) & (4);

$2y = 14$ or $y = 7$ **[½]**

Put $y = 7$ in equation (3); $x = 8$ **[½]**

14. For equal roots;

D = 0

$[-2(a^2 - bc)]^2 - 4[(c^2 - ab)(b^2 - ac)] = 0$ **[½]**

$4[a^2 - bc]^2 - 4[c^2b^2 - ac^3 - ab^3 + a^2bc] = 0$ **[½]**

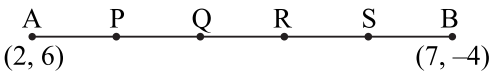
$4[a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc] = 0$ **[½]**

$\Rightarrow$ $[a^4 - 2a^2bc + ac^3 + ab^3 - a^2bc] = 0$ **[½]**

$a[a^3 + b^3 + c^3 - 3abc] = 0$ **[½]**

$\therefore$ $a = 0$ or $a^3 + b^3 + c^3 = 3abc$ **[½]**

15.



AP = PQ = QR = RS = SB = x

$\therefore \frac{AP}{PB} = \frac{x}{4x} = \frac{1}{4}$ **[½]**

Coordinates of P $= \left[\frac{1 \times 7 + 4 \times 2}{1 + 4}, \frac{1 \times -4 + 4 \times 6}{1 + 4}\right]$ **[½]**

$= \left[\frac{15}{5}, \frac{20}{5}\right]$

Coordinates of P = [3, 4] **[½]**

Coordinates of R $= \left[\frac{3 \times 7 + 2 \times 2}{3 + 2}, \frac{3 \times -4 + 2 \times 6}{3 + 2}\right]$ **[½]**

$= \left[\frac{25}{5}, \frac{0}{5}\right]$ **[½]**

= [5, 0] **[½]**

16. $\frac{1}{\cos\theta} + \cos\theta = 2$

$1 + \cos^2\theta = 2\cos\theta$ **[½]**

$\cos^2\theta - 2\cos\theta + 1 = 0$ **[½]**

$(\cos\theta - 1)^2 = 0$ **[½]**

$\therefore$ $\cos\theta = 1$ **[½]**

$\theta = 0°$. **[½]**

$\therefore$ $\sec^5\theta + \cos^5\theta = \cos^5\theta\ (0°) + \cos^5\theta\ (0°)$ **[½]**

$1 + 1 = 2$.

OR

Given,

$$\sec\theta - \tan\theta = \sqrt{2}\tan\theta$$

$\Rightarrow$ $\sec\theta = \sqrt{2}\tan\theta + \tan\theta$ **[½]**

$\Rightarrow$ $\sec\theta = (\sqrt{2}+1)\tan\theta$ **[½]**

$\Rightarrow$ $\frac{1}{(\sqrt{2}+1)}\sec\theta = \tan\theta$ **[½]**

$\Rightarrow$ $\tan\theta = \frac{1}{(\sqrt{2}+1)}\frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}\sec\theta$

$\Rightarrow$ $\tan\theta = (\sqrt{2}-1)\sec\theta$ **[½]**

Consider $\sec\theta + \tan\theta = \sec\theta + (\sqrt{2}-1)\sec\theta$ **[½]**

$= \sec\theta(1+\sqrt{2}-1)$

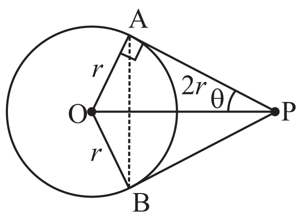
$= \sqrt{2}\sec\theta$ **[½]**

17. Given length of OP = diameter

$= 2r$

OA $\perp$ AP (Radius drawn in $\perp$ to the tangent at point of contact)

$\therefore$ In rt. $\angle$'d ΔOAP



$\sin\theta = \frac{OA}{OP} \Rightarrow \sin\theta = \frac{r}{2r}$ **[½]**

$\Rightarrow$ $\sin\theta = \frac{1}{2} \Rightarrow \sin\theta = \sin 30°$ **[½]**

$\Rightarrow$ $\theta = 30°$

$\because$ $\Delta OAP \cong \Delta OBP$

$\therefore$ $\angle OPB = 30°$

$\Rightarrow$ $\angle APB = 60°$ ...(1) **[1]**

As AP = PB (Length of tangents from an outside point)

$\Rightarrow$ $\angle ABP = \angle PAB$ ...(2)

(Isosceles triangle theorem)

$\therefore$ $\angle ABP + \angle PAB + \angle APB = 180°$

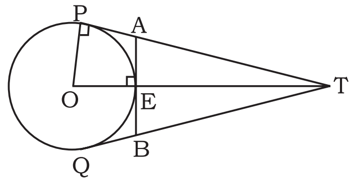
$\Rightarrow$ $\angle ABP + \angle ABP + 60° = 180°$ [from (1) and (2)] **[1]**

$\Rightarrow$ $\angle ABP = 60°$

$\therefore$ ΔAPB is an equilateral Δ. **[1]**

OR

In ΔOPT; PT = 12cm [By Pythagoras theorem]



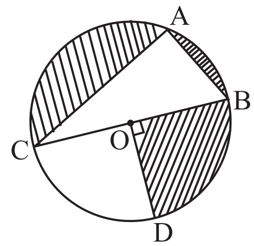
Let AP = AE = x

$\therefore$ ΔAET; $(12 - x)^2 = (x)^2 + (8)^2$

$\therefore$ $x = \frac{10}{3}$

$AB = \frac{20}{3}$ cm.

18. Area of shaded region



= area of semicircle – ar (ΔABC) + $\frac{1}{4}$ area of circle.

$= \pi r^2 - \frac{1}{2} AC \times AB + \frac{1}{4} \pi r^2$ **[½]**

Here, $r = \frac{AB}{2}$

and $AB = \sqrt{AC^2 + AB^2} = \sqrt{24^2 + 7^2} = 25$ **[½]**

$r = \frac{AB}{2} = \frac{AB}{2} = 12.5$ **[½]**

$= \frac{\pi r^2}{2} + \frac{\pi r^2}{4} - \frac{1}{2} B \times H$ **[½]**

$= \pi r^2 \left(\frac{1}{2} + \frac{1}{4}\right) - \frac{1}{2} \times 24 \times 7$ **[½]**

$= \frac{1.57 \times 25 \times 28 \times 3}{2 \times 4} - 84$ **[½]**

$= 351.45 - 84 = 267.45$ cm^2

19. Circumference of the circular field = 360 km

Distances covered by the three cyclists per day = 60 km, 72 km, 90 km

Number of days taken by the three cyclists to cover the complete field

$= \frac{360}{60}, \frac{360}{70}, \frac{360}{90} = 6$ days, 5 days, 4 days **[1]**

Three cyclists will meet again at the starting point at the lowest common multiple of 6, 5 and 4 days **[1]**

i.e., after 60 days. **[1]**

20. Let Monday be day 1, Tuesday be day 2 and so on.

Therefore, we have total of 36 possible outcomes

(i) P(visit the shop on same day) $= \frac{6}{36} = \frac{1}{6}$ **[½]**

(ii) P(visit the shop on different days) $= \frac{30}{36} = \frac{5}{6}$ **[½]**

(iii) P(visit the shop on consecutive days) $= \frac{10}{36} = \frac{5}{18}$ **[½]**

21. LHS: $S_{12} = 6[2a + (11)d]$

RHS: $3(S_8 - S_4) = 3\{4[2a + 7d] - 2[2a + 3d]\}$

= 3[8a + 28d – 4a – 6d] **[1]**

= 3[4a + 22d] **[1]**

⇒ 6[2a + 11d] **[1]**

∴ LHS = RHS.

OR

RHS = $3[S_2 - S_1]$

$$3\left\{\frac{2n}{2}[2a+(2n-1)d]-\frac{n}{2}[2a+n-1]d\right\}$$

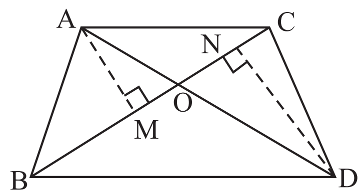
$$\Rightarrow \frac{3n}{2}[4a+4nd-2d-2a-nd+d]$$

$$=\frac{3n}{2}[2a+3nd-d]$$

$$=\frac{3n}{2}[2a+(3n-1)d]$$

= LHS.

22. **Construction:** Draw AM ⊥ BC and DN ⊥ BC.



Proof: In ΔAOM and ΔDON, we have

∠AOM = ∠DON ...[Vertical opp. ∠s]

∠AMO = ∠DNO ...[Each = 90°]

∴ ΔAOM ~ ΔDON ...[AA similarity] **[½]**

$\Rightarrow \frac{AM}{DN}=\frac{AO}{DO}$ ...[Corresponding sides of similar triangles] **[½]**

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{\frac{1}{2}\times BC\times AM}{\frac{1}{2}\times BC\times DN}=\frac{AM}{DN}$ **[½]**

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$ ...$\left[\because \frac{AM}{DN}=\frac{AO}{DO}\right]$ **[1]**

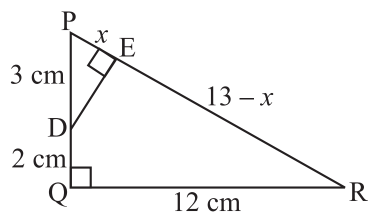
OR

In right ΔPQR,

$PR^2 = PQ^2 + QR^2$

= 25 + 144 = 169

∴ PR = 13 cm **[½]**



Let PE = x, then ER = 13 – x

In ΔPQR and ΔPED, we have

∠P = ∠P ...[Common]

∠PQR = ∠PED ...[Each = 90°] **[½]**

$\therefore$ $\Delta PQR \sim \Delta PED$ ...[AA similarity]

$\therefore$ $\frac{PQ}{PE} = \frac{QR}{ED} = \frac{PR}{PD}$

$\frac{5}{x} = \frac{12}{ED} = \frac{13}{3}$ **[1]**

$x = \frac{5 \times 3}{13} = \frac{15}{13} = 1\frac{2}{13}$ cm and $ED = \frac{13 \times 3}{13} = \frac{36}{13} = 2\frac{10}{13}$ cm **[1]**

23. Area of Δ

$= \frac{1}{2}[(p + 1)(3 - 2p) + (2p + 1)(2p - 1) + (2p + 2)(1 - 3)] = 0$ **[1]**

$= [(p + 1)(3 - 2p) + (2p + 1)(2p - 1) + (2p + 2)(-2)] = 0$

$= [3p - 2p^2 + 3 - 2p + 4p^2 - 1 - 4p - 4] = 0$ **[1]**

$= [2p^2 - 3p - 2] = 0$

$= [2p^2 - 4p + p - 2] = 0$ **[1]**

$= 2p(p - 2) + 1(p - 2) = 0$

$= (p - 2)(2p + 1) = 0$

$p = 2$ & $p = -\frac{1}{2}$ **[1]**

OR

Let $O = (0, 0)$ and $A = (0, 2\sqrt{3})$.

Let the third vertex of the triangle be $B(x, y)$

Now, $OB = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$ **[½]**

$OA = \sqrt{(0-0)^2 + (0-2\sqrt{3})^2} = \sqrt{12}$ **[½]**

and $AB = \sqrt{(x-0)^2 + (y-2\sqrt{3})^2} = \sqrt{x^2 + y^2 - 4\sqrt{3}y + 12}$ **[½]**

Since the triangle OAB is equilateral, we have

$OB = OA = AB$

$\therefore$ $OB^2 = OA^2 = AB^2$ **[½]**

$\Rightarrow$ $x^2 = y^2 = 12 = x^2 + y^2 - 4\sqrt{3y} + 12$ **[½]**

Thus $x^2 + y^2 = 12$

and $x^2 + y^2 = x^2 + y^2 - 4\sqrt{3y} + 12$ or $4\sqrt{3}y - 12 = 0$ ...(i) **[½]**

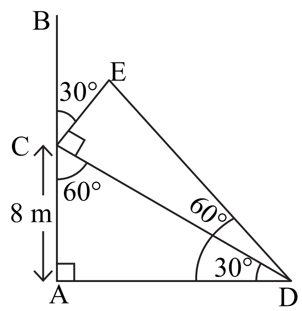
or $y = \sqrt{3}$ ...(ii) **[½]**

From (i) and (ii), we get

$x = \pm 3$ **[½]**

$\therefore$ third vertex is $(3, \sqrt{3})$ or $(-3, \sqrt{3})$ **[½]**

24. In given figure



AC = 8 m [Height of window] **[1]**

CE = Length of pole

$\angle ACD = 180° - [90° + 30°]$

$\angle ACD = 60°$

$\therefore$ $\angle DCE = 90°$ **[1]**

In ΔACD,

$\sin 30° = \frac{AC}{CD}$

$\frac{1}{2} = \frac{8}{CD} \Rightarrow CD = 16 \text{ m}$ **[1]**

In ΔCDE,

$\tan 30° = \frac{CE}{CD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CE}{16}$

$\frac{16}{\sqrt{3}} = CE$ or $\frac{16\sqrt{3}}{3} \text{m} = CE$

$\therefore$ Length of pole is $\frac{16\sqrt{3}}{3}$m. **[1]**

25. $\sin\theta + \sin^2\theta = 1$

$\sin\theta = \cos^2\theta$ **[½]**

$\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$ **[½]**

$\Rightarrow \quad \sin^6\theta + 3\sin^5\theta + 3\sin^4\theta + \sin^3\theta + 2\sin^2\theta + 2\cos^2\theta - 2$ **[1]**

$\Rightarrow \quad (\sin^2\theta + \sin\theta)^3 + 2(\sin^2\theta + \cos^2\theta) - 2$ **[½]**

$\Rightarrow \quad (\sin^2\theta + \cos^2\theta)^3 + 2(1) - 2$ **[½]**

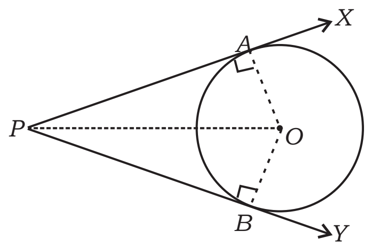
$= 1 + 2 - 2$ **[½]**

$= 1$ **[½]**

26. Given: A circle with centre *O* and pair of tangents drawn from point *P*.

To prove: *PA* = *PB*

Construction: Join *OA*, *OB* and *OP*.



Proof: In Δ*OAP* and Δ*OBP*.

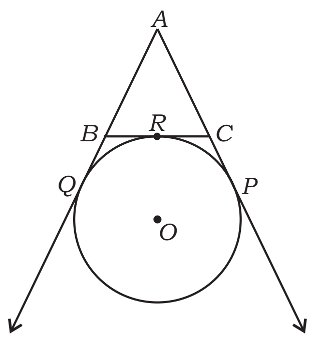
OA = *OB* [radii of same circle]

OP = OP [Common]

$\angle$OAP = $\angle$OBP [Both 90°] **[1]**

$\therefore$ ΔOAP $\cong$ ΔOBP [By RHS congruency]

$\Rightarrow$ PA = PB [cpct] **[1]**



Perimeter ΔABC

= AB + BC + AC

= [AQ – BQ + BR + CR + AP – CP] **[1]**

= [AQ + AP]

[$\because$ tangents from an external point are equal in length]

= [2AQ]

= 10 cm. **[1]**

27. X divides PQ in ratio 2 : 1,

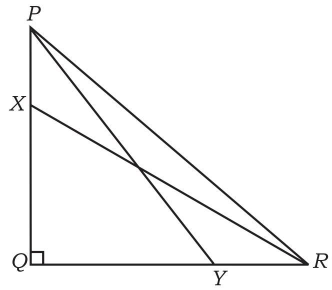
$\Rightarrow$ XQ = 2 PX

$\Rightarrow \quad XQ = \frac{2}{3}PQ$ ...(i)

Also, Y divides QR in ratio 2 : 1,

$\Rightarrow$ QY = 2 YR

$\Rightarrow \quad QY = \frac{2}{3}QR$ ...(ii)



[1]

In right ΔPQR, $PY^2 = PQ^2 + QY^2$ [Pythagoras Theorem]

$$PY^2 = PQ^2 + \left(\frac{2}{3}QR\right)^2$$ [Using (ii)]

$$PY^2 = PQ^2 + \frac{4}{9}QR^2$$

$9PY^2 = 9PQ^2 + 4QR^2$ ...(iii) **[1]**

In right ΔXQR, $XR^2 = XQ^2 + QR^2$ [Pythagoras Theorem]

$$XR^2 = \left(\frac{2}{3}PQ\right)^2 + QR^2$$ [Using (i)]

$$XR^2 = \frac{4PQ^2}{9} + QR^2$$

$9XR^2 = 4PQ^2 + 9QR^2$ ...(iv) **[1]**

Adding (iii) and (iv), we get

$9PY^2 + 9XR^2 = 9PQ^2 + 4QR^2 + 4PQ^2 + 9QR^2$

$= 13PQ^2 + 13QR^2 = 13(PQ^2 + QR^2)$

Hence, $9(PY^2 + XR^2) = 13PR^2$ [$\because PQ^2 + QR^2 = PR^2$]

28. $\sum_{i=1}^{n}(x_i = 50) - 10$ [Given]

and $\sum_{i=1}^{n}(x_i - 46) = 70$ [Given]

$\therefore$ $(x_1 - 50) + (x_2 - 50) + \ldots + (x_n - 50) = -10$

or $(x_1 + x_2 + \ldots + x_n) - 50n = -10$ ...(i) **[½]**

and $(x_1 + x_2 + \ldots + x_n) - 46n = 70$ ...(ii) **[½]**

Subtracting (ii) from (i); **[½]**

$-50n + 46n = -10 - 70$

$-4n = -80$

$\therefore$ $n = 20$ **[½]**

Put $n = 20$ in equation (i) **[½]**

$(x_1 + x_2 + \ldots + x_n) - 1000 = -10$

$(x_1 + x_2 + \ldots + x_n) = 990$ **[½]**

$\therefore$ Mean $(\bar{x}) = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{990}{20}$ **[½]**

$= 49.5$ **[½]**

OR

Let the unknown frequencies be x, y and z.

Class interval	Frequency	Cumulative frequency
0 – 10	4	4
10 – 20	16	20
20 – 30	60	80
30 – 40	x	$80 + x$
40 – 50	y	$80 + x + y$
50 – 60	z	$80 + x + y + z$
60 – 70	4	$84 + x + y + z$
	$N = 230$	

[1]

Since median is 33.5, median class is 30 – 40

Now, median $M = l + \frac{\frac{N}{2} - C}{f} \times h$ **[½]**

Here, $l = 30, \frac{N}{2} = \frac{230}{2}$ = 115, C = 80, f = x, h = 10, M = 33.5

∴ $33.5 = 30 + \frac{115 - 80}{x} \times 10$

or $3.5 = \frac{350}{x}$

∴ x = 100 **[½]**

Again, since mode is 34, therefore, modal class is 30 – 40

Again, mode $M_0 = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$ **[½]**

Here, M_0 = 34, l = 30, f_1 = x = 100, f_0 = 60, f_2 = y, h = 10

∴ $34 = 30 + \frac{100 - 60}{200 - 60 - y} \times 10$ **[½]**

or $4 = \frac{400}{140 - y}$

or $140 - y = 100$

∴ y = 40 **[½]**

Again, since total frequency N = 230

∴ $84 + x + y + z = 230$

or $84 + 100 + 40 + z = 230$

∴ z = 6 **[½]**

Hence, unknown frequencies are 100, 40 and 6.

29. Let the speed of train = x km/hr

and let scheduled time = y hr

Case I:

$(x + 10)(y - 2) = xy$

⇒ $xy - 2x + 10y - 20 = xy$

$-2x + 10y = 20$

$-x + 5y = 10$ ...(i) **[1]**

Case II:

$(x - 10)(y + 2) = xy$

$xy + 3x - 10y - 30 = xy$

$3x - 10y = 30$ ...(ii) **[1]**

From (i) and (ii),

$-3x + 15y = 30$

$3x - 10y = 30$

⇒ $5y = 60$

∴ y = 12 **[½]**

Put y = 12, in (i)

$-x + 60y = 10$

x = 50 **[½]**

∴ Distance = 600 km.

Time management reduce the stress and increase the efficiency of a person.

OR

Let the digit at units place = x

digit tens place = y **[½]**

∴ Given number = $x + 10y$

and reverse number = $10x + y$ **[½]**

ATQ

$x + 10y + 10x + y = 110$

$11x + 11y + 110$

or $x + y = 10$ ...(i) **[½]**

and $x + 10y - 10 = 5(x + y) + 4$

$x + 10y - 10 = 5x + 5y + 4$

$-4x + 5y = 14$ ...(ii) **[½]**

From (i) and (ii)

$4x + 4y = 40$

$-4x + 5y = 14$

∴ $9y = 54$

$y = 6.$

Put y = 6 in equation (i), we have **[½]**

$x = 4$

∴ Amount is Rs. 64. **[½]**

Yoga is essential as it improve and maintain a balanced metabolism.

30. Radius of the cylindrical tank R = 6 m **[½]**

Depth of the cylindrical tank, H = 2.5 m

Volume of water in the tank = $\pi R^2 H = \pi \times 6 \times 6 \times 2.5\ m^3 = 90\ \pi m^3$ **[½]**

Radius of the cylindrical pipe,

$$r = \frac{25}{2}\text{cm} = \frac{25}{2 \times 100}\text{m} = \frac{1}{8}\text{m}$$

Length of water flown in 1 h

= 3.6 km = 3.6 × 1000 m = 3600 m **[½]**

Volume of water flown in 1 h

$$= \pi r^2 h = \pi \times \frac{1}{8} \times \frac{1}{8} \times 3600\ \text{m}^3 = \frac{225}{4}\pi\ \text{m}^3$$ **[½]**

Time taken to fill the tank

$$= \frac{\text{Volume of water in the tank}}{\text{Volume of water flown in 1 h}} = \frac{90\pi}{\frac{225}{4}\pi}\text{h}$$ **[½]**

$$= \frac{90 \times 4}{225}\text{h} = \frac{8}{5}\text{h} = 1\text{ hour 30 minutes}$$ **[½]**

Cost of water at the rate of ₹0.07/m^3

$$= ₹90 \times \frac{22}{7} \times 0.07 = ₹19.80$$ **[1]**

OR

Radius of the top of the frustum R = 20 cm

Radius of the bottom of the frustum r = 12 cm **[½]**

Volume of the frustum = 12308.8 cm^3

Let the height of the bucket = h cm **[½]**

Now, volume of the bucket = Volume of frustum of the cone

$\therefore \quad \frac{1}{3}\pi h(R^2 + r^2 + rR) = 12308.8$

$\Rightarrow \quad \frac{1}{3} \times 3.14 \times h\,[(20)^2 + (12)^2 + 20 \times 12] = 12308.8$ **[½]**

$\Rightarrow \quad 784h = \frac{12308.8}{3.14}$

$\Rightarrow \quad h = \left(\frac{12308.8 \times 3}{3.14 \times 784}\right) = 15$ **[½]**

If l be the slant height of the bucket, then

$l = \sqrt{h^2 + (R - r)^2}$ units

$= \sqrt{(15)^2 + (20 - 12)^2}$ cm **[½]**

$= \sqrt{(15)^2 + 8^2}\ cm = \sqrt{225 + 64}\ cm$

$= \sqrt{289}\ cm = 17\ cm$ **[1]**

Now, area of the metal sheet used = (curved surface area) + (area of the bottom)

= $[\pi l(R + r) + \pi r^2]$ sq units **[1]**

= $[3.14 \times 17 \times (20 + 12) + 3.14 \times 12 \times 12]$ cm^2

= $[3.14 \times (17 + 32) + 3.14 \times 144]$ cm^2

= (3.14×688) cm^2 = 2160.32 cm^2 **[1]**