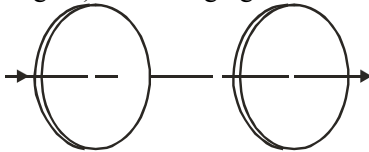


1. Figure shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.



- (a) Calculate the capacitance and the rate of change of potential difference between the plates.  
 (b) Obtain the displacement current across the plates.  
 (c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

**Sol.** (a) Here,  $r = 12 \text{ cm} = 0.12 \text{ m}$ ,  $d = 5 \text{ cm} = 0.05 \text{ m}$ ,  $I = 0.15 \text{ A}$

$$\text{Capacitance of the capacitor} = C = \frac{\epsilon_0 \pi r^2}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 3.14 \times (0.12)^2}{5 \times 10^{-2}} = 80.1 \text{ pF}$$

$$Q = CV$$

$$\frac{dQ}{dt} = C \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{0.15}{80.1 \times 10^{-12}} = 1.87 \times 10^9 \text{ Vs}^{-1}$$

- (b) The displacement current is due to time-varying electric field and is given by

$$i_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d(EA)}{dt} \quad (\because \phi_E = EA)$$

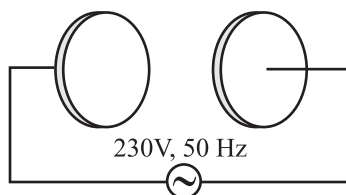
$$\therefore i_D = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 A \frac{d}{dt} \left( \frac{Q}{\epsilon_0 A} \right) \quad \left( \because E = \frac{Q}{\epsilon_0 A} \right)$$

$$= \frac{dQ}{dt}$$

$$\Rightarrow i_D = i = 0.15 \text{ A.}$$

- (c) Yes, the total current is the sum of conduction current and the displacement current.

2. A parallel plate capacitor (Figure) made of circular plates each of radius  $R = 6.0 \text{ cm}$  has a capacitance  $C = 100 \text{ pF}$ . The capacitor is connected to a 230 V ac supply with a (angular) frequency of  $300 \text{ rad s}^{-1}$ .



- (a) What is the rms value of the conduction current?  
 (b) Is the conduction current equal to the displacement current?  
 (c) Determine the amplitude of  $\vec{B}$  at point 3.0 cm from the axis between the plates.

**Sol.** (a) Given  $r = 6.0 \text{ cm} = 6 \times 10^{-2} \text{ m}$ ,  $C = 100 \text{ pF} = 10^{-10} \text{ F}$

$$V_{\text{rms}} = 230 \text{ V}, \omega = 300 \text{ rad/s}$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{V_{\text{rms}}}{\frac{1}{\omega C}} = V_{\text{rms}} \times \omega C$$

$$I_{\text{rms}} = 230 \times 300 \times 10^{-10} = 6.9 \times 10^{-6} \text{ A} = 6.9 \mu\text{A}$$

(b)  $I_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}(EA) \quad [\because \phi_E = EA]$

$$= \epsilon_0 A \frac{dE}{dt} = \epsilon_0 A \frac{d}{dt} \left( \frac{Q}{\epsilon_0 A} \right) \quad \left[ \because E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \right]$$

$$I_D = \epsilon_0 A \times \frac{1}{\epsilon_0 A} \frac{dQ}{dt} = I$$

$\therefore$  Conduction current = Displacement current.

(c)  $B = \frac{\mu_0}{2\pi} \cdot \frac{r}{R^2} \cdot I_D \quad [\because \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_D \Rightarrow 2\pi r B = \mu_0$   
 (current passing through the area of the loop)]

$$B \cdot 2\pi r = \frac{\mu_0}{2\pi} \frac{r}{R^2} I_D \cdot 2\pi r$$

$$B \cdot 2\pi r = \frac{\mu_0 r^2}{R^2} I_D \times \frac{\pi}{\pi}$$

$\therefore$  Amplitude of  $B$  = Max. value of  $B$

$$= \frac{\mu_0 r I_0}{2\pi R^2} = \frac{\mu_0 r \sqrt{2} I_{\text{rms}}}{2\pi R^2} = \frac{[4\pi \times 10^{-7} \times 0.03 \sqrt{2} \times 6.9 \times 10^{-6}]}{2 \times 3.14 \times (0.06)^2}$$

$$= 1.63 \times 10^{-11} \text{ T.}$$

$$I_0 = 2I_{\text{rms}}$$

3. What physical quantity is the same for X-rays of wavelength  $10^{-10}$  m, red light of wavelength  $6800 \text{ \AA}$  and radio waves of wavelength  $500 \text{ m}$ ?

**Sol.** The speed in vacuum is same for all of them. It is given by  $c = 3 \times 10^8 \text{ m/s}$ .

4. A plane electromagnetic wave travels in vacuum along  $z$ -direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is  $30 \text{ MHz}$ , what is its wavelength?

**Sol.** Here  $\vec{E}$  and  $\vec{B}$  are mutually perpendicular in  $X$ - $Z$  and  $Y$ - $Z$  planes.

$$\therefore c = v\lambda \Rightarrow \lambda = \frac{c}{v} = \frac{3 \times 10^8}{30 \times 10^6} = 10 \text{ m}$$

5. A radio can tune in to any station in the  $7.5 \text{ MHz} - 12 \text{ MHz}$  band. What is the corresponding wavelength band?

**Sol.** By relation,  $c = v\lambda \Rightarrow \lambda = \frac{c}{v}$

$$\lambda_1 = \frac{3 \times 10^8}{7.5 \times 10^6} = 40 \text{ m} \quad \text{and} \quad \lambda_2 = \frac{3 \times 10^8}{12 \times 10^6} = 25 \text{ m}$$

The corresponding wavelength band is  $25-40 \text{ m}$ .

6. A charged particle oscillates about its mean equilibrium position with a frequency of  $10^9 \text{ Hz}$ . What is the frequency of the electromagnetic waves produced by the oscillator?

**Sol.** The electromagnetic waves will have same frequency as that of the oscillating charged particle (i.e.  $10^9 \text{ Hz}$ ).

7. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is  $B_0 = 510 \text{ nT}$ . What is the amplitude of the electric field part of the wave?

**Sol.**  $\frac{E_0}{B_0} = c \Rightarrow E_0 = cB_0 = 3 \times 10^8 \times 510 \times 10^{-9} = 153 \text{ N/C}$

8. Suppose that the electric field amplitude of an electromagnetic wave is  $E_0 = 120 \text{ N/C}$  and that its frequency is  $\nu = 50.0 \text{ MHz}$ .

(a) Determine,  $B_0, \omega, k,$  and  $\lambda$ .

(b) Find expressions for  $\vec{E}$  and  $\vec{B}$

**Sol.**  $E_0 = 120 \text{ N/C}, \nu = 50 \times 10^6 \text{ Hz}$

(a) (i) Now,  $E_0/B_0 = c$ ;  $\therefore B_0 = \frac{E_0}{c} = \frac{120}{3 \times 10^8} = 4 \times 10^{-7} \text{ T}$   
 $= 400 \times 10^{-9} \text{ T} = 400 \text{ nT}$

(ii)  $\omega = 2\pi\nu = 2\pi \times 50 \times 10^6 = \pi \times 10^8 \text{ rad/s}$

(iii)  $k = \frac{2\pi}{\lambda} = \frac{2\pi}{c/\nu} = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} = \frac{\pi}{3} = 1.05 \text{ rad/m.}$

(iv)  $c = \lambda \cdot \nu \Rightarrow \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{50 \times 10^6} = 6 \text{ m.}$

(b) Suppose the wave is travelling in  $x$ -direction. Electric field is oscillating in  $x$ - $y$  plane and magnetic field is oscillating in  $y$ - $z$  plane.

$$\therefore \vec{E} = E_0 \sin[kx - \omega t] \hat{j} = 120 \sin[1.05x - \pi \times 10^8 t] \hat{j} \text{ N/C}$$

$$= 120 \text{ N/C} \sin \{ (1.05 \text{ rad/m})x - (\pi \times 10^8 \text{ rad/s})t \} \hat{j} \text{ and}$$

$$\vec{B} = B_0 \sin(kx - \omega t) \hat{k} = (400 \text{ nT}) \sin \{ (1.05 \text{ rad/m})x - (\pi \times 10^8 \text{ rad/s})t \} \hat{k}$$

9. The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula  $E = h\nu$  (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

**Sol.** Formula is,  $E = h\nu = \frac{hc}{\lambda}$   $\text{J} = \frac{6.6 \times 10^{-34} \text{ J} \cdot \text{s} \times 3 \times 10^8 \text{ m/s}}{1.6 \times 10^{-19} \text{ C}}$   $\text{eV}$ ,  $h = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}$   
 For radio waves,  $\lambda = 1 \text{ m}$

$$\text{Photon energy} = 12.375 \times 10^{-7} \text{ eV} = 1.2 \times 10^{-6} \text{ eV}$$

These energy values are related with spacing between energy levels of their sources.

Part of spectrum	Mean Frequency $\nu$ (MHz)	Energy of photon (E)	
		$\nu$ (MHz)	eV
$\gamma$ -rays	$3 \times 10^{14}$	$19.8 \times 10^{-14}$	$12.375 \times 10^5$
X-rays	$3 \times 10^{12}$	$19.8 \times 10^{-16}$	$12.375 \times 10^3$
UV		$10^9$	$6.6 \times 10^{-19} \cdot 4.125$
Visible	$6 \times 10^8$	$39.6 \times 10^{20}$	$24.75 \times 10^{-1}$
Infrared	$10^7$	$6.6 \times 10^{-21}$	$4.125 \times 10^{-2}$
Microwaves	$10^5$	$6.6 \times 10^{-23}$	$4.125 \times 10^{-4}$
Radio waves	$3 \times 10^2$	$19.8 \times 10^{-26}$	$12.375 \times 10^{-7}$

- 10.** In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of  $2.0 \times 10^{10}$  Hz and amplitude  $48 \text{ V m}^{-1}$ .
- What is the wavelength of the wave?
  - What is the amplitude of the oscillating magnetic field?
  - Show that the average energy density of the  $\vec{E}$  field equals the average energy density of the  $\vec{B}$  field. [ $c = 3 \times 10^8 \text{ m s}^{-1}$ .]

**Sol.** Here  $c = 3 \times 10^8 \text{ m/s}$ ,  $\nu = 2 \times 10^{10} \text{ Hz}$ ,  $E_0 = 48 \text{ V/m}$

$$\lambda = ?, B_0 = ?, \vec{u}_E = ?, \vec{u}_B = ?$$

$$(a) \quad \therefore c = \nu\lambda \Rightarrow \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{ m}$$

$$(b) \quad \frac{E_0}{B_0} = c \Rightarrow B_0 = \frac{48}{3 \times 10^8} = 16 \times 10^{-8} \text{ T.}$$

$$(c) \quad \text{Energy density due to electric field, } u_E = \frac{1}{2} \epsilon_0 E^2$$

$$\text{Energy density due to magnetic field, } u_B = \frac{1}{2} \mu_0 B^2$$

$$\text{We have } E = cB \text{ and } c^2 = \frac{1}{\mu_0 \epsilon_0}$$

Subtract these values in eqn (i)

$$U_E = \frac{1}{2} \epsilon_0 (cB)^2 = \frac{1}{2} \epsilon_0 \left( \frac{1}{\mu_0 \epsilon_0} \right) B^2 = \frac{1}{2 \mu_0} B^2 = U_B.$$

- 11.** Suppose that the electric field part of an electromagnetic wave in vacuum is

$$\vec{E} = \left\{ (3.1 \text{ N/C}) \cos [1.8 \text{ rad/m}]y + (5.4 \times 10^8 \text{ rad/s})t \right\} \hat{i}$$

- What is the direction of propagation?
- What is the wavelength  $\lambda$ ?
- What is the frequency  $\nu$ ?
- What is the amplitude of the magnetic field part of the wave?
- Write an expression for the magnetic field part of the wave.

**Sol.**  $E = \left\{ (3.1 \text{ N/C}) \cos [1.8 \text{ rad/m}]y + (5.4 \times 10^8 \text{ rad/s})t \right\} \hat{i}$

- The wave is propagating in the  $-y$ -direction.
- Comparing above equation with the standard equation for the EM wave.

$$E = E_0 \sin \left[ 2\pi \left( \frac{y}{\lambda} - \nu t \right) \right]$$

$$\text{we have } \frac{2\pi}{\lambda} = 1.8 \Rightarrow \lambda = \frac{2\pi}{1.8} = \frac{2 \times 22}{7 \times 1.8} = 3.488 \Rightarrow \lambda = 3.5 \text{ m}$$

$$(c) \text{ Frequency } = \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{3.5} = 0.85 \times 10^8 \text{ Hz}$$

$$= 8.6 \times 10^7 \text{ Hz} = 86 \text{ MHz.}$$

$$(d) \text{ Amplitude of magnetic field } = B_0$$

$$= \frac{E_0}{c} = \frac{3.1}{3 \times 10^8} = 1.03 \times 10^{-8} \approx 10.3 \times 10^{-9} = 10.3 \text{ nT}$$

$$(e) \text{ Expression for magnetic field } B$$

$$= \left[ (10.3 \text{ nT}) \cos \left\{ (1.8 \text{ rad/m})y + (5.4 \times 10^8 \text{ rad/s})t \right\} \right] \hat{k}$$

12. About 5% of the power of a 100 W light bulb is converted to visible radiation.

What is the average intensity of visible radiation

(a) at a distance of 1 m from the bulb?

(b) at a distance of 10 m?

Assume that the radiation is emitted isotropically and neglect reflection.

**Sol.** Here,  $P = 100 \text{ W}$ , visible radiation = 5% of 100 W

(a) Average intensity at a distance of 1 m from the bulb  $\Rightarrow r = 1 \text{ m}$

$$I = \frac{\text{Power}}{\text{area}} = \frac{0.05 \times 100}{4\pi r^2} = \frac{5 \times 7}{4 \times 22 \times 1} = 0.4 \text{ W/m}^2$$

(b) at a distance of 10 m,  $I = \frac{0.05 \times 100}{4\pi \times (10)^2} = 0.004 \text{ W/m}^2$

13. Use the formula  $\lambda_m T = 0.29 \text{ cm K}$  to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?

**Sol.** Given  $\lambda_m T = 0.29 \text{ cmK}$

Electromagnetic spectrum ranges from,  $\lambda_E = 10^{-14} \text{ m}$  to  $\lambda_0 = 6 \times 10^6 \text{ m}$

$$T_E = \frac{0.29 \text{ cmK}}{\lambda_E} = \frac{0.29}{10^{-14} \times 100} = 0.29 \times 10^{12} \text{ K} = 2.9 \times 10^{11} \text{ K}$$

$$T_0 = \frac{0.29 \text{ cmK}}{\lambda_0} = \frac{0.29}{6 \times 10^6 \times 100} = 4.8 \times 10^{-9} \text{ K}$$

Similarly, for other values of  $\lambda$  in the EM spectrum, temperature of the source can be found.

$$\lambda_1 = 10^{-6} \text{ m} \Rightarrow T_1 = \frac{0.29 \text{ cmK}}{\lambda} = \frac{0.29}{10^{-6}} = 2.9 \times 10^3 \text{ K} \Rightarrow T_1 = 2900 \text{ K}$$

$$\lambda_2 = 5 \times 10^{-7} \Rightarrow T_2 = \frac{0.29 \text{ cmK}}{\lambda_2} = \frac{0.29}{5 \times 10^{-7} \times 10^2} = 6000 \text{ K} \Rightarrow T_2 = 6000 \text{ K}$$

14. Given below are some famous numbers associated with electromagnetic radiations in different contexts in physics. State the part of the electromagnetic spectrum to which each belongs.

- 21 cm (wavelength emitted by atomic hydrogen in interstellar space).
- 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen, known as Lamb shift).
- 2.7 K [temperature associated with the isotropic radiation filling all space—thought to be a relic of the ‘big-bang’ origin of the universe].
- 5890 Å – 5896 Å [double lines of sodium]
- 14.4 keV [energy of a particular transition in  $^{57}\text{Fe}$  nucleus associated with a famous high resolution spectroscopic method (Mossbauer spectroscopy)].

- Sol.**
- Radio waves (short wavelength end)
  - Radio waves (short wavelength end)
  - Microwave
  - Visible (yellow)
  - X-rays (or soft  $\gamma$ -rays) region.

15. Answer the following questions:

- Long distance radio broadcasts use short-wave bands. Why?
- It is necessary to use satellites for long distance TV transmission. Why?
- Optical and radiotelescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?
- The small ozone layer on top of the stratosphere is crucial for human survival. Why?
- If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?
- Some scientists have predicted that a global nuclear war on the earth would be followed by a severe ‘nuclear winter’ with a devastating effect on life on earth. What might be the basis of this prediction?

- Sol.**
- Ionosphere reflects waves in these bands.
  - Television signals are not properly reflected by the ionosphere.

Therefore, reflection is effected by satellites.

- (c) Atmosphere absorbs X-rays, while visible and radio waves can penetrate it.
- (d) It absorbs ultraviolet radiations from the sun and prevents it from reaching the earth's surface and causing damage to life.
- (e) The temperature of the earth would be lower because the Greenhouse effect of the atmosphere would be absent.
- (f) The clouds produced by global nuclear war would perhaps cover substantial parts of the sky preventing solar light from reaching many parts of the globe. This would cause a 'winter'.

