NCERT TEXTUAL EXERCISE (SOLVED)

EXERCISE: 16.1

1. Describe the sample space for the indicated experiment: a coin is tossed three times.

Sol. When a coin is tossed three times, the sample space is given by:

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

2. Describe the sample space for the indicated experiment: a die is thrown two times.

Sol. When a die is thrown twice, the sample space is given by:

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

3. Describe the sample space for the indicated experiment: a coin is tossed four times.

Sol. When a coin is tossed four times, the sample space is given by:

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, TTHH, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT}

4. Describe the sample space for the indicated experiment: a coin is tossed and a die is thrown.

Sol. When a coin is tossed and a die is thrown. The sample space S associated with the experiment is

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

5. Describe the sample space for the indicated experiment: a coin is tossed and then a die is rolled only in case a head is shown on the coin.

Sol. When a coin is tossed and then a die is rolled only in case a head is shown on the coin, the sample space is given by:

S = {H1, H2, H3, H4, H5, H6, T}

6. 2 boys and 2 girls are in a Room X, and 1 boy and 3 girls in another Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Sol. Let us denote 2 boys and 2 girls in room X as B_1, B_2 and G_1, G_2 respectively. Let us denote 1 boy and 3 girls in room Y as B_3 and G_3, G_4, G_5 respectively. The sample space is given as

S = {XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5}

7. One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Sol. When a die is selected and then rolled, the sample space is given by

S = {R1, R2, R3, R4, R5, R6, W1, W2, W3, W4, W5, W6, B1, B2, B3, B4, B5, B6}.

8. An experiment consists of recording boy-girl composition of families with 2 children.

(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?

(ii) What is the sample space if we are interested in the number of girls in the family?

Sol. (i) The sample space S, in knowing whether it is a boy or a girl in the order of their birth in composition of families is

S = {BB, BG, GB, GG}

(ii) The sample space S, in knowing the number of girls in a family is

S = {0, 1, 2}

9. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Sol. There are 1 red and 3 identical white balls in a box. The sample space S for selected two balls at random in succession without replacement is S = {RW, WR, WW}.

10. An Experiment consists of throwing a coin and then tossing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

Sol. The sample space S, for tossing a coin and then tossing it second time if a head occurs, if a tail occurs on the first toss, the die is tossed once, then

S = { HH, HT, T1, T2, T3, T4, T5, T6}

11. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

Sol. The sample spaces S for selecting three bulbs at random from a lot is

S = {DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

where D indicates a defective bulb and N a non-defective bulb.

12. A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?

Sol. A coin is tossed. If the result is a head a die is thrown. If the die shows up an even number, the die is thrown again. The sample space S for this experiment is

S = {T, H1, H3, H5, H21, H22, H23, H24, H25, H26, H41, H42, H43, H44, H45, H46, H61, H62, H63, H64, H65, H66}

13. The number 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

Sol. Four slips marked as 1, 2, 3 and 4 are in a box. Two slips are drawn from it one after the other without replacement. The sample space S for the experiment is

S = {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}

14. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Sol. An experiment consisting of rolling a die and then tossing a coin once, if the number on the die is even. If the number is odd, the coin is tossed twice. The sample space S for this experiment is given by

S = {1HH, 1TH, 1HT, 1TT, 2H, 2T, 3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T}

15. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment?

Sol. An experiment consists of tossing a coin. If it shows a tail, a ball is drawn from a box which contains 2 red and 3 black balls. If it shows head, a die is thrown. The sample space S for this experiment is

S = $\{TR_1, TR_2, TB_1, TB_2, TB_3, H1, H2, H3, H4, H5, H6\}$

16. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

Sol. Let N denotes the event that 6 does not appear.

$\therefore$ Sample space = {6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6), (1, 3, 6), (1, 4, 6), (1, 5, 6), (2, 1, 6), (2, 2, 6), ..., (2, 5, 6) ..., (5, 1, 6), (5, 2, 6), ...}.

EXERCISE: 16.2

1. A die is rolled. Let E be the event 'die shows 4' and F be the event 'die shows even number'. Are E and F mutually exclusive?

Sol. When we throw a die, it can result in any one of the six numbers 1, 2, 3, 4, 5 and 6.

Also, $S = \{1, 2, 3, 4, 5, 6\}$

E (die shows 4) = $\{4\}$

F (die shows an even number)= $\{2, 4, 6\}$

$\therefore \quad E \cap F = \{4\} \Rightarrow E \cap F \neq \phi$

$\Rightarrow$ E and F are not mutually exclusive.

2. A die is thrown. Describe the following events:

(i) A : a number less than 7 (ii) B : a number greater than 7

(iii) C : a multiple of 3 (iii) D : a number less than 4

(v) E : an even number greater than 4

(vi) F : a number not less than 3

Also find $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, F', E \cap F'$.

Sol. When we throw a die, it can result in any one of the six numbers 1, 2, 3, 4, 5 and 6.

Also, $S = \{1, 2, 3, 4, 5, 6,\}$

(i) A : a number less than 7 = $\{1, 2, 3, 4, 5, 6\}$

(ii) B : a number greater than 7 = $\{\ \} = \phi$

(iii) C : a multiple of 3 = $\{3, 6\}$

(iv) D : a number, less than 4 = $\{1, 2, 3\}$

(v) E : an even number greater than 4 = $\{6\}$

(vi) F : a number not less than 3 = $\{3, 4, 5, 6\}$

Now, $A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \phi = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$

$B \cup C = \{3, 6\}$

$E \cap F = \{6\}$

$D \cap E = \{1, 2, 3\} \cap \{6\} = \{\phi\}$

$A - C = \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$

$D - E = \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$

$F = \{3, 4, 5, 6\}$

$F' = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$

$E \cap F' = \{6\} \cap \{1,2\} = \phi$

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A : the sum is greater than 8

B : 2 occurs on either die

C : the sum is at least 7 and a multiple of 3

Which pairs of these events are mutually exclusive?

Sol. When two dice are thrown, there are $6 \times 6 = 6^2$ possible outcomes and

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Now, A : the sum is greater than 8

= {(3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

B : 2 occurs on either die

= {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6)}

C : The sum is at least 7 and a multiple of = {(3, 6), (6, 3), (5, 4), (4, 5), (6, 6)}

$A \cap B = \phi$, $B \cap C = \phi$

and $A \cap C$ = {(3, 6), (6, 3), (5, 4), (4, 5), (6, 6)}

(i) Since $A \cap B = \phi$, so, events A and B are mutually exclusive events

(ii) Since $B \cap C = \phi$, so, events B and C are mutually exclusive events

(iii) Since $A \cap C \neq \phi$, so, events A and C are not mutually exclusive events

4. Three coins are tossed once. Let A denotes the event 'three heads shows', B denotes the event 'two heads and one tail show', C denotes the event 'three tails show' and D denotes the event 'a head shows on the first coin'. Which events are

(i) mutually exclusive? (ii) simple?

(iii) compound?

Sol. When three coins are tosses then the simple space S is

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Now, A: Three heads show = {HHH}

B : Two heads and one tail show = {HHT, HTH, THH}

C : Three tails show = {TTT}

D : A head show on the first coin = {HHH, HHT, HTH, HTT}

(i) Since $A \cap B = \phi$, $A \cap C = \phi$, $B \cap C = \phi$, $C \cap D = \phi$, $A \cap B \cap C = \phi$

$\Rightarrow$ The events A and B, A and C; B and C, C and D are mutually exclusive.

(ii) The events A and C are simple events.

(iii) The events B and D are compound events.

5. Three coins are tossed. Describe the following.

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii) Two events which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Sol. When three coin are tossed, then the sample space S is given by

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(i) Two events A and B which are mutually exclusive are

A : 'getting at least two heads'

= {HHH, HHT, HTH, THH}

B : 'getting at least two tails'

= {TTT, TTH, THT, HTT}

It is clear that the events A and B are mutual exclusive events.

(ii) Three events A, B and C which are mutually exclusive and exhaustive are

A : 'getting at most one head'

= {TTT, HTT, THT, TTH}

B : 'getting exactly two heads'

= {HHT, HTH, THH}

C : 'getting exactly three heads'

= {HHH}

It is clear that the events A, B and C are mutually exclusive and exhaustive events.

(iii) Two events which are not mutually exclusive are

A : Getting at least one head.

= {HHH, HHT, HTH, THH, HTT, THT, TTH}

B : Getting exactly two heads.

= {HHT, HTH, THH}

It is clear that the events A and B are not mutually exclusive events.

(iv) Two events which are mutually exclusive but not exhaustive are

A : Getting exactly one head.

= {HTT, THT, TTH}

B : Getting exactly two heads.

= {HHT, HTH, THH}

It is clear that the events A and B are mutually exclusive but not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive are

A : Getting exactly three heads.

= {HHH}

B : Getting exactly two heads.

= {HHT, HTH, THH}

C : Getting exactly three tail.

= {TTT}

It is clear that the events A, B and C are mutually exclusive but not exhaustive events.

6. Two dice are thrown. The events A, B and C are as follows:

A : getting an even number on the first die.

B : getting an odd number on the first die.

C : getting the sum of the numbers on the die ≤ 5.

Describe the events:

(i) A' (ii) not B

(iii) A or B (iv) A and B

(v) A but not C (vi) B or C

(vii) B and C (viii) $A \cap B' \cap C'$

Sol. When two dice are thrown, there are $6 \times 6 = 36$ possible outcomes.

$\therefore$ Possible outcomes are

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

A : getting an even number on the first die

= {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

B : getting an odd number on the first die

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

C : getting the sum of the numbers on the dice ≤ 5

= {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

(i) A' : getting an odd number on the first die = B

(ii) not B : getting an even number on the first die = A

(iii) A : getting an even number on the first die.

B : getting an odd number on the first die.

A or B = $A \cup B = S$ = {1, 2, 3, 4, 5, 6} appear on the first die as well as on the second die.

$A \cup B$ = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1) (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(iv) A and B = $A \cap B = \phi$

(v) A but not C

A : getting are even number on the first die

not C : getting the sum of numbers on the dice > 5

= {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

A but not C : {(2, 4), (4, 2), (2, 5), (4, 3), (6, 1), (2, 6), (4, 4), (6, 2), (4, 5), (6, 3), (4, 6), (6, 4), (6, 5), (6, 6)}

(vi) B : getting an odd number on first die

C : {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

B or C = $B \cup C$

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

(vii) B and C = $B \cap C$ = {(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)}

(viii) A : getting an even number on the first die.

B' = getting an even number on the first die.

C' = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

$A \cap B' \cap C'$ = {(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

7. Two dice are thrown. The events A, B and C are as follows:

A : getting an even number on the first die.

B : getting an odd number on the first die.

C : getting the sum of the numbers on the dice ≤ 5

State true or false (give reason for your answer):

(*i*) A and B are mutually exclusive events

(*ii*) A and B are mutually exclusive and exhaustive events

(*iii*) A = B'

(*iv*) A and C are mutually exclusive events

(*v*) A and B' are mutually exclusive events

(vi) A', B', C are mutually exclusive and exhaustive events

Sol. **A** = {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

C = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

(*i*) It is clear that the given statement is true.

(*ii*) It is clear that the given statement is true.

(*iii*) It is clear that the given statement is true.

(*iv*) It is clear that the given statement is false.

(*v*) It is clear that the given statement is false.

(vi) It is clear that the given statement is false.

NCERT TEXTUAL EXERCISE (SOLVED)

EXERCISE: 16.3

1. Which of the following cannot be valid assignment of probabilities for outcomes of sample space S = $\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignments	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	–0.1	0.2	0.3	0.4	–0.2	0.1	0.3
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Sol. (a) Sum of probabilities

$= P(\omega_1) + P(\omega_2) + P(\omega_3) + P(\omega_4) + P(\omega_5) + P(\omega_6) + P(\omega_7)$

$= 0.1 + 001 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6$

$= 1$

Thus, the assignment is valid.

(b) Sum of Probabilities

$= P(\omega_1) + P(\omega_2) + P(\omega_3) + P(\omega_4) + P(\omega_5) + P(\omega_6) + P(\omega_7)$

$= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = 7 \times \frac{1}{7} = 1$

Thus, the assignment is valid.

(c) Sum of Probabilities

$= P(\omega_1) + P(\omega_2) + P(\omega_3) + P(\omega_4) + P(\omega_5) + P(\omega_6) + P(\omega_7)$

$= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$

$= 2.8 \neq 1.$

Thus, the assignment is not valid.

(d) Here, $P(\omega_1)$ and $P(\omega_5)$ are negative.

Hence, the assignment is not valid.

(e) Here, $P(\omega_7) = \frac{15}{14} > 1$

Hence, the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Sol. When a coin is tossed twice, the sample space is given by

S = {HH, HT, TH, TT}

Let A be the event of the occurrence of at least one tail, i.e.

A = {HT, TH, TT}

$$\therefore \quad P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

$$= \frac{n(A)}{n(S)}$$

$$= \frac{3}{4}$$

2. A die is thrown, find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear

(iii) A number less than or equal to 1 will appear

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

Sol. The sample space of the given experiment is given by

S = {1, 2, 3, 4, 5, 6}

(i) Let A be the event of the occurrence of a prime number, i.e.

A = {2, 3, 5}

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) Let B be the event of the occurrence of a number greater than or equal to 3. Accordingly,

B = {3, 4, 5, 6}

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

(iii) Let C be the event of the occurrence of a number less than or equal to 1, i.e.

C = {1}

$$\therefore \quad P(C)=\frac{n(C)}{n(S)}=\frac{1}{6}$$

(iv) Let D be the event of the occurrence of a number greater than 6, i.e. $D = \phi$.

$$\therefore \quad P(D)=\frac{n(D)}{n(S)}=\frac{0}{6}=0$$

(v) Let E be the event of the occurrence of a number less than 6, i.e. Accordingly,

$E = \{1, 2, 3, 4, 5\}$

$$\therefore \quad P(E)=\frac{n(E)}{n(S)}=\frac{5}{6}.$$

4. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the cards is (i) an ace and (ii) black card.

Sol. (a) There are 52 points in the sample space.

(b) Let A be the event in which the card drawn is an ace of spades.

$$\therefore P(A) = \frac{n(A)}{n(S)}=\frac{1}{52}$$

(c) (i) Let E be the event in which the card drawn is an ace.

$$\therefore P(E) = \frac{n(E)}{n(S)}=\frac{4}{52}=\frac{1}{13}$$

(ii) Let F be the event in which the card drawn is black.

$$\therefore P(F) = \frac{n(F)}{n(S)}=\frac{26}{52}=\frac{1}{2}$$

5. A fair coin with 1 marked on one face and 6 on the other, and a fair die are tossed together. Find the probability that the sum of numbers that turn up is (i) 3, and (ii) 12.

Sol. The sample space is given by

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(i) Let A be the event in which the sum of numbers that turn up is 3, i.e.

A = {(1, 2)}

∴ $P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$

(ii) Let B be the event in which the sum of number that turn up is 12, i.e.

B = {(6, 6)}

∴ $P(B) = \frac{n(B)}{n(S)} = \frac{1}{12}$

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Sol. There are 10 members of the council. Any one may be selected for a committees.

∴ Number of all possible outcomes = 10

One woman out of 6 may be selected in 6 ways.

∴ Number of favourable cases = 6

∴ Probability of selection of a woman = $\frac{6}{10} = 0.6$

7. A fair coin is tossed four times, and a person wins ₹ 1 for each head and lose ₹1.50 for each tail that turns up.

From the sample space, calculate how many different amounts of money the person can have after four tosses and the probability of having each of these amounts.

Sol. (i) No head and 4 tail appear.

Money lost = ₹ 4 × 1.50 = ₹ 6.00

There is only 1 way when, TTTT occurs

Number of exhaustive cases = $2^4 = 16$

∴ Probability of getting no head or 4 tails $= \frac{1}{16}$

(ii) When 1 head and 3 tails appear.

Money lost = ₹ (–1 × 1 + 3 × 1.50) = ₹ 3.50

These are 4 ways when 1 head and 3 tails occur, i.e.

HTTT, THTT, TTHH, TTTH

∴ 1 head and 3 tails appear in 4 ways.

Probability of getting 1 head and 3 tails $= \frac{4}{16} = \frac{1}{4}$

(iii) When 2 heads and 2 tails appear.

Money lost = ₹ (2 × 1.5 – 1 × 2) = ₹ (3 – 2) = ₹ 1

2 heads and 2 tails may occur as

HHTT, HTHT, HTTH, THHT, THTH, TTHH

Thus, 2 heads and 2 tails may appear in 6 ways,

Probability of getting 2 heads and 2 tails $= \frac{6}{16} = \frac{3}{8}$

(iv) When 3 heads and 1 tail appear,

Money gained ₹ (3 × 1 – 1 × 1.5) = ₹1.50

3 heads and 1 tails may occurs as HHHT, HHTH, HTHH, THHH

∴ 3 heads and 1 tail appear in 4 ways.

∴ Probability of getting 3 heads and 1 tail $= \frac{4}{16} = \frac{1}{4}$

(v) When all the heads appear,

Money gained = ₹ 4 × 1 = ₹ 4

4 heads occur as HHHH, i.e. in one way.

∴ Probability of getting 4 heads $= \frac{1}{16}$

8. Three coins are tossed once. Find the probability of getting

(i) 3 heads
(ii) 2 head
(iii) at least 2 head
(iv) atmost 2 heads
(v) no head
(vi) 3 tails
(vii) exactly two tails
(viii) no tail
(ix) atmost two tails.

Sol. When three coins are tossed once, the sample space S is given by

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Number of all possible outcomes = 8

(i) There is only one favourable cases, HHH

∴ P (3 heads) $= \frac{1}{8}$

(ii) There are three favourable cases when two heads occur, viz., HHT, HTH, THH.

$\therefore$ P (exactly 2 heads) = $\frac{3}{8}$

(iii) At least 2 heads $\Rightarrow$ 2 or 3 heads

There are 4 favourable case HHT, HTH, THH, HHH

$\therefore$ P (at least 2 heads) = $\frac{4}{8} = \frac{1}{2}$

(iv) P(at most 2 heads) = P (not 3 heads) = 1– P (3 heads) = $1 - \frac{1}{8} = \frac{7}{8}$

(v) No head means all tails are obtained. There is only one favourable case, TTT.

$\therefore$ P (no heads) = $\frac{1}{8}$

(vi) There is only one favourable case, TTT.

$\therefore$ P (3 tails) = $\frac{1}{8}$

(vii) There are 3 favourable cases: HTT, THT, TTH.

$\therefore$ P (exactly 2 tails) = $\frac{3}{8}$

(viii) No tail means all heads are obtained. There is only one favourable case, HHH.

$\therefore$ P (all heads) = $\frac{1}{8}$

(ix) Atmost two tails $\Rightarrow$ All the 3 tails do not occur, i.e. 3 tails occur is 1 way.

$\therefore$ Probability that 3 tails appear = $\frac{1}{8}$

Probability that 3 tails do not appear = $1 - \frac{1}{8} = \frac{7}{8}$

$\therefore$ Probability of getting atmost two tails = $\frac{7}{8}$

9. If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Sol. Let $P(A) = \frac{2}{11}$

Then P(not A) = 1 – P(A)

$$=1-\frac{2}{11}=\frac{9}{11}$$

10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that the letter is (i) a vowel, and (ii) a consonant.

Sol. The word 'ASSASSINATION' has 13 letters;

∴ n (S) = 13

No. of vowels AAAIIO = 6

∴ Probability of choosing a vowel = $\frac{6}{13}$

No. of consonants SSSSNNT = 7

∴ Probability of choosing a consonant = $\frac{7}{13}$

11. In a lottery, a person choses six different natural numbers at random from 1 to 20 and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game.

Sol. There are 20 natural numbers from which 6 numbers may be chosen in $^{20}C_6$ ways.

$$^{20}C_6=\frac{20\times19\times18\times17\times16\times15}{1\times2\times3\times4\times5\times6}=38,760$$

There is only 1 favourable case which is being fixed by the lottery committee.

Probability of winning the lottery = $\frac{1}{38760}$

12. Check whether the following probabilities P(A) and P(B) are consistently defined

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

Sol. (i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

Clearly, P(A ∩ B) > P(A).

Hence, P(A) and P(B) are not consistently defined.

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

Clearly, P(A ∪ B) > P(A) and P(A ∪ B) > P(B)

Hence, P(A) and P(B) are consistently defined.

13. Fill in the blanks in following table:

	P(A)	P(B)	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	....
(ii)	0.35	...	0.25	0.6
(iii)	0.5	0.35	...	0.7

Sol. (*i*) Here, $P(A)=\frac{1}{3},\ P(B)=\frac{1}{5},\ P(A\cap B)=\frac{1}{15}$

We know that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore\ P(A\cup B)=\frac{1}{3}+\frac{1}{5}+\frac{1}{15}=\frac{7}{15}$

(*ii*) Here, $P(A) = 0.35$, $P(A \cap B) = 0.25$, $P(A \cup B) = 0.6$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow\ 0.6 = 0.35 + P(B) - 0.25$

$\Rightarrow P(B) = 0.5$

(*iii*) Here, $P(A) = 0.5$, $P(B) = 0.35$, $P(A \cup B) = 0.7$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow\ 0.7 = 0.5 + 0.35 - P(A \cap B)$

$P(A \cap B) = 0.15$

14. Given $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

Sol. When A and B are mutually exclusive events, then

$P(A\cup B)=P(A)+P(B)$

[as $P(A\cup B)=P(A)+P(B)$ for mutually exclusive]

$P(A \cup B) = P(A) + P(B)$

$=\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$

15. If E and F are two events such that $P(E)=\frac{1}{4},\ P(F)=\frac{1}{2}$ and P(E and F) $=\frac{1}{8}$, find

(i) P(E or F) (ii) P(not E and not F).

Sol. (i) $P(E \text{ or } F)=P(E\cup F)$

$$= P(E) + P(F) - P(E \cap F)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

(ii) P(not E and not F) = $E' \cap F' = (E \cup F)'$

$\therefore$ P (not E and not F) $= P(E \cup F)'$

$$= 1 - P(E \cup F)$$

$$= 1 - \frac{5}{8} = \frac{3}{8}$$

16. Two events E and F are such that P(not E or not F) = 0.25. State whether E and F are mutually exclusive.

Sol. It is given that P (not E or not F) = 0.25, i.e.

$P(E' \cup F') = 0.25$

$\Rightarrow$ $P(E \cap F)' = 0.25$ [$\because E' \cup F' = (E \cap F)'$]

Now $P(E \cap F) = 1 - P(E \cap F)'$ [De Morgan's' Law]

$\Rightarrow$ $P(E \cap F) = 1 - 0.25$

$\Rightarrow$ $P(E \cap F) = 0.75 \neq 0$

$\Rightarrow$ $E \cap F \neq \phi$

Thus, the events E and F are not mutually exclusive.

17. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (i) P (not A), (ii) P (not B) and (iii) P(A or B).

Sol. (i) P(not A) = 1 – P(A) = 1 – 0.42 = 0.58

(ii) P(not B) = 1 – P(B) = 1 – 0.48 = 0.52

(iii) We know that P(A or B) = P(A) + P(B) – P(A and B)

$\therefore$ P(A or B) = 0.42 + 0.48 – 0.16 = 0.74

18. In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Sol. Let E be the event in which the selected student studies Mathematics and F be the event in which the selected student studies Biology.

Accordingly,

$P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$

$P(F) = 30\% = \frac{30}{100} = \frac{3}{10}$

$P(E \text{ and } F) = 10\% = \frac{10}{100} = \frac{1}{10}$

We know that P(E or F) = P(E) + P(F) – P(E and F)

$\therefore \quad P(E \text{ or } F) = \frac{2}{5} + \frac{3}{10} - \frac{1}{10} = \frac{6}{10} = 0.6$

Thus, the probability that the selected student will be studying mathematics or biology is 0.6.

19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Sol. Let A and B be the events of passing I and II examinations respectively.

$\therefore \quad P(A) = 0.8, P(B) = 0.7$

Probability of passing at least one examination

$= 1 - P(A' \cap B') = 0.95$ ...(1)

Now $A' \cap B' = (A \cup B)'$ (De Morgan's Law)

$P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

Putting this value in relation (i), we get

$1 - [1 - P(A \cup B)] = 0.95$

or $\quad P(A \cup B) = 0.95$

Further $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.8 + 0.7 - 0.95 = 1.5 - 0.95 = 0.55$

Thus, probability that the student will pass in both the examinations = 0.55.

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Sol. Let E and F be the events of passing English and Hindi examinations respectively.

It is given that P(E and F) = 0.5

P(not E and not F) = 0.1, i.e. $P(E' \cap F') = 0.1 \Rightarrow P(E \cup F)' = 0.1$

P(E) = 0.75

Now, $P(E \cup F) = 1 - P(E \cup F)' = 1 - 0.1 = 0.9$

We know that

P(E or F) = P(E) + P(F) – P(E and F)

$\Rightarrow$ P(F) = 0.9 – 0.75 + 0.5

$\Rightarrow$ P(F) = 0.65

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these student is selected at random, find the probability that

(i) the student opted for NCC and NSS.

(ii) the student has opted neither NCC and NSS.

(iii) the student has opted NSS but not NCC.

Sol. In a class of 60 students, 30 students opted for NCC.

$\therefore$ Probability of opting NCC $= \dfrac{30}{60}$

Let A be the event that a student opts NCC. Then $n(A) = 30$

$\therefore$ $P(A) = \dfrac{30}{60}$

If B be the event that a student opts NSS. Then $n(B) = 32$

$\therefore$ $P(B) = \dfrac{32}{60}$

24 students opt NCC and NSS both. Then $n(A \cap B) = 24$.

$$P(A \cap B) = \frac{24}{60}$$

(i) Probability that a student opts NSS or NCC = $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60}$$

$$= \frac{30 + 32 - 24}{60} = \frac{38}{60} = \frac{19}{30} = 0.63$$

(ii) Probability that the student has opted neither NCC nor NSS

$= P(A \cap B)'$

$= 1 - P(A \cup B)$

$= 1 - 0.63 = 0.37$

(iii) Probability that the student has opted NSS but not NCC

$= P(A' \cap B)$

$= P(B) - P(A \cap B)$

$= \frac{32}{60} - \frac{24}{60}$

$= \frac{8}{60} = \frac{2}{15}$