

EXERCISE 1.1

1. Which of the following are sets? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

Sol. SET

Reason: It is a well-defined collection of objects because one can definitely identify a month that belongs to this collection.

(ii) The collection of ten most talented writers of India.

Sol. Not a SET

Reason: It is not a well-defined collection because the criteria for determining a writer's talent may vary from person to person.

(iii) A team of eleven best-cricket batsmen of the world.

Sol. Not a SET

Reason: It is not a well-defined collection because the criteria for determining a batsman's talent may vary from person to person.

(iv) The collection of all boys in your class.

Sol. SET

Reason: It is a well-defined collection of objects because one can definitely identify a boy who belongs to this collection.

(v) The collection of all natural numbers less than 100.

Sol. SET

Reason: It is well-defined collection because one can definitely identify a number that belongs to this collection.

(vi) A collection of novels written by Munshi Prem Chand.

Sol. SET

Reason: It is well-defined collection because one can definitely identify a book that belongs to this collection.

(vii) The collection of all even integers.

Sol. SET

Reason: Well-defined collection of objects because one can definitely identify a number that belongs to this collection.

(viii) The collection of questions in this chapter.

Sol. SET

Reason: Well-defined collection because one can definitely identify a question that belongs to the chapter.

(ix) A collection of most dangerous animals in the world.

Sol. Not a SET.

Reason: Not well-defined collection because the criteria for determining if the animal is dangerous can vary from person to person.

2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol \in or \notin in the blank spaces:

(i) $5 \underline{\quad} A$

(ii) $8 \underline{\quad} A$

(iii) $0 \underline{\quad} A$

(iv) $4 \underline{\quad} A$

(v) $2 \underline{\quad} A$

(vi) $10 \underline{\quad} A$

Sol. (i) \in

(ii) \notin

(iii) \notin

(iv) \in

(v) \in

(vi) \notin

3. Write the following sets in roster form:

(i) $A = \{x : x \text{ is an integer and } -3 < x < 7\}$

Sol. $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = \{x : x \text{ is a natural number less than } 6\}$

Sol. $B = \{1, 2, 3, 4, 5\}$

(iii) $C = \{x : x \text{ is a two-digit natural number such that the sum of its digits is } 8\}$

Sol. $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

(iv) $D = \{x : x \text{ is a prime number which is divisor of } 60\}$

Sol. $D = \{2, 3, 5\}$

(v) $E = \text{The set of all letters in the word TRIGONOMETRY}$

Sol. $E = \{T, R, I, G, O, N, M, E, Y\}$

(vi) $F = \text{The set of all letters in the word BETTER.}$

Sol. $F = \{B, E, T, R\}$

4. Write the following sets in the set-builder form:

(i) $A = \{3, 6, 9, 12\}$

Sol. $A = \{x : x = 3n, n \in \mathbf{N}, n \leq 4\}$

(ii) $B = \{2, 4, 8, 16, 32\}$

Sol. $B = \{x : x = 2^n, n \in \mathbf{N}, n \leq 5\}$

(iii) $C = \{5, 25, 125, 625\}$

Sol. $C = \{x : x = 5^n, n \in \mathbf{N}, n \leq 4\}$

(iv) $D = \{2, 4, 6, \dots\}$

Sol. $D = \{x : x = 2n, n \in \mathbf{N}\}$

(v) $E = \{1, 4, 9, \dots, 100\}$

Sol. $E = \{x : x = n^2, n \in \mathbf{N}, n \leq 10\}$

5. List all the elements of the following sets:

(i) $A = \{x : x \text{ is an odd natural number}\}$

Sol. $A = \{1, 3, 5, 7, \dots\}$

(ii) $B = \{x : x \text{ is an integer, } -1/2 < x < 9/2\}$

Sol. $B = \{0, 1, 2, 3, 4\}$

(iii) $C = \{x : x \text{ is an integer, } x^2 \leq 4\}$

Sol. $C = \{-2, -1, 0, 1, 2\}$

(iv) $D = \{x : x \text{ is a letter in the word LOYAL}\}$

Sol. $D = \{L, O, Y, A\}$

(v) $E = \{x : x \text{ is a month of a year not having 31 days}\}$

Sol. $E = \{\text{February, April, June, September, November}\}$

(vi) $F = \{x : x \text{ is a consonant in the English alphabet which precedes } k\}$

Sol. $F = \{b, c, d, f, g, h, j\}$

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form :

(i) $\{1, 2, 3, 6\}$

(a) $\{x : x \text{ is a prime number and a divisor of } 6\}$

(ii) $\{2, 3\}$

(b) $\{x : x \text{ is an odd natural number less than } 10\}$

(iii) $\{M, A, T, H, E, I, C, S\}$

(c) $\{x : x \text{ is natural number and divisor of } 6\}$

(iv) $\{1, 3, 5, 7, 9\}$

(d) $\{x : x \text{ is a letter of the word MATHEMATICS}\}$

Sol. (i) \rightarrow (c)

(ii) \rightarrow (a)

(iii) \rightarrow (d)

(iv) \rightarrow (b)

EXERCISE 1.2

1. Which of the following are examples of the null set?

(i) Set of odd natural numbers divisible by 2.

Sol. Null set

(ii) Set of even prime numbers

Sol. Not a Null set

(iii) $\{x : x \text{ is a natural number, } x < 5 \text{ and } x > 7\}$

Sol. Null set

(iv) $\{y : y \text{ is a point common to any two parallel lines}\}$

Sol. Null set

2. Which of the following sets are finite or infinite?

(i) The set of months of a year

Sol. Finite

(ii) $\{1, 2, 3, \dots\}$

Sol. Infinite

(iii) $\{1, 2, 3, \dots, 99, 100\}$

Sol. Finite

(iv) The set of positive integers greater than 100

Sol. Infinite

(v) The set of prime numbers less than 99

Sol. Finite

3. State whether each of the following set is finite or infinite.

(i) The set of lines which are parallel to the x-axis

Sol. Infinite

(ii) The set of letters in the English alphabet

Sol. Finite

(iii) The set of numbers which are multiple of 5

Sol. Infinite

(iv) The set of animals living on the earth

Sol. Finite

(v) The set of circles passing through the origin $(0, 0)$

Sol. Infinite

4. In the following, state whether $A = B$ or not.

(i) $A = \{a, b, c, d\}$ $B = \{d, c, b, a\}$

Sol. $A = B$

(ii) $A = \{4, 8, 12, 16\}$ $B = \{8, 4, 16, 18\}$

Sol. $A \neq B$

(iii) $A = \{2, 4, 6, 8, 10\}$ $B = \{x : x \text{ is positive even integer and } x \leq 10\}$

Sol. $A = B$

(iv) $A = \{x : x \text{ is a multiple of } 10\}$ $B = \{10, 15, 20, 25, 30, \dots\}$

Sol. $A \neq B$

5. Are the following pair of sets equal? Give reasons.

(i) $A = \{2, 3\}$ $B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}$

Sol. Equation $x^2 + 5x + 6 = 0$ can be written as

$$(x + 2)(x + 3) = 0$$

$$\Rightarrow x = -2, x = -3$$

Therefore, $A = \{2, 3\}$ $B = \{-2, -3\}$ Hence, $A \neq B$

(ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\}$

$B = \{y : y \text{ is a letter in the word WOLF}\}$

Sol. $A = \{F, O, L, W\}$, $B = \{W, O, L, F\}$

Therefore, $A = B$

6. From the sets given below, select equal sets:

$A = \{2, 4, 8, 12\}$, $B = \{1, 2, 3, 4\}$ $C = \{4, 8, 12, 14\}$

$D = \{3, 1, 4, 2\}$ $E = \{-1, 1\}$ $F = \{0, a\}$

$G = \{1, -1\}$ $H = \{0, 1\}$

Sol. (i) $B = D$, (ii) $E = G$

EXERCISE 1.3

1. Make correct statements by filling in the symbols \subset or $\not\subset$ in the blank spaces:

(i) $\{2, 3, 4\} \underline{\hspace{1cm}} \{1, 2, 3, 4, 5\}$ (ii) $\{a, b, c\} \underline{\hspace{1cm}} \{b, c, d\}$

(iii) $\{x : x \text{ is a student of Class XI of your school}\} \underline{\hspace{1cm}} \{x : x \text{ is student of your school}\}$

(iv) $\{x : x \text{ is a circle in the plane}\} \underline{\hspace{1cm}} \{x : x \text{ is a circle in the same plane with radius 1 unit}\}$

(v) $\{x : x \text{ is a triangle in a plane}\} \underline{\hspace{1cm}} \{x : x \text{ is a rectangle in the plane}\}$

(vi) $\{x : x \text{ is an equilateral triangle in a plane}\} \underline{\hspace{1cm}} \{x : x \text{ is a triangle in the same plane}\}$

(vii) $\{x : x \text{ is an even natural number}\} \underline{\hspace{1cm}} \{x : x \text{ is an integer}\}$

Sol. (i) \subset (ii) \subset
 (iii) \subset (iv) $\not\subset$
 (v) $\not\subset$ (vi) \subset (vii) \subset

2. Examine whether the following statements are true or false:

(i) $\{a, b\} \not\subset \{b, c, a\}$

(ii) $\{a, e\} \subset \{x : x \text{ is a vowel in the English alphabet}\}$

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$

(iv) $\{a\} \subset \{a, b, c\}$

(v) $\{a\} \in \{a, b, c\}$

(vi) $\{x : x \text{ is an even natural number less than 6}\} \subset \{x : x \text{ is a natural number which divides 36}\}$

Sol. (i) False (ii) True (iii) False
 (iv) True (v) False (vi) True

3. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$

Sol. The statement $\{3, 4\} \subset A$ is incorrect because $3 \in \{3, 4\}$; but, $3 \notin A$.

(ii) $\{3, 4\} \in A$

Sol. The statement $\{3, 4\} \in A$ is correct because $\{3, 4\}$ is an element of A .

(iii) $\{\{3, 4\}\} \subset A$

Sol. The statement $\{\{3, 4\}\} \subset A$ is correct because $\{3, 4\} \in A$

(iv) $1 \in A$

Sol. The statement $1 \in A$ is correct as 1 is element of set A .

(v) $1 \subset A$

Sol. The statement $1 \subset A$ is incorrect because 1 is element of A and not a subset.

(vi) $\{1, 2, 5\} \subset A$

Sol. The statement $\{1, 2, 5\} \subset A$ is correct because each element of $\{1, 2, 5\}$ is also an element of A .

(vii) $\{1, 2, 5\} \in A$

Sol. The statement $\{1, 2, 5\} \in A$ is incorrect because $\{1, 2, 5\}$ is not an element of A .

(viii) $\{1, 2, 3\} \subset A$

Sol. The statement $\{1, 2, 3\} \subset A$ is incorrect because 3 is not an element of A .

(ix) $\phi \in A$

Sol. The statement $\phi \in A$ is incorrect because null set is not the element of set A .

(x) $\phi \subset A$

Sol. The statement $\phi \in A$ is correct because null set is subset of every set.

(xi) $\{\phi\} \subset A$

Sol. The statement $\{\phi\} \subset A$ is incorrect because $\phi \notin A$.

4. Write down all the subsets of the following sets:

(i) $\{a\}$

Sol. $\{a\}, \phi$

(ii) $\{a, b\}$

Sol. $\{a\}, \{b\}, \{a, b\}, \phi$

(iii) $\{1, 2, 3\}$

Sol. $\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \phi$

(iv) ϕ **Sol.** ϕ 5. How many elements has $P(A)$, if $A = \phi$?**Sol.** $A = \phi$

$$n(A) = 0$$

$$n[P(A)] = 2^0 = 1$$

6. Write the following as intervals:

(i) $\{x : x \in \mathbf{R}, -4 < x \leq 6\}$ (ii) $\{x : x \in \mathbf{R}, -12 < x < -10\}$ (iii) $\{x : x \in \mathbf{R}, 0 \leq x < 7\}$ (iv) $\{x : x \in \mathbf{R}, 3 \leq x \leq 4\}$ **Sol.** (i) $(-4, 6]$ (ii) $(-12, -10)$ (iii) $[0, 7)$ (iv) $[3, 4]$

7. Write the following intervals in set-builder form:

(i) $(-3, 0)$ (ii) $[6, 12]$ (iii) $(6, 12]$ (iv) $[-23, 5)$ **Sol.** (i) $\{x : x \in \mathbf{R}, -3 < x < 0\}$ (ii) $\{x : x \in \mathbf{R}, 6 \leq x \leq 12\}$ (iii) $\{x : x \in \mathbf{R}, 6 < x \leq 12\}$ (iv) $\{x : x \in \mathbf{R}, -23 \leq x < 5\}$

8. What universal set(s) would you propose for each of the following:

(i) The set of right triangles. (ii) The set of isosceles triangles.

Sol. (i) The set of triangles in the same plane.

(ii) The set of triangles in the same plane.

9. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C?(i) $\{0, 1, 2, 3, 4, 5, 6\}$ (ii) ϕ (iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **Sol.** (iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

EXERCISE 1.4

1. Find the union of each of the following pairs of sets:

(i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$ **Sol.** $X \cup Y = \{1, 2, 3, 5\}$ (ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$ **Sol.** $A \cup B = \{a, b, c, e, i, o, u\}$ (iii) $A = \{x : x \text{ is a natural number and multiple of } 3\}$ $B = \{x : x \text{ is a natural number less than } 6\}$

Sol. $A \cup B = \{x : x = 1, 2, 4, 5 \text{ or a multiple of } 3\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$

$B = \{x : x \text{ is a natural number and } 6 < x < 10\}$

Sol. $A \cup B = \{x : 1 < x < 10, x \in \mathbf{N}\}$

(v) $A = \{1, 2, 3\}, B = \phi$

Sol. $A \cup B = \{1, 2, 3\}$

2. Let $A = \{a, b\}, B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Sol. Yes. $A \cup B = \{a, b, c\} = B$

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Sol. $A \cup B = B$

4. If $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$

Sol. $\{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C$

Sol. $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C$

Sol. $\{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D$

Sol. $\{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C$

Sol. $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D$

Sol. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D$

Sol. $\{3, 4, 5, 6, 7, 8, 9, 10\}$

5. Find the intersection of each of the following pairs of sets:

(i) $X = \{1, 3, 5\}, Y = \{1, 2, 3\}$

Sol. $X \cap Y = \{1, 3\}$

(ii) $A = \{a, e, i, o, u\} \quad B = \{a, b, c\}$

Sol. $A \cap B = \{a\}$

(iii) $A = \{x : x \text{ is a natural number and multiple of } 3\}$


$B = \{x : x \text{ is a natural number less than } 6\}$

Sol. $A \cap B = \{3\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$

$B = \{x : x \text{ is a natural number and } 6 < x < 10\}$

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Sol. $A \cap B = \phi$

(v) $A = \{1, 2, 3\}$, $B = \phi$

Sol. $A \cap B = \phi$

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

- | | |
|-----------------------------------|----------------------------------|
| (i) $A \cap B$ | (ii) $B \cap C$ |
| (iii) $A \cap C \cap D$ | (iv) $A \cap C$ |
| (v) $B \cap D$ | (vi) $A \cap (B \cup C)$ |
| (vii) $A \cap D$ | (viii) $A \cap (B \cup D)$ |
| (ix) $(A \cap B) \cap (B \cup C)$ | (x) $(A \cup D) \cap (B \cup C)$ |

Sol. (i) $\{7, 9, 11\}$

(ii) $\{11, 13\}$

(iii) ϕ

(iv) $\{11\}$

(v) ϕ

(vi) $\{7, 9, 11\}$

(vii) ϕ

(viii) $\{7, 9, 11\}$

(ix) $\{7, 9, 11\}$

(x) $\{7, 9, 11, 15\}$

7. If $A = \{x : x \text{ is a natural number}\}$, $B = \{x : x \text{ is an even number}\}$, $C = \{x : x \text{ is an odd natural number}\}$, and $D = \{x : x \text{ is a prime number}\}$, find

- | | |
|------------------|-----------------|
| (i) $A \cap B$ | (ii) $A \cap C$ |
| (iii) $A \cap D$ | (iv) $B \cap C$ |
| (v) $B \cap D$ | (vi) $C \cap D$ |

Sol. (i) B

(ii) C

(iii) D

(iv) ϕ

(v) $\{2\}$

(vi) $\{x : x \text{ is an odd prime number}\}$

8. Which of the following pairs of sets are disjoint:

(i) $\{1, 2, 3, 4\}$ and $\{x : x \text{ is a natural number and } 4 \leq x \leq 6\}$

Sol. $A \cap B = \{4\}$, not disjoint.

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

Sol. $A \cap B = \{e\}$, not disjoint.

(iii) $\{x : x \text{ is an even integer}\}$ and $\{x : x \text{ is an odd integer}\}$

Sol. $A \cap B = \phi$, disjoint.

9. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$,

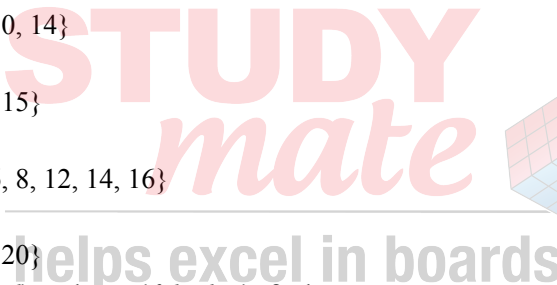
$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ and $D = \{5, 10, 15, 20\}$; find

(i) $A - B$

Sol. $\{3, 6, 9, 15, 18, 21\}$

(ii) $A - C$

Sol. $\{3, 9, 15, 18, 21\}$

(iii) $A - D$ **Sol.** $\{3, 6, 9, 12, 18, 21\}$ (iv) $B - A$ **Sol.** $\{4, 8, 16, 20\}$ (v) $C - A$ **Sol.** $\{2, 4, 8, 10, 14, 16\}$ (vi) $D - A$ **Sol.** $\{5, 10, 20\}$ (vii) $B - C$ **Sol.** $\{20\}$ (viii) $B - D$ **Sol.** $\{4, 8, 12, 16\}$ (ix) $C - B$ **Sol.** $\{2, 6, 10, 14\}$ (x) $D - B$ **Sol.** $\{5, 10, 15\}$ (xi) $C - D$ **Sol.** $\{2, 4, 6, 8, 12, 14, 16\}$ (xii) $D - C$ **Sol.** $\{5, 15, 20\}$ **10.** If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find(i) $X - Y$ (ii) $Y - X$ (iii) $X \cap Y$ **Sol.** (i) $\{a, c\}$ (ii) $\{f, g\}$ (iii) $\{b, d\}$ **11.** If \mathbf{R} is the set of real numbers and \mathbf{Q} is the set of rational numbers, then what is $\mathbf{R} - \mathbf{Q}$?**Sol.** Set of irrational numbers = \mathbf{T} **12.** State whether each of the following statement is true or false. Justify your answer.(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.(iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.(iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.**Sol.** (i) False; since $\{2, 3, 4, 5\} \cap \{3, 6\} = 3$ 

(ii) False; since $\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

(iii) True; since $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \phi$

(iv) True; since $\{2, 6, 10\} \cap \{3, 7, 11\} = \phi$

EXERCISE 1.5

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find

(i) A'

Sol. $A' = \{5, 6, 7, 8, 9\}$

(ii) B'

Sol. $B' = \{1, 3, 5, 7, 9\}$

(iii) $(A \cup C)'$

Sol. $(A \cup C)' = \{1, 2, 3, 4, 5, 6\}' = \{7, 8, 9\}$

(iv) $(A \cup B)'$

Sol. $(A \cup B)' = \{1, 2, 3, 4, 6, 8\}' = \{5, 7, 9\}$

(v) $(A')'$

Sol. $A' = \{5, 6, 7, 8, 9\}' = \{1, 2, 3, 4\} = A$

(vi) $(B - C)'$

Sol. $(B - C)' = \{2, 8\}' = \{1, 3, 4, 5, 6, 7, 9\}$

2. If $U = \{a, b, c, d, e, f, g, h\}$, find the complements of the following sets :

(i) $A = \{a, b, c\}$

Sol. $A' = \{d, e, f, g, h\}$

(ii) $B = \{d, e, f, g\}$

Sol. $B' = \{a, b, c, h\}$

(iii) $C = \{a, c, e, g\}$

Sol. $C' = \{b, d, f, h\}$

(iv) $D = \{f, g, h, a\}$

Sol. $D' = \{b, c, d, e\}$

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) $A = \{x : x \text{ is an even natural number}\}$

Sol. $A' = \{x : x \text{ is an odd natural number}\}$

(ii) $B = \{x : x \text{ is an odd natural number}\}$

Sol. $B' = \{x : x \text{ is an even natural number}\}$

(iii) $C = \{x : x \text{ is a positive multiple of 3}\}$



Sol. $C' = \{x : x \in \mathbf{N} \text{ and } x \text{ is not a multiple of } 3\}$

(iv) $D = \{x : x \text{ is a prime number}\}$

Sol. $D' = \{x : x \text{ is a positive composite number or } x = 1\}$

(v) $E = \{x : x \text{ is a natural number divisible by } 3 \text{ and } 5\}$

Sol. $E' = \{x : x \text{ is a positive integer which is not divisible by } 3 \text{ or not divisible by } 5\}$

(vi) $F = \{x : x \text{ is a perfect square}\}$

Sol. $F' = \{x : x \in \mathbf{N} \text{ and } x \text{ is not a perfect square}\}$

(vii) $G = \{x : x \text{ is a perfect cube}\}$

Sol. $G' = \{x : x \in \mathbf{N} \text{ and } x \text{ is not a perfect cube.}\}$

(viii) $H = \{x : x + 5 = 8\}$

Sol. $H' = \{x : x \in \mathbf{N} \text{ and } x \neq 3\}$

(ix) $I = \{x : 2x + 5 = 9\}$

Sol. $I' = \{x : x \in \mathbf{N} \text{ and } x \neq 2\}$

(x) $J = \{x : x \geq 7\}$

Sol. $J' = \{x : x < 7\}$

(xi) $K = \{x : x \in \mathbf{N} \text{ and } 2x + 1 > 10\}$

Sol. $K' = \{x : x \in \mathbf{N} \text{ and } x \leq 9/2\}$

4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Sol. $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$, $A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$,

(i) $LHS = (A \cup B)' = \{1, 9\}$,

$RHS = A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$

Therefore, $LHS = RHS$... (Hence Verified)

(ii) Here, $A \cap B = \{2\}$, $A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$,

$LHS = (A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$RHS = A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} =$

$\{1, 3, 4, 5, 6, 7, 8, 9\}$

Therefore, $LHS = RHS$... (Hence Verified)

5. Draw appropriate venn diagram for each of the following:

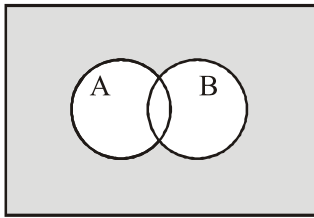
(i) $(A \cup B)'$

(ii) $A' \cap B'$

(iii) $(A \cap B)'$

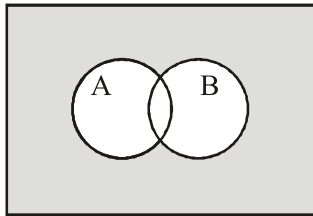
(iv) $A' \cup B'$

Sol. (i)



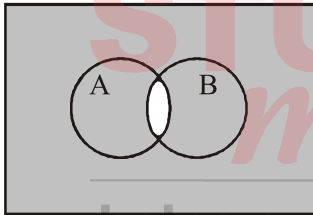
$$A \cup B$$

(ii)



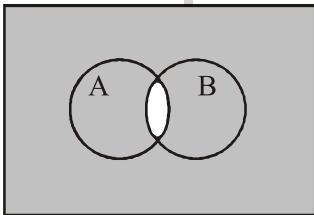
$$A' \cap B'$$

(iii)



$$(A \cap B)'$$

(iv)



$$A' \cup B'$$

6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60° . What is A' ?

Sol. A' is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement:

- | | |
|---------------------|--------------------------|
| (i) $A \cup A'$ | (ii) $\emptyset' \cap A$ |
| (iii) $A \cap A'$ | (iv) $U' \cap A$ |
| Sol. (i) U | (ii) A |
| (iii) ϕ | (iv) ϕ |

EXERCISE 1.6

1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Sol. $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

Here, $n(X) = 17$, $n(Y) = 23$, $n(X \cup Y) = 38$

$$38 = 17 + 23 - n(X \cap Y)$$

$$n(X \cap Y) = 40 - 38 = 2$$

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements, how many elements does $X \cap Y$ have?

Sol. $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

Here, $n(X) = 8$, $n(Y) = 15$, $n(X \cup Y) = 18$

$$18 = 8 + 15 - n(X \cap Y)$$

$$n(X \cap Y) = 23 - 18 = 5$$

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Sol. Let H be the set of people who speak Hindi, and E be the set of people who speak English

Here, $n(H \cup E) = 400$, $n(H) = 250$, $n(E) = 200$, $n(H \cap E) = ?$

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$

$$400 = 250 + 200 - n(H \cap E)$$

$$n(H \cap E) = 450 - 400 = 50$$

Thus, 50 people can speak both Hindi and English.

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

Sol. It is given that:

$$n(S) = 21, n(T) = 32, n(S \cap T) = 11, n(S \cup T) = ?$$

$$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$

$$n(S \cup T) = 21 + 32 - 11 = 42$$

Thus, the set $S \cup T$ has 42 elements.

5. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Sol.

It is given that:

$$n(X) = 40, n(Y) = ?, n(X \cap Y) = 10, n(X \cup Y) = 60$$

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$60 = 40 + n(Y) - 10$$

$$n(Y) = 30$$

Thus, the set Y have 30 elements.

6. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Sol .

Let C denote the set of people who like coffee, and T denotes the set of people who like tea

$$n(C) = 37, n(T) = 52, n(C \cap T) = ?, n(C \cup T) = 70$$

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$70 = 37 + 52 - n(C \cap T)$$

$$n(C \cap T) = 89 - 70 = 19$$

Thus, 19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Sol .

Let C denotes the set of people who like cricket, and T denote the set of people who like tennis.

$$n(C \cup T) = 65, n(C) = 40, n(C \cap T) = 10, n(T - C) = ?, n(T) = ?$$

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$65 = 40 + n(T) - 10$$

$$n(T) = 65 - 30 = 35$$

Thus, 35 people like tennis.

No. of people like tennis only and not cricket = $n(T - C)$

$$= n(T) - n(C \cap T) = 35 - 10 = 25$$

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Sol. Let F denote the set of people who speak French, and S denote the set of people who speak Spanish.

$$n(F) = 50, n(S) = 20, n(F \cap S) = 10, n(F \cup S) = ?$$

$$n(F \cup S) = n(F) + n(S) - n(F \cap S)$$

$$n(F \cup S) = 50 + 20 - 10$$

$$n(F \cup S) = 60$$

Thus, 60 people speak at least one of these two languages.