NCERT TEXTUAL EXERCISE (SOLVED)

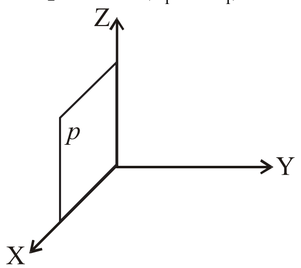
EXERCISE: 12.1

1. A point is on the x-axis, what are its y-coordinate and z-coordinate?

Sol. Any point on x-axis is $(x, 0, 0)$, *i.e.* y-coordinate and z-coordinate of this point are 0 and 0 respectively.

2. A point is in the XZ-plane what can you say about its y-coordinate ?

Sol. Any point lying on XZ-plane is $(x_1, 0, z_1)$, *i.e.* its y-coordinate is zero.



3. Name the octants in which the following points lie:

(1, 2, 3), (4, –2, 3), (4, –2, –5), (4, 2, –5), (–4, 2, –5), (–4, 2, 5), (–3, –1, 6), (2, –4, –7).

Sol. I, IV, VIII, V, VI, II, III, VIII

4. Fill in the blanks.

(i) The x-axis and y-axis taken together determine a plane known as...

(ii) The coordinates of points in the XY-plane are of the form...

(iii) coordinates planes divide the space into... octants.

Sol. (i) xy-plane (ii) $(x, y, 0)$

(iii) 8

EXERCISE: 12.2

1. Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1)

(ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4)

(iv) (2, –1, 3) and (–2, 1, 3)

Sol. The distance between the points $P(x_1, y_1\ z_1)$ and $Q(x_2, y_2\ z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between the points (2, 3, 5) and (4, 3, 1)

$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

(ii) Distance between the points (–3, 7, 2) and (2, 4, –1)

$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$

$= \sqrt{43}$

(iii) Distance between the points (–1, 3, –4) and (1, –3, 4)

$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$

$\sqrt{104} = 2\sqrt{26}$

(iv) Distance between the points (2, –1, 3) and (–2, 1, 3)

$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

2. Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Sol. Let the given points be A(–2, 3, 5), B(1, 2, 3) and C(7, 0, –1).

$AB = \sqrt{(-2-1)^2 + (3-2)^2 + (5-3)^2} = \sqrt{9+1+4} = \sqrt{14}$

$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$

$CA = \sqrt{(-2-7)^2 + (3-0)^2 + (5+1)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$

Now AB + BC = $\sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$

Thus AB + BC = AC

Hence, the points A, B and C are collinear.

3. Verify the following:

(i) The points (0, 7, –10), (1, 6, –6) and (4, 9, –6) are the vertices of an isosceles triangle.

(ii) The points (0, 7, –10), (–1, 6, 6) and (–4, 9, –6) are the vertices of a right angled triangle.

(iii) The points (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4)are the vertices of an parallelogram.

Sol. (i) Let the points (0, 7, –10), (1, 6, –6), and (4, 9, –6) be denoted by A, B, and C respectively,

Now, $AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2}$

$= \sqrt{(1)^2 + (-1)^2 + (4)^2}$

$= 3\sqrt{2}$

$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{18} = 3\sqrt{2}$

and $CA = \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2}$

$= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} = 6$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let the points (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) be denoted by A, B, and C respectively.

Now, $AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + (-4)^2}$

$= 3\sqrt{2}$

$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$

$= \sqrt{(-3)^2 + (3)^2 + (0)^2} = 3\sqrt{2}$

and $CA = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$

$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$

$= 6$

Now, $AB^2 + BC^2 = \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 = 18 + 18 = 36 = AC^2$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let the points (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) be denoted by A, B, C and D respectively.

Now, $AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$

$= \sqrt{4+16+16}$

$= 6$

$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$

$= \sqrt{9+25+9} = \sqrt{43}$

$CD = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$

$= \sqrt{4+16+16}$

$= 6$

and $DA = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$

$= \sqrt{9+25+9} = \sqrt{43}$

Here, AB = Cd = 6, BC = AD = $\sqrt{43}$

Therefore, ACD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

4. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Sol. Let P(*x, y, z*) be the point that is equidistant form points A(1, 2, 3) and B(3, 2, –1). Accordingly,

PA = PB

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9$

$= x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow \quad -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow \quad 4x - 8z = 0$

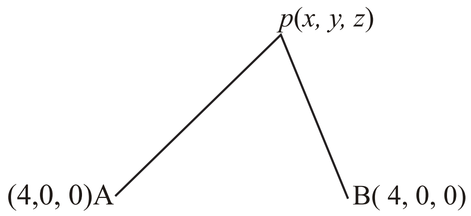
$\Rightarrow \quad x = 2z = 0$

Thus, the required equation is $x - 2z = 0$.

5. Find the equation of set of all points P, the sum of whose distance from A(4, 0, 0) and B(–4, 0, 0) is equal to 10.

Sol. The given points are A(4, 0, 0) and B (–4, 0, 0).

Let the point P be (*x, y, z*).



We have, PA + PB = 10

$\therefore \quad \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$

$$\sqrt{(x-4)^2+y^2+z^2}=10-\sqrt{(x+4)^2+y^2+z^2}$$

Squaring both sides, we get

$$(x-4)^2+y^2+z^2=100+(x+4)^2+y^2+z^2-20\sqrt{(x+4)^2+y^2+z^2}$$

$$x^2-8x+16=100+x^2+8x+16-20\sqrt{(x+4)^2+y^2+z^2}$$

$$-16x-100=-20\sqrt{(x+4)^2+y^2+z^2}$$

$$4x+25=5\sqrt{(x+4)^2+y^2+z^2}$$

Squaring again, we get

$$16x^2+200x+625=25(x^2+y^2+z^2+8x+16)$$

$$9x^2+25y^2+25z^2=625-400=225$$

i.e. $\quad 9x^2+25y^2+25z^2=225$

EXERCISE: 12.3

1. Find the coordinates of the point which divides the line segment joining the points (–2, 3, 5) and (1, –4, 6) in the ratio (i) 2 : 3 internally, (ii) 2 : 3 externally.

Sol. (i) Let P(x, y, z) be the point that divides the line segment joining the points (–2, 3, 5) and (1, –4, 6) internally in the ratio 2 : 3.

Now, $x=\dfrac{2(1)+3(-2)}{2+3}, y\dfrac{2(-4)+3(3)}{2+3}$ and $z=\dfrac{2(6)+3(5)}{2+3}$

i.e. $x=\dfrac{-4}{5}, y=\dfrac{1}{5}$ and $z=\dfrac{27}{5}$

Thus, the coordinates of the required point are $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

(ii) Let P(x, y, z) be the point that divides the line segment joining the points (–2, 3, 5) and (1, –4, 6) externally in the ratio 2 : 3.

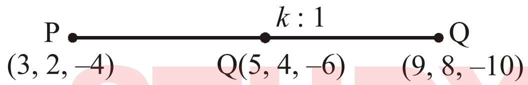
Now, $x = \frac{2(1)-3(-2)}{2-3}, y = \frac{2(-4)-3(3)}{2-3}$ and $z = \frac{2(6)-3(5)}{2-3}$

i.e., $x = -8$, $y = 17$ and $z = 3$

Thus, the coordinates of the required point are (–8, 17, 3).

2. Given that P(3, 2, –4), Q(5, 4, –6) and R(9, 8, –10) are collinear. Find the ratio in which Q divides PR.

Sol. Let Q divides PR in the ratio k : 1



Now, x-coordinate of Q $= \frac{9 \times k + 3}{k+1} = \frac{9k+3}{k+1} = 5$

or $9k + 3 = 5(k+1) = 5k + 5$

or $4k = 2 \Rightarrow k = \frac{1}{2}$

y-coordinate of Q $= \frac{8k+2}{k+1} = 4$

or $8k + 2 = 4k + 4$

or $4k = 2 \Rightarrow k = \frac{1}{2}$

and z-coordinate of $Q = \frac{-10 \times k - 1 \times 4}{k+1} = \frac{-10k-4}{k+1} = -6$

or $-10k - 4 = -6k - 6$

or $4k = 2,$

$\Rightarrow k = \frac{1}{2}$

This shows that the point Q lies on it and it divides PR in the ratio 1 : 2.

3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

Sol. Let the YZ-plane divides the line segment joining the points (–2, 4, 7) and (3, –5, 8) in the ratio k : 1.

Hence, by section formula, the coordinates of point of intersection are given by $\left(\dfrac{k(3)-2}{k+1}, \dfrac{k(-5)+4}{k+1}, \dfrac{k(8)+7}{k+1}\right)$.

On the YZ-plane, the x-coordinates of any point is zero.

$$\frac{3k-2}{k+1} = 0$$

$$\Rightarrow \qquad k = \frac{2}{3}$$

Thus, the YZ-plane divides the line segment formed by joining the given points in the ratio 2 : 3.

4. Using section formula, show that the points A(2, –3, 4), B(–1, 2, 1) and $C\left(0, \dfrac{1}{3}, 2\right)$ are collinear.

Sol. The given points are A(2, –3, 4), B(–1, 2, 1), and $C\left(0, \dfrac{1}{3}, 2\right)$.

Let P be a point that divides AB in the ratio k : 1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\dfrac{-k+2}{k+1} = 0$, we obtain $k = 2$.

For $k = 2$, the coordinates of point P are $\left(0, \dfrac{1}{3}, 2\right)$.

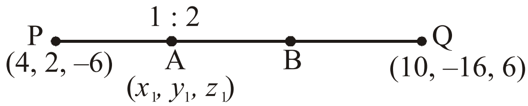
i.e. $\left(0, \dfrac{1}{3}, 2\right)$ is a point same as point P.

Hence, the points A, B and C are collinar.

5. Find the coordinates of the points which trisect the line segment PQ formed by joining the points P(4, 2, –6) and Q(10, –16, 6).

Sol. Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ trisect the line segment PQ.

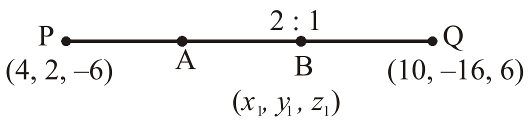
$\Rightarrow$ The point A divides PQ in the ratio 1 : 2.



$\therefore$ The coordinates of $A(x_1, y_1, z_1)$ are

$$\left(\frac{10+8}{1+2}, \frac{-16+4}{1+2}, \frac{6-12}{1+2}\right)$$

or (6, –4, –2)



Further B divides PQ in the ratio 2 : 1

∴ The coordinates of $B(x_2, y_2, z_2)$ are

$$\left(\frac{2\times10+4}{2+1}, \frac{-32+2}{2+1}, \frac{12-6}{2+1}\right)$$

i.e. (8, –10, 2)

Thus the points A(6, –4, –2) and B(8, –10, 2) trisect the line segment PQ.