EXERCISE 7.1

1. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that

(i) repetition of the digits is allowed?

(ii) repetition of the digits is not allowed?

Sol. (i) There are as many ways as there are ways of filling 3 vacant places ___ ___ ___ in succession by the given five digits. Therefore, the unit's place can be filled in by any of the given five digits.

Similarly, ten's and hundred's digits can be filled in by any of the given five digits.

Thus, by the Fundamental Principle of Counting (FPC), the number of ways in which three-digit numbers can be formed from the given digits is $5 \times 5 \times 5 = 125$

(ii) In this case, repetition of digits is not allowed. Here, if unit's place is filled in first, then it can be filled by any of the given five digits. Therefore, the number of ways of filling the unit's place of the three-digit number is 5.

Then, the ten's place can be filled with any of the remaining four digits and the hundred's place can be filled with any of the remaining three digits.

Thus, by the FPC, the number of ways in which three-digit numbers can be formed without repeating the given digits is $5 \times 4 \times 3 = 60$

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2. How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Sol. There are as many ways as there are ways of filling 3 vacant places ___ ___ ___ in succession by the given six digits. In this case, the unit's place can be filled by 2 or 4 or 6 only. The unit's place can be filled in 3 ways. The ten's place can be filled by any of the 6 digits in 6 different ways and also the hundred's place can be filled by any of the 6 digits in 6 different ways, as the digits can be repeated.

Therefore, by FPC, the required number of three digit even numbers is $3 \times 6 \times 6 = 108$

3. How many 4-letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

Sol. There are as many codes as there are ways of filling 4 vacant places ___ ___ ___ ___ in succession by the first 10 letters of the English alphabet.

The first place can be filled in 10 different ways by any of the first 10 letters of the English alphabet following which, the second place can be filled in by any of the remaining letters in 9 different ways. The third place can be filled in 8 different ways and the fourth place can be filled in 7 different ways.

Therefore, by FPC, the required numbers of ways in which 4 vacant places can be filled is $10 \times 9 \times 8 \times 7 = 5040$

4. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

Sol. It is given that the 5-digit telephone numbers always start with 67.

Therefore, there are as many phone numbers as there are ways of filling 3 vacant places $\underline{6}\ \underline{7}\ \_\_\ \_\_\ \_\_$ by the digits 0 – 9.

The unit's place can be filled by any of the digits from 0 – 9, except digits 6 and 7.

Therefore, the unit's place can be filled in 8 different ways following which, the ten's place can be filled in 7 different ways, and the hundred's place can be filled in by any of the remaining in 6 different ways.

Therefore, by FPC, the required number of ways in which 5-digit telephone numbers can be constructed is $8 \times 7 \times 6 = 336$.

5. A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Sol. When a coin is tossed once, the number of outcomes is 2 (head and tail).

Thus, by FPC, the required number of possible outcomes is $2 \times 2 \times 2 = 8$.

6. Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Sol. Each signal requires the use of 2 flags.

There will be as many flags as there are ways of filling in 2 vacant places ⊟ in succession by the given 5 flags of different colours.

The upper vacant place can be filled in 5 different ways by any one of the 5 flags following which, the lower vacant place can be filled in 4 different ways.

Thus, by FPC, the number of different signals that can be generated is $5 \times 4 = 20$.

EXERCISE 7.2

1. Evaluate:

(i) 8! (ii) 4! – 3!

Sol. (i) $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

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(ii) $4! = 4 \times 3 \times 2 \times 1 = 24$

$3! = 3 \times 2 \times 1 = 6$

$\therefore \quad 4! - 3! = 24 - 6 = 18$

2. Is $3! + 4! = 7!$?

Sol. $3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$\therefore \quad 3! + 4! = 6 + 24 = 30$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$\therefore \quad 3! + 4! \neq 7!$

3. Compute $\frac{8!}{6! \times 2!}$

Sol. $\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = \frac{8 \times 7}{2} = 28.$

4. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x.

Sol. $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$

$\Rightarrow \quad \frac{1}{6!} + \frac{1}{7 \times 6!} = \frac{x}{8 \times 7 \times 6!}$

$\Rightarrow \quad \frac{1}{6!}\left(1 + \frac{1}{7}\right) = \frac{x}{8 \times 7 \times 6!}$

$\Rightarrow \quad 1 + \frac{1}{7} = \frac{x}{8 \times 7}$

$\therefore \quad x = 64.$

5. Evaluate $\frac{n!}{(n-r)!}$, when

(i) $n = 6, r = 2$ (ii) $n = 9, r = 5$

Sol. (i) When $n = 6, r = 2$, $\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 30$

(ii) When $n = 9, r = 5$, $\frac{n!}{(n-r)!} = \frac{9!}{(9-5)!} = \frac{9!}{4!}$

$= 9 \times 8 \times 7 \times 6 \times 5 = 15120.$

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EXERCISE 7.3

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Sol. There will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, required number of 3-digit numbers $= {}^9P_3 = \dfrac{9!}{(9-3)!} = \dfrac{9!}{6!}$

$= 9 \times 8 \times 7 = 504$

2. How many 4-digit numbers are there with no digit repeated?

Sol. The thousand's place of the 4-digit number is to be filled with any of the digits from 1 to 9 as the digit 0 cannot be included. Therefore, the number of ways in which thousands place can be filled is 9.

The hundred's, ten's, and unit's place can be filled by any of the digits from 0 to 9.

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However, the digits cannot be repeated in the 4-digit numbers and thousand's place is already occupied with a digit. The hundred's, ten's, and unit's place is to be filled by the remaining 9 digits.

Hence, by FPC, total number of numbers $= 9 \times 9 \times 8 \times 7 = 4536$.

3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

Sol. The unit's digit can be filled in 3 ways by any of the digits, 2, 4, or 6.

Since the digits cannot be repeated in the 3-digit numbers and unit's place is already occupied with a digit, the hundred's and ten's place is to be filled by the remaining 5 digits.

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The required number of 3-digit numbers is

$3 \times 5 \times 4 = 60$

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

Sol. 4-digit numbers are to be formed using the digits, 1, 2, 3, 4, and 5.

There will be as many 4-digit numbers as there are permutations of 5 different digits taken 4 at a time.

Therefore, required number of 4 digit numbers $= {}^5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!}$

$= 5 \times 4 \times 3 \times 1 \times 1 = 120$

Among the 4-digit numbers formed by using the digits, 1, 2, 3, 4, 5, even numbers end with either 2 or 4.

The number of ways in which unit's place is filled with digits is 2.

Since the digits are not repeated and the unit's place is already occupied with a digit, the remaining places are to be filled by the remaining 4 digits.

Therefore, the number of ways in which the remaining places can be filled is the permutation of 4 different digits taken 3 at a time.

Number of ways of filling the remaining places $= {}^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!}$

$= 4 \times 3 \times 2 \times 1 = 24$

Thus, by FPC, the required number of even numbers is $24 \times 2 = 48$

5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

Sol. From a committee of 8 persons, a chairman and a vice chairman are to be chosen in such a way that one person cannot hold more than one position.

Hence, by FPC, total numbers of ways $= 8 \times 7 = 56$.

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6. Find n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$.

Sol. ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$

$$\Rightarrow \frac{{}^{n-1}P_3}{{}^nP_4} = \frac{1}{9}$$

$$\Rightarrow \frac{\left[\frac{(n-1)!}{(n-1-3)!}\right]}{\left[\frac{n!}{(n-4)!}\right]} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$$

$\Rightarrow \quad \frac{1}{n} = \frac{1}{9}$

$\Rightarrow \quad n = 9.$

7. Find r if (i) ${}^5P_r = 2\,{}^6P_{r-1}$ (ii) ${}^5P_r = {}^6P_{r-1}$.

Sol. (i) ${}^5P_r = 2\,{}^6P_{r-1}$

$\Rightarrow \quad \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(6-r+1)!}$

$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{2 \times 6!}{(7-r)!}$

$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{2 \times 6 \times 5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow \quad (7-r)(6-r) = 12$

$\Rightarrow \quad 42 - 6r - 7r + r^2 = 12$

$\Rightarrow \quad r^2 - 13r + 30 = 0$

$\Rightarrow \quad (r-3)(r-10) = 0$

$\Rightarrow \quad (r-3) = 0$ or $(r-10) = 0$

$\Rightarrow \quad r = 3$ or $r = 10$

It is known that, ${}^nP_r = \frac{n!}{(n-r)!}$, where $0 < r \le n$

$\therefore \quad 0 < r \le 5$

Hence, $r \ne 10$

$\therefore \quad r = 3$

(ii) ${}^5P_r = {}^6P_{r-1}$

$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$

$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{6 \times 5!}{(7-r)!}$

$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{(7-r)(6-r)(5-r)!}$

$\Rightarrow \quad 42 - 7r - 6r + r^2 - 6 = 0$

$\Rightarrow \quad r^2 - 4r - 9r + 36 = 0$

$\Rightarrow \quad (r-4) = 0$ or $(r-9) = 0$

$\Rightarrow \quad r = 4$ or $r = 9$

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It is known that,

$\therefore$ $0 < r \leq 5$

Hence, $r \neq 9$

$\therefore$ $r = 4$.

8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Sol. The number of words that can be formed using all the letters of the word EQUATION, using each letter exactly once, is the number of permutations of 8 different objects taken 8 at a time, which is ${}^8P_8 = 8! = 40320$.

9. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

(i) 4 letters are used at a time, (ii) all letters are used at a time,

(iii) all letters are used but first letter is a vowel?

Sol. There are 6 different letters in the word MONDAY.

(i) Number of 4-letter words that can be formed from the letters of the word MONDAY, without repetition of letters, is the number of permutations of 6 different objects taken 4 at a time, which is ${}^6P_4$.

Thus, required number of words that can be formed using 4 letters at a time is $^6P_4 = \dfrac{6!}{(6-4)!} = \dfrac{6!}{2!} = 360$

(ii) Number of words that can be formed by using all the letters of the word MONDAY at a time is the number of permutations of 6 different objects taken 6 at a time, which is ${}^6P_6 = 6! = 720$.

(iii) First letter can be choosen in 2 ways, *i.e.* O and A.

Thus, in this case, required number of words that can be formed is $5! \times 2 = 120 \times 2 = 240$

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Sol. In the given word MISSISSIPPI, I appears 4 times, S appears 4 times, P appears 2 times, and M appears 1 time.

Therefore, number of distinct permutations of the letters in the given word

$$= \frac{11!}{4!4!2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1 \times 2 \times 1} = 34650$$

There are 4 I's in the given word. When they occur together, they are treated as a single object $\boxed{\text{IIII}}$ for the time being. This single object together with the remaining 7 objects will account for 8 objects.

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Number of arrangements where all I's occur together = $\frac{8!}{4!2!} = 840$

Thus, number of distinct permutations of the letters in MISSISSIPPI in which four I's do not come together = 34650 – 840 = 33810

11. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) words start with P and end with S,

(ii) vowels are all together,

(ii) there are always 4 letters between P and S?

Sol. In the word PERMUTATIONS, there are 2 Ts and all the other letters appear only once.

(i) If P and S are fixed at the extreme ends, then 10 letters are left

P											S

Hence, in this case, required number of arrangements = $\frac{10!}{2!} = 1814400$

(ii) There are 5 vowels in the given word, each appearing only once.

Since they have to always occur together, they are treated as a single object for the time being. This single object together with the remaining 7 objects will account for 8 objects. These 8 objects in which they are arranged in $\frac{8!}{2!}$ ways. .

Corresponding to each of these arrangements, the 5 different vowels can be arranged in 5! ways.

Therefore, by FPC, required number of arrangements in this case $\frac{8!}{2!} \times 5! = 2419200$

(iii) The letters have to be arranged in such a way that there are always 4 letters between P and S.

Total number of letters to be arranged = 12

There are always 4 letters between P and S, *i.e.*

	1	2	3	4	5	6	7	8	9	10	11	12
I^{st} way	P					S						
II^{nd} way	___	P	___	___	___	___	S	___	___	___	___	___

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III[rd] way ___ ___ [P] ___ ___ ___ ___ [S] ___ ___ ___ ___

IV[th] way ___ ___ ___ [P] ___ ___ ___ ___ [S] ___ ___ ___

V[th] way ___ ___ ___ ___ [P] ___ ___ ___ ___ [S] ___ ___

VI[th] way ___ ___ ___ ___ ___ [P] ___ ___ ___ ___ [S] ___

VII[th] way ___ ___ ___ ___ ___ ___ [P] ___ ___ ___ ___ [S]

$\therefore$ Hence, P and S having 4 places between them can be filled in 7 ways.
Hence, P and S or S and P can be filled in 7 + 7 = 14 ways.
Remaining 10 letters (in which T repeated twice) can be filled in

$$=\frac{10!}{2!} \text{ ways}$$

Hence, by FPC total number of ways

$$=14\times\frac{10!}{2!}=\frac{14\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2}$$

$$=\frac{14\times3628800}{2}=25401600$$

Therefore, by FPC, required number of arrangements in this case $\frac{10!}{2!}\times14=25401600.$

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EXERCISE 7.4

1. If ${}^nC_8={}^nC_4$, find ${}^nC_2$.

Sol. It is known that, ${}^nC_x={}^nC_y \Rightarrow x=y$ or $n=x+y$

Therefore, ${}^nC_8={}^nC_2, \Rightarrow n=8+2=10$

$$\therefore \quad {}^nC_2={}^{10}C_2 \Rightarrow \frac{10!}{2!(10-2)!}=\frac{10!}{2!8!}=45$$

2. Determine n if

(i) ${}^{2n}C_3={}^nC_3=12:1$ (ii) ${}^{2n}C_3:{}^nC_3=11:1$

Sol. (i) $\frac{{}^{2n}C_3}{{}^nC_3}=\frac{12}{1}$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!}\times\frac{3!(n-3)!}{n!}=\frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 12$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)!} = 12$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{(2n-1)}{(n-2)} = 3$$

$\Rightarrow$ $2n - 1 = 3n - 6$

$\Rightarrow$ $n = 5$

(ii) $\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{11}{1}$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 11$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 11$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)}{n-2} = 11$$

$\Rightarrow$ $8n - 4 = 11n - 22$

$\Rightarrow$ $n = 6$.

3. How many chords can be drawn through 21 points on a circle?

Sol. For drawing one chord on a circle, only 2 points are required.

Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.

Thus, required number of chords = ${}^{21}C_2 = \frac{21!}{2!(21-2)} = \frac{21!}{2!19!} = \frac{21 \times 20}{2} = 210$

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

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Sol. A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.

3 boys can be selected from 5 boys in ${}^5C_3$ ways.

3 girls can be selected from 4 girls in ${}^4C_3$ ways.

Therefore, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!} = 40.$

5. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Sol. There are a total of 6 red balls, 5 white balls, and 5 blue balls.

9 balls have to be selected in such a way that each selection consists of 3 balls of each colour.

Thus, required number of ways of selecting 9 balls

$${}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!2!}$$

$$= \frac{6\times5\times4\times3!}{3!\times3\times2} \times \frac{5\times4\times3!}{3!\times3\times2} \times \frac{5\times4\times3!}{3!\times2\times1} = 20\times10\times10 = 2000$$

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Sol. In a deck of 52 cards, there are 4 aces. A combination of 5 cards has to be made in which there is exactly one ace.

Then, one ace can be selected in ${}^4C_1$ ways and the remaining 4 cards can be selected out of the 48 cards in ${}^{48}C_4$ ways.

Thus required number of 5 card combinations

$$= {}^{48}C_4 \times {}^4C_1 = \frac{48!}{4!44!} \times \frac{4!}{1!3!}$$

$$= \frac{48\times47\times46\times45}{4\times3\times2\times1} \times 4 = 778320.$$

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Sol. A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.

4 bowlers can be selected in ${}^5C_4$ ways and the remaining 7 players can be selected out of the 12 players in ${}^{12}C_7$ ways.

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Thus, required number of ways of selecting cricket team

$$= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!1!} \times \frac{12!}{7!5!} = 3960.$$

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Sol. 2 black balls can be selected out of 5 black balls in ${}^5C_2$ ways and 3 red balls can be selected out of 6 red balls in ${}^6C_3$ ways.

Thus, required number of ways of selecting 2 black and 3 red balls

$$= {}^5C_2 \times {}^6C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!} = \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 10 \times 20 = 200.$$

9. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Sol. There are 9 courses available out of which, 2 specific courses are compulsory for every student.

Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^7C_3$ ways.

Thus, required number of ways of choosing the programme =

$${}^7C_3 = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35.$$

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