NCERT TEXTUAL EXERCISE (SOLVED)

EXERCISE 2.1

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

Sol. $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

$\frac{x}{3}+1=\frac{5}{3}, \quad y-\frac{2}{3}=\frac{1}{3}$

$x = 2, y = 1$

2. If the set A has 3 elements and the set $B = \{3, 4, 5\}$, then find the number of elements in $(A \times B)$?

Sol. $n(A) = 3$ $\qquad n(B) = 3$

$\therefore n(A \times B)=n(A) \times n(B)$

$\therefore n(A \times B)=3 \times 3=9$

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G

Sol. $G \times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

$H \times G=\{(5,7),(4,7),(2,7),(5,8),(4,8),(2,8)\}$

4. State whether each of the following statements are true or false. If the statement is false, rewrite the statement correctly.

(i) If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q = \{(m, n), (n, m)\}$

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \in$ A and $y \in$ B.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ ϕ) = ϕ.

Sol. (i) False

Correct statement

If $P=\{m,n\}$ and $Q=\{n,m\}$, then $P \times Q=\{(m,n),(n,m),(m,m),(n,n)\}$

(ii) True.

(iii) True.

5. If A = {–1, 1}, find A × A × A.

Sol. A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1), (1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

6. If A × B = {(a, x), (a, y) (b, x) (b, y)}. Find A and B.

Sol. A = {a, b} and B = {x, y}

7. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) $A \times C$ is a subset of $B \times D$

Sol. (i) $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \phi$

LHS = $A \times (B \cap C) = A \times \phi = \phi$

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

RHS = $(A \times B) \cap (A \times C) = \phi$

LHS = RHS

(ii) $A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

All the elements of set $A \times C$ are the elements of set $B \times D$.

Therefore, $A \times C$ is a subset of $B \times D$.

8. Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have?

Sol. $A = \{1, 2\}$ and $B = \{3, 4\}$

Therefore, $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\} \Rightarrow n(A \times B) = 4$

No. of subsets of $A \times B = 2^{n(A \times B)} = 2^4 = 16$

9. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$.
If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B, where x, y and z are distinct elements.

Sol. A = Set of first elements of the ordered pair elements of $A \times B$

B = Set of second elements of the ordered pair elements of $A \times B$

$A = \{x, y, z\}, B = \{1, 2\}$

10. The cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

Sol. $n(A \times A) = 9$

$n(A) \times n(A) = 9 \Rightarrow n(A) = 3$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the 9 elements of $A \times A$.

Therefore, $-1, 0, 1$ are elements of A.

The remaining elements of set $A \times A$ are

$(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$

EXERCISE 2.2

1. Let $A = \{1, 2, 3, \ldots, 14\}$. Define a relation R from A to A by $R = \{(x, y): 3x - y = 0$, where $x, y \in A\}$. Write down its domain, codomain and range.

Sol. The relation R from A to A is given as

R = {(x, y) : $3x - y = 0$, wehre $x, y \in$ A}

R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Domain of R = {1, 2, 3, 4}

Range of R = {3, 6, 9, 12}

Co-domain of R = A = {1, 2, 3, .., 14}

2. Define a relation R on the set N of natural numbers by

R = {(x, y) : $y = x + 5$, x is a natural number less than 4, $x, y \in$ N}.

Depict this relationship using roster form. Write down domain and the range.

Sol. The relation R from N to N is given as

R = {(x, y) : $y = x + 5$, $x < 4$, $x, y \in$ N}

R = {(1, 6), (2, 7), (3, 8)}

Domain of R = {1, 2, 3}

Range of R = {6, 7, 8}

3. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by

R = {(x, y) : the difference between x and y is odd, $x \in$ A, $y \in$ B}.

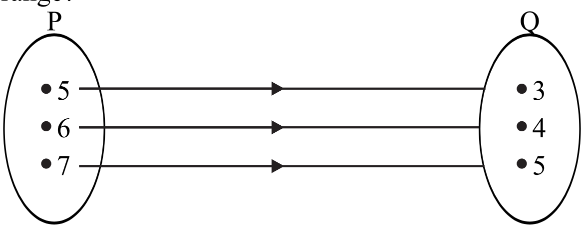
Write R in roster form.

Sol. A = {1, 2, 3, 5} and B = {4, 6, 9}

Difference of two numbers is odd if one is even and other is odd.

So, R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

4. The given figure shows a relationship between the sets P and Q. Write this relation (i) in set-builder form (ii) in roster form. What is its domain and range?



Sol. According to given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) R = {(x, y) : $y = x - 2$; $x \in$ P, $y \in$ Q}

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

{(a, b): $a, b \in$ A, b is exactly divisible by a}

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Sol. A = {1, 2, 3, 4, 6}

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

6. Determine the domain and range of the relation R defined by
R = $\{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

Sol. R = $\{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$
R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
Domain of R = {0, 1, 2, 3, 4, 5}
Range of R = {5, 6, 7, 8, 9, 10}

7. Write the relation R = $\{(x, x^3)$: x is a prime number less than 10}

Sol. R = {(2, 8), (3, 27), (5, 125), (7, 343)}

8. Let A = $\{x, y, z\}$ and B = {1, 2}. Find the number of relations from A to B.

Sol. A = $\{x, y, z\}$, B = {1, 2}
Therefore, $n(A) = 3$, $n(B) = 2$, $n(A \times B) = 6$
Number of relation of A × B = $2^{n(A \times B)} = 2^6 = 64$

9. Let R be the relation on Z defined by R = $\{(a, b) : a, b \in Z, a - b$ is an integer}. Find the domain and range of R.

Sol. Difference between any two integers is always an integer.
Therefore Domain of R = Z and Range of R = Z

EXERCISE 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2,1), (5,1), (8,1), (11,1), (14, 1), (17, 1)}

(ii) {(2,1), (4,2), (6,3), (8,4), (10, 5), (12, 6), (14, 7)}

(iii) {(1,3), (1,5), (2,5)}

Sol. (i) Every element has one and only one image, this relation is a function.
Domain: {2, 5, 8, 11, 14, 17} : Range : {1}

(ii) Every element has one and only one image, this relation is a function.

Domain: {2, 4, 6, 8, 10, 12, 14} : Range : {1, 2, 3, 4, 5, 6, 7}

(iii) Since 1 corresponds to two different images 3 and 5, this relation is not a function.

2. Find the domain and range of the following real functions:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9-x^2}$

Sol. (i) $f(x) = -|x|$

Since $f(x)$ is defined for $x \in$ R, the domain of $f(x)$ is **R**.

Range of f is $(-\infty, 0)$

(ii) $f(x) = \sqrt{9-x^2}$

Domain:

$f(x)$ is defined and real if

$9 - x^2 \geq 0$

$3^2 - x^2 \geq 0$

$(3 - x)(3 + x) \geq 0$

Case 1 : Both Positive

$(3-x) \geq 0$ and $(3+x) \geq 0$

$x \leq 3$ and $x \geq -3$

$\Rightarrow -3 \leq x \leq 3$

Case 2 : Both negative

$(3 - x) \leq 0$ and $(3 + x) \leq 0$

$x \leq -3$ and $x \geq 3$

Not possible.

Domain of f is $[-3, 3]$.

Range:

Let $f(x) = y$

$y = \sqrt{9-x^2}$

We know that square root is non-negative.

$\therefore y \geq 0$ ...(i)

Again, $9 - x^2 \leq 9$

$\Rightarrow \sqrt{9-x^2} \leq 3$

$\Rightarrow y \leq 3$ ...(ii)

$\therefore$ from (1) & (2)

$O \leq y \leq 3$

Range of f is $[0, 3]$

3. A function f is defined by $f(x) = 2x - 5$. Write down the value of

(i) $f(0)$ (ii) $f(7)$

(iii) $f(-3)$

Sol. (i) $f(0) = 2(0) - 5 = -5$

(ii) $f(7) = 2(7) - 5 = 9$

(iii) $f(-3) = 2(-3) - 5 = -11$

4. The function t which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$

Find

(i) $t(0)$ (ii) $t(28)$

(iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$.

Sol. (i) $t(0) = \frac{9(0)}{5} + 32 = 32$

(ii) $t(28) = \frac{9(28)}{5} + 32 = \frac{252+160}{5} = \frac{412}{5}$

(iii) $t(-10) = \frac{9(-10)}{5} + 32 = -18 + 32 = 14$

(iv) $t(C) = 212 \Rightarrow \frac{9C}{5} + 32 = 212 \Rightarrow \frac{9C}{5} = 180 \Rightarrow C = 100$

5. Find the range of each of the following functions.

(i) $f(x) = 2 - 3x$, $x \in R$, $x > 0$

(ii) $f(x) = x^2 + 2$, x is a real number.

(iii) $f(x) = x$, x is a real number.

Sol. (i) $f(x) = 2 - 3x$

$x > 0 \Rightarrow 3x > 0 \Rightarrow -3x < 0 \Rightarrow 2 - 3x < 2 \Rightarrow f(x) < 2$

Range of f is $(-\infty, 2)$.

(ii) $f(x) = x^2 + 2, \Rightarrow x^2 + 2 \geq 2 \Rightarrow f(x) \geq 2$

Range of f is $[2, \infty)$.

(iii) $f(x) = x$

$x \in R \Rightarrow f(x) \in R$

Range of f is $(-\infty, \infty)$ or **R**.